Analyzing `hasDuplicate`

```
public boolean hasDuplicate1(int[] array) {
   for (int i=0; i < array.length; i++) { // 1
      for (int j=0; j < array.length; j++) { // 1
         if (i != j && array[i] == array[j]) { // 1
            return true;                  // 1
         }
      }
   }
   return false;                          // 1
}
```

n, n $n \cdot n + 1$

$(2N + 1)(N + 1)$

```
public boolean hasDuplicate2(int[] array) {
   for (int i=0; i < array.length - 1; i++) { // 1
      if (array[i] == array[i+1]) {         // 1
         return true;                      // 1
      }
   }
   return false;                           // 1
}
```

$2N + 1$

We'd like to be able to compare two functions. Intuitively, we want an operation like "$\leq$" (e.g. $4 \leq 5$), but for functions.

If we have f and $4f$, we should consider them the same:

$f \leq g$ when...

$n \geq n_0 \rightarrow f(n) \leq c\,g(n)$

Asymptotics

We'd like to be able to compare two functions. Intuitively, we want an operation like "$\leq$" (e.g. $4 \leq 5$), but for functions.

If we have f and $4f$, we should consider them the same:

$f \leq g$ when...

$f \leq cg$ where c is a constant and $c \neq 0$.

We also care about **all values of the function** that are **big enough**:

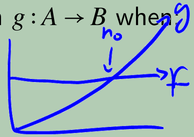
$f \leq g$ when...

For all n "large enough", $f(n) \leq cg(n)$, where $c \neq 0$
For some $n_0 \geq 0$, for all $n \geq n_0$, $f(n) \leq cg(n)$, where $c \neq 0$
For some $c \neq 0$, for some $n_0 \geq 0$, for all $n \geq n_0$, $f(n) \leq cg(n)$

Definition (Big-Oh)

We say a function $f : A \to B$ **is dominated by** a function $g : A \to B$ when:

$$\exists (c, n_0 > 0).\ \forall (n \geq n_0).\ f(n) \leq cg(n)$$



Formally, we write this as $f \in \mathcal{O}(g)$.

Big-Oh Proofs

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Formally, we write this as $f \in \mathcal{O}(g)$.

We want to prove $4 + 3n \in \mathcal{O}(n)$. That is, we want to prove:

$$\exists(c, n_0 > 0).\ \forall(n \geq n_0).\ 4 + 3n \leq cn$$

Proof Strategy

SW $3n \leq 4 + 3n \leq cn$

$\leq 4 + 3n \leq 4n + 3n \leq cn$

$7n \leq cn$

Big-Oh Proofs 2

Definition (Big-Oh)

We say a function $f : A \to B$ **is dominated by** a function $g : A \to B$ when:

$$\exists(c, n_0 > 0).\ \forall(n \geq n_0).\ f(n) \leq cg(n)$$

Formally, we write this as $f \in \mathcal{O}(g)$.

We want to prove $4 + 3n + 4n^2 \in \mathcal{O}(n^3)$.

WTS: $4 + 3n + 4n^2 \leq cn^3$

$4 + 3n + 4n^2 \leq 4n^3 + 3n^3 + 4n^3 \leq cn^3$

$n_0 = 1$