CSE 332

JULY 19TH – SORTING 2

ASSORTED MINUTIAE

- Exams graded

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  - Should have in hand

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- **New exercise out tonight**

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  - Regrading also available after class Friday

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  - Collection of Comparable data

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  - Sorting v. Maintaining sortedness

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    - The most recent sort will always be the primary

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    - Bogo sort is not a comparison sort
    - Comparison sorts are $\Omega(n \log n)$, they cannot do better than this

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      - When you have your lowest candidate, do not replace with an element that ties.
    - In place? Can be, but can also create a separate collection (if we only want the top 5, for example)

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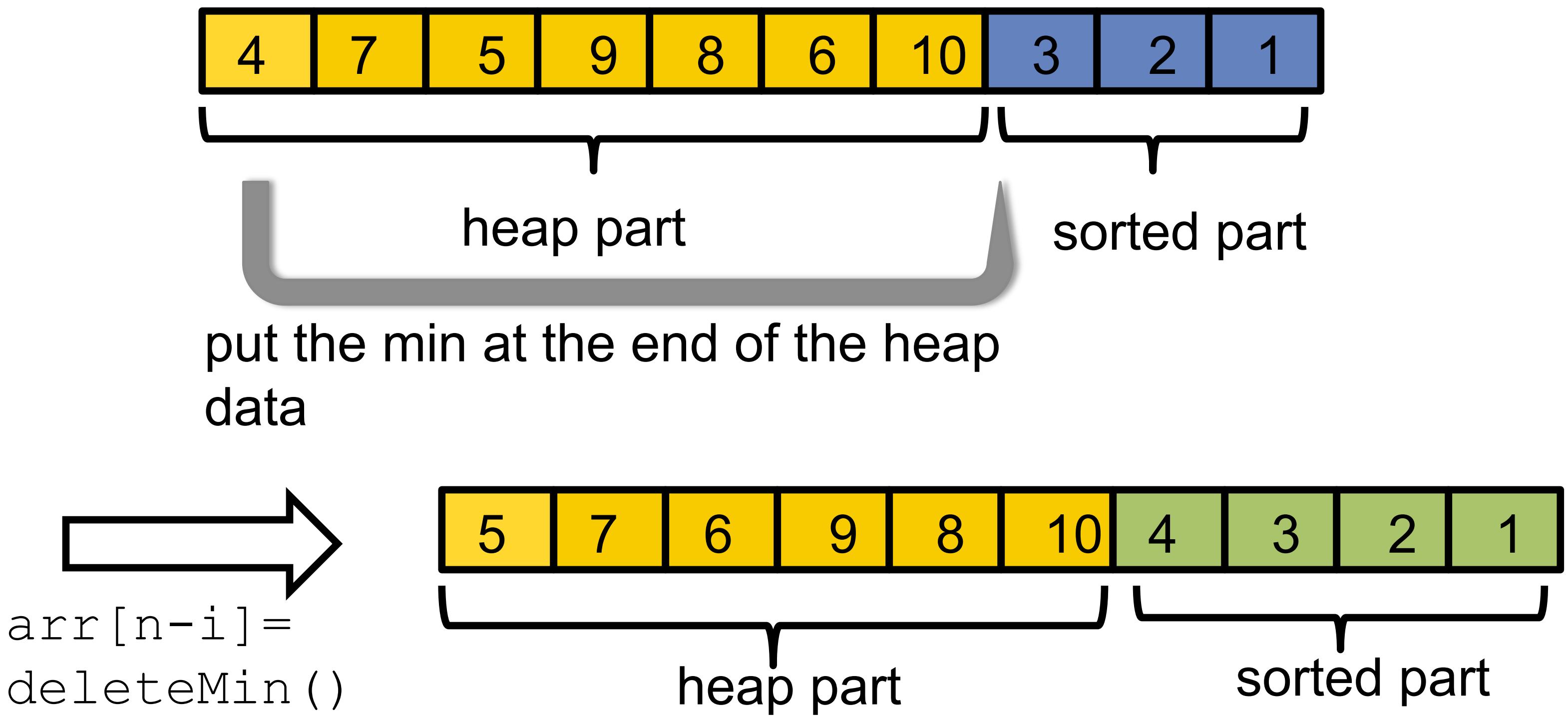
    - In-place? Can be easily. Since not interruptable, having a duplicate array is only necessary if you don't want the original array to be mutated

IN-PLACE HEAP SORT

- Treat the initial array as a heap (via **`buildHeap`**)
- When you delete the **`i`**th element, put it at **`arr[n-i]`**
  - That array location isn't needed for the heap anymore!



DIVIDE AND CONQUER

Divide-and-conquer is a useful technique for solving many kinds of problems (not just sorting). It consists of the following steps:

1. Divide your work up into smaller pieces (recursively)
2. Conquer the individual pieces (as base cases)
3. Combine the results together (recursively)

```
algorithm(input) {
    if (small enough) {
        CONQUER, solve, and return input
    } else {
        DIVIDE input into multiple pieces
        RECURSE on each piece
        COMBINE and return results
    }
}
```

DIVIDE-AND-CONQUER SORTING

Two great sorting methods are fundamentally divide-and-conquer

Mergesort:

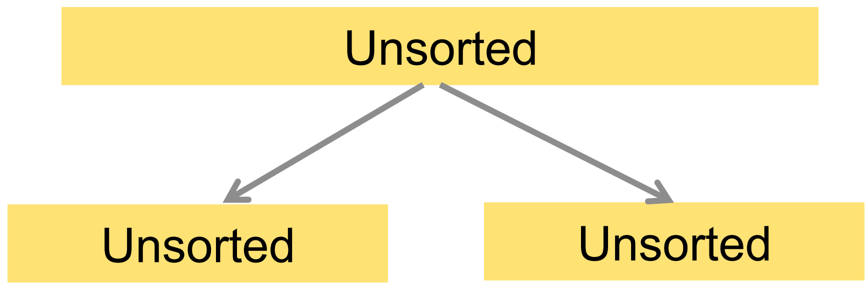
Sort the left half of the elements (recursively)
Sort the right half of the elements (recursively)
Merge the two sorted halves into a sorted whole

Quicksort:

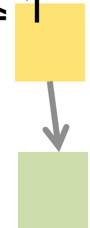
Pick a "pivot" element
Divide elements into less-than pivot and greater-than pivot
Sort the two divisions (recursively on each)
Answer is: sorted-less-than....pivot....sorted-greater-than

MERGE SORT

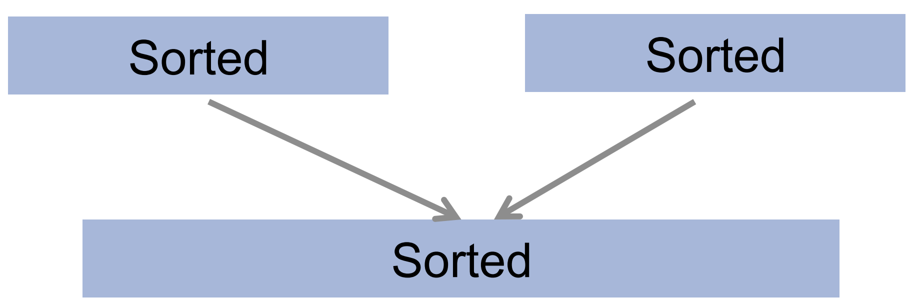
Divide: Split array roughly into half



Conquer: Return array when length ≤ 1



Combine: Combine two sorted arrays using merge

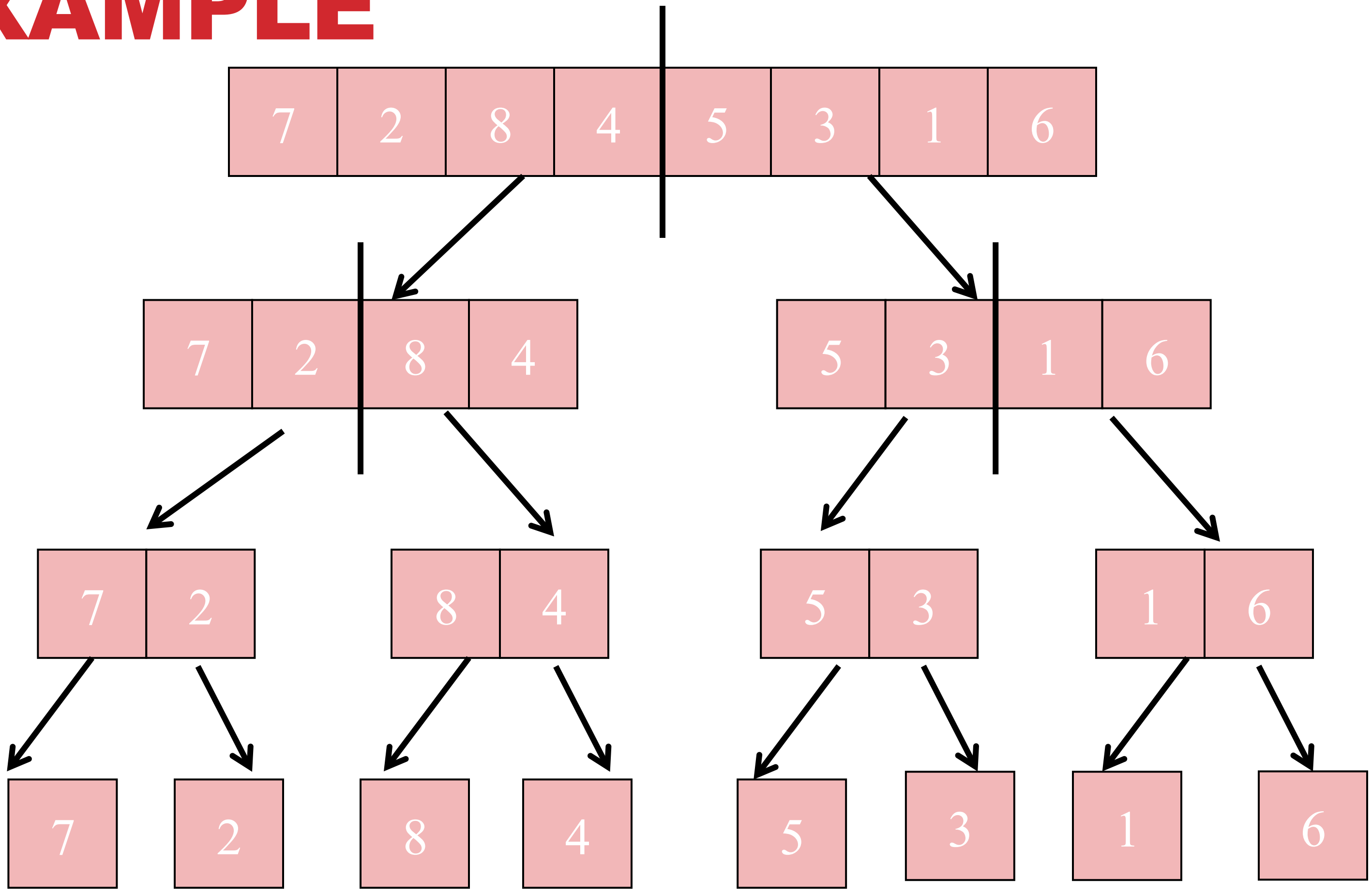


MERGE SORT: PSEUDOCODE

Core idea: split array in half, sort each half, merge back together. If the array has size 0 or 1, just return it unchanged

```
mergesort(input) {
    if (input.length < 2) {
        return input;
    } else {
        smallerHalf = sort(input[0, ..., mid]);
        largerHalf = sort(input[mid + 1, ...]);
        return merge(smallerHalf, largerHalf);
    }
}
```

MERGE SORT EXAMPLE



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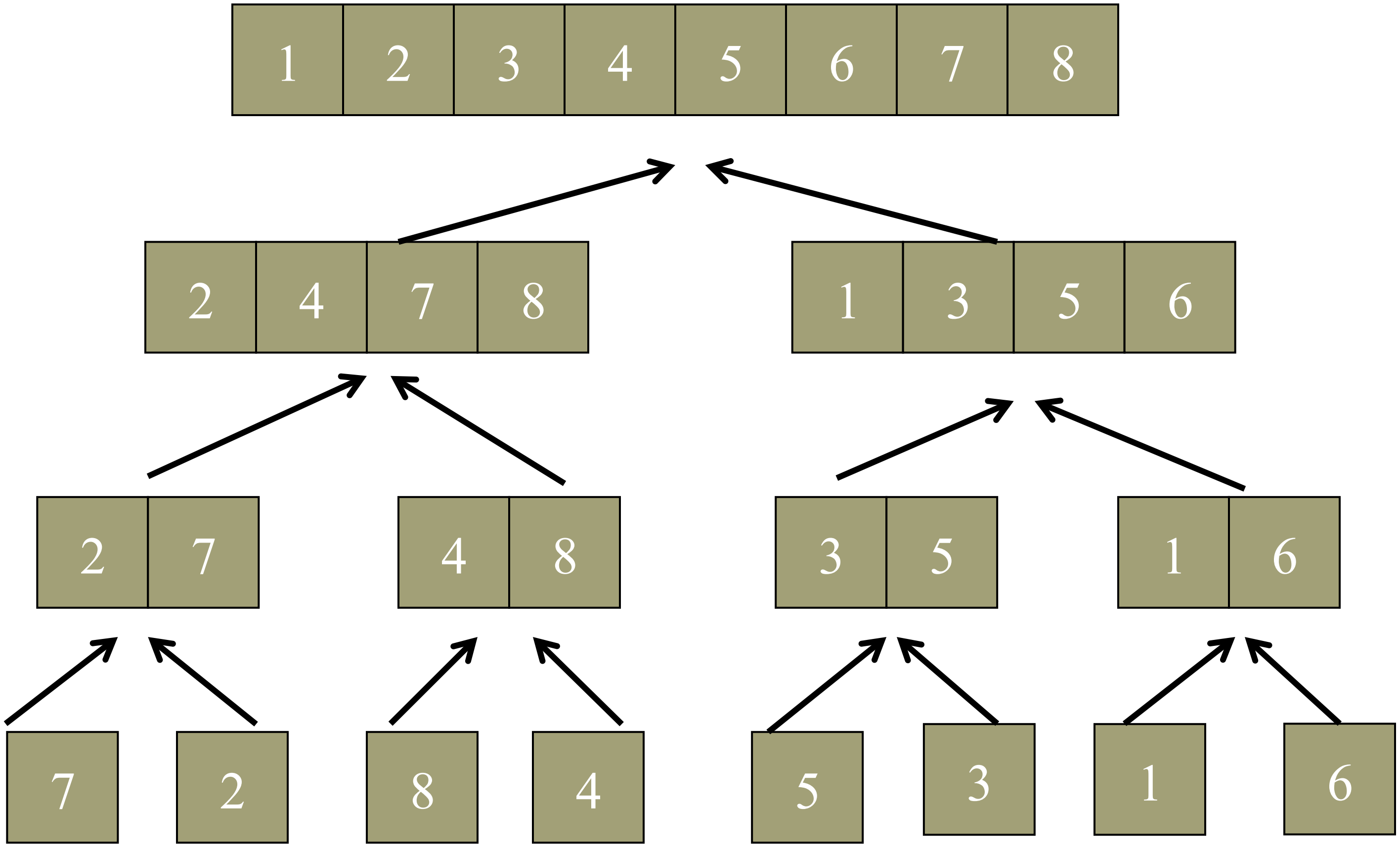
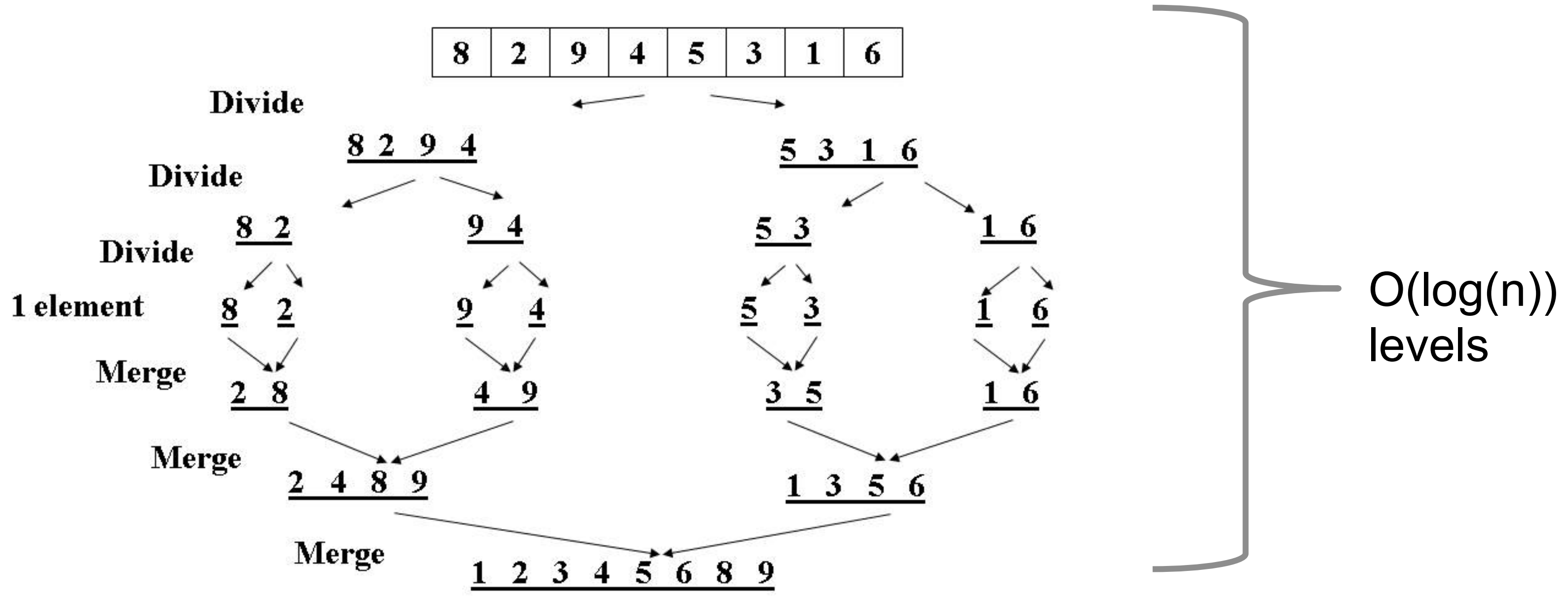


MERGE SORT ANALYSIS

Runtime:

- subdivide the array in half each time: O(log(n)) recursive calls
- merge is an O(n) traversal at each level

So, the best and worst case runtime is the same: O(n log(n))



MERGE SORT ANALYSIS

Stable?

Yes! If we implement the merge function correctly, merge sort will be stable.

In-place?

No. Unless you want to give yourself a headache. Merge must construct a new array to contain the output, so merge sort is not in-place.

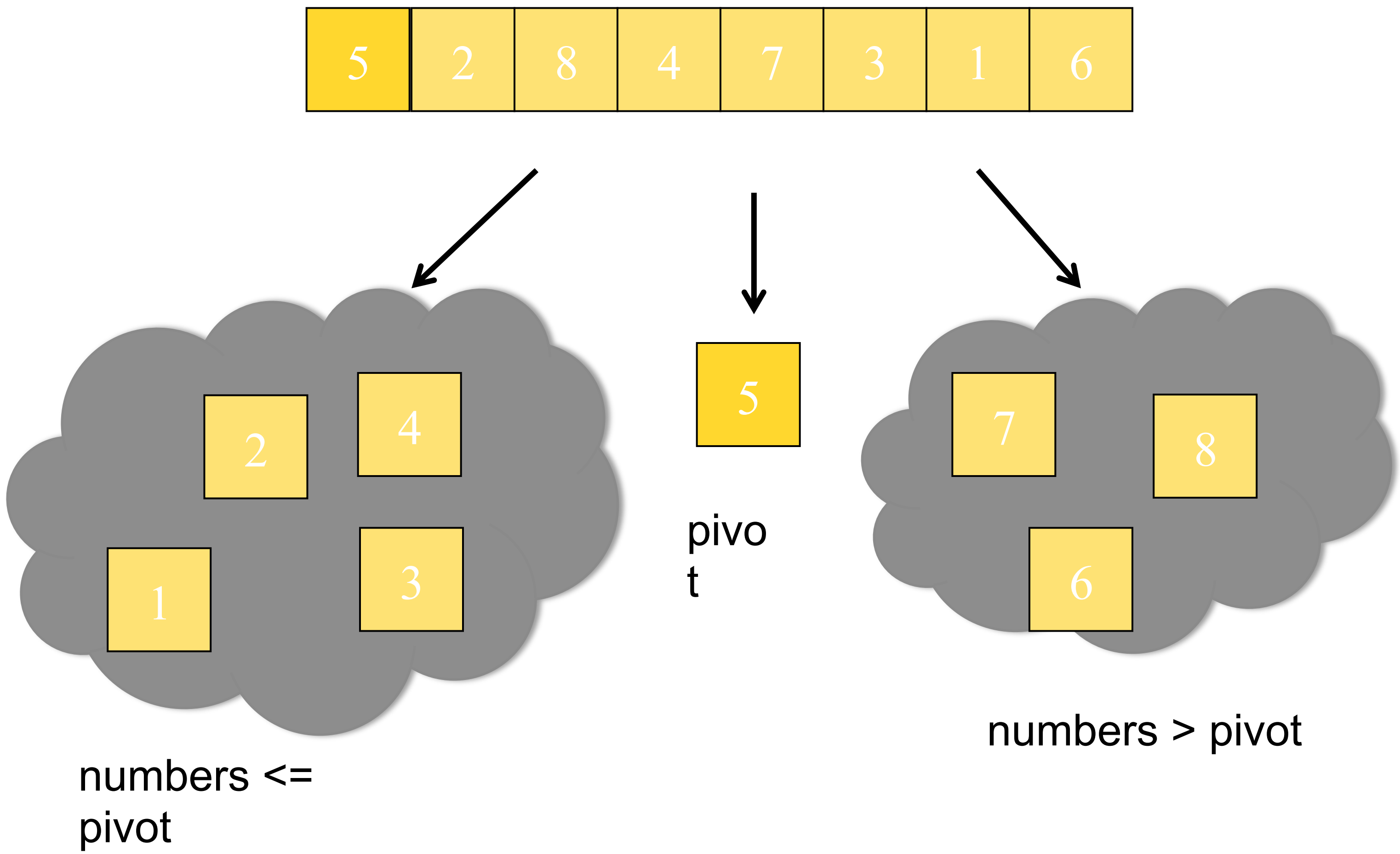
We're constantly copying and creating new arrays at each level...

One Solution: (less of a headache than actually implementing in-place) create a single auxiliary array and swap between

it and the original on each level.

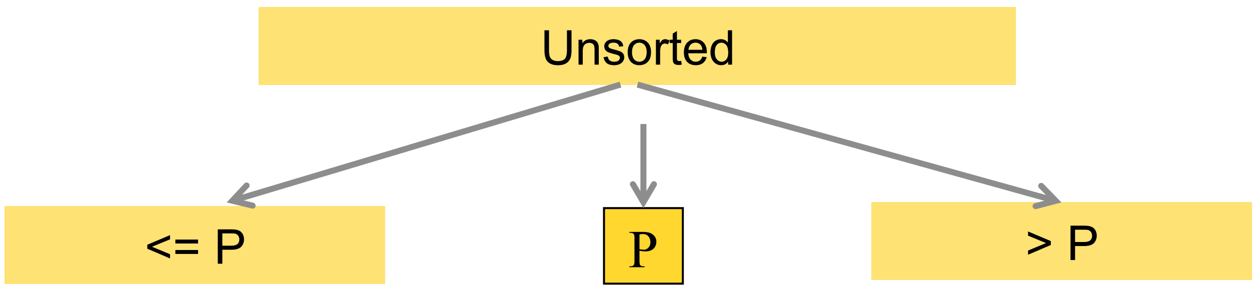
QUICK SORT

Divide: Split array around a 'pivot'

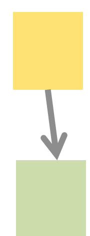


QUICK SORT

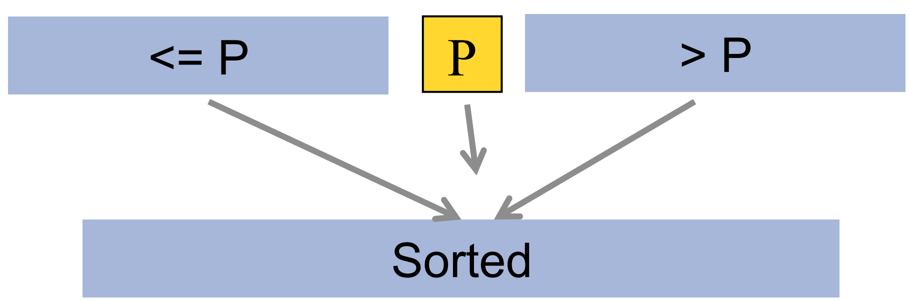
Divide: Pick a pivot, partition into groups



Conquer: Return array when length ≤ 1



Combine: Combine sorted partitions and pivot

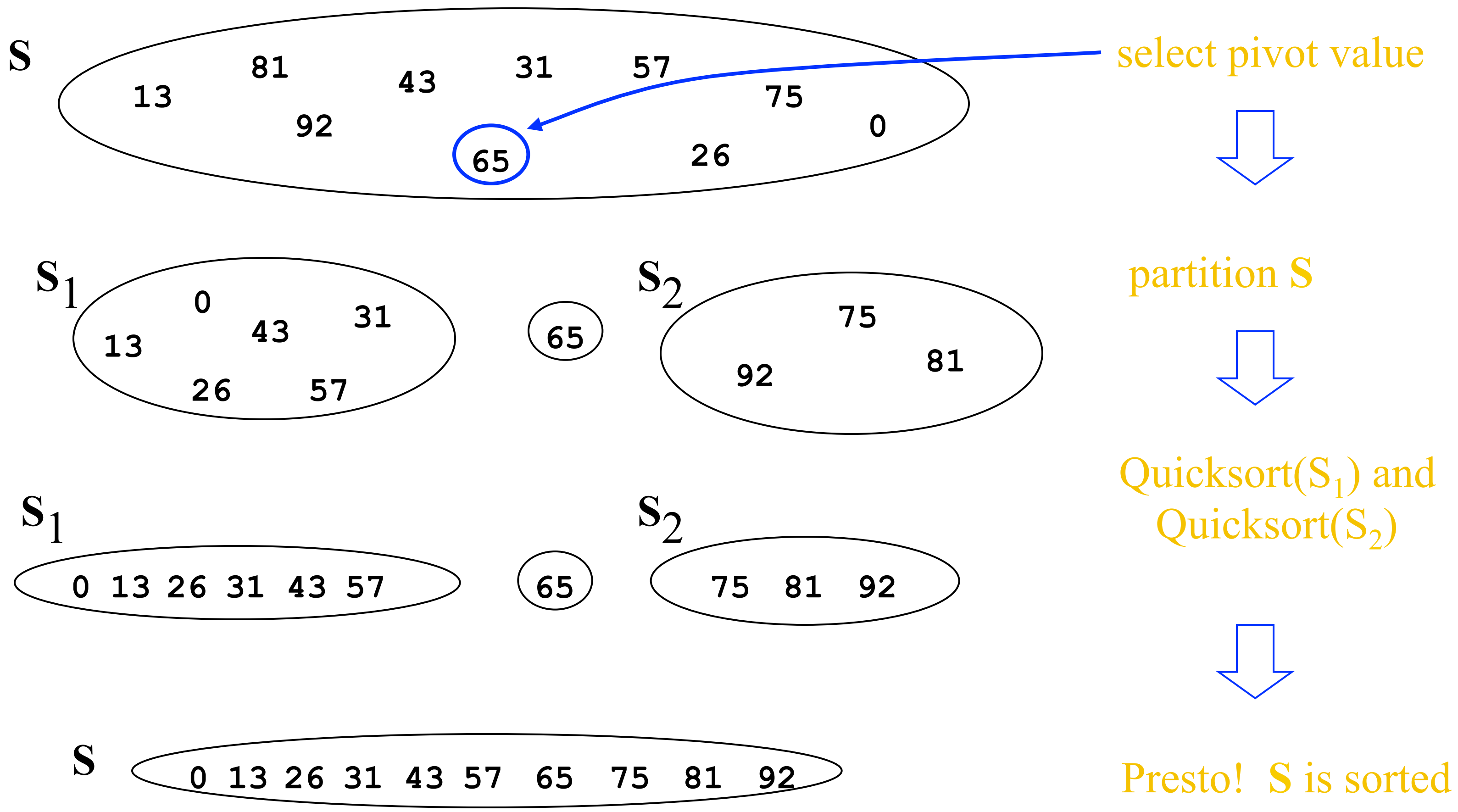


QUICK SORT PSEUDOCODE

Core idea: Pick some item from the array and call it the pivot. Put all items smaller in the pivot into one group and all items larger in the other and recursively sort. If the array has size 0 or 1, just return it unchanged.

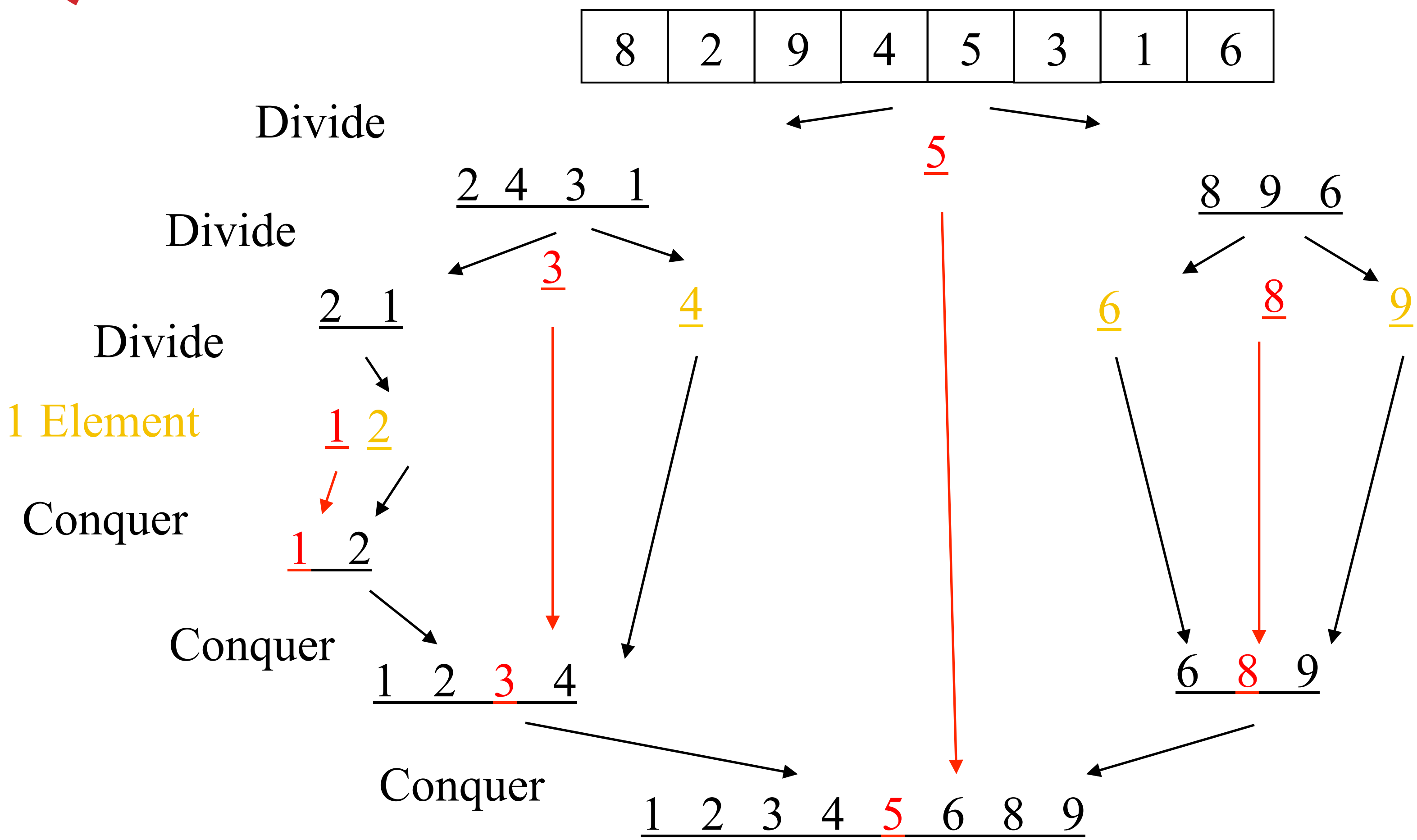
```
quicksort(input) {
    if (input.length < 2) {
        return input;
    } else {
        pivot = getPivot(input);
        smallerHalf = sort(getSmaller(pivot, input));
        largerHalf = sort(getBigger(pivot, input));
        return smallerHalf + pivot + largerHalf;
    }
}
```

QUICKSORT



[Weiss]

QUICKSORT



8 2 9 4 5 3 1 6
Divide
5
2 4 3 1
8 9 6
Divide
3
4
2 1
6
8
9
Divide
1 Element
1 2
Conquer
1 2
Conquer
1 2 3 4
6 8 9
Conquer
1 2 3 4 5 6 8 9

DETAILS

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How to pick the pivot element

- Any choice is correct: data will end up sorted
- But as analysis will show, want the two partitions to be about equal in size

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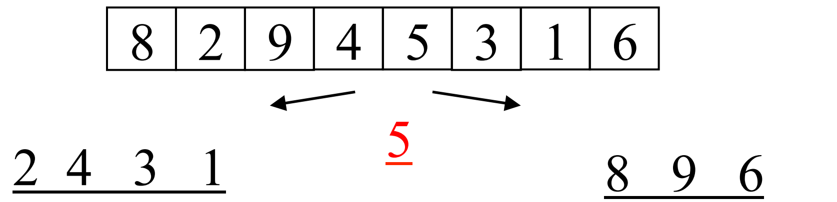
How to implement partitioning

- In linear time
- In place

PIVOTS

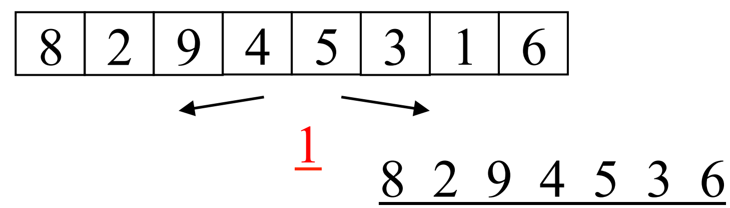
Best pivot?

- Median
- Halve each time



Worst pivot?

- Greatest/least element
- Problem of size n - 1
- $O(n^2)$



POTENTIAL PIVOT RULES

While sorting `arr` from `lo` (inclusive) to `hi` (exclusive)...

Pick `arr[lo]` or `arr[hi-1]`

- Fast, but worst-case occurs with mostly sorted input

Pick random element in the range

- Does as well as any technique, but (pseudo)random number generation can be slow
- Still probably the most elegant approach

Median of 3, e.g., `arr[lo], arr[hi-1], arr[(hi+lo)/2]`

- Common heuristic that tends to work well

PARTITIONING

Conceptually simple, but hardest part to code up correctly

- After picking pivot, need to partition in linear time in place

One approach (there are slightly fancier ones):

1. Swap pivot with **`arr[lo]`**
2. Use two counters **`i`** and **`j`**, starting at **`lo+1`** and **`hi-1`**
3. **`while (i < j)`**

```
    if (arr[j] > pivot) j--
    else if (arr[i] < pivot) i++
    else swap arr[i] with arr[j]
```

4. Swap pivot with **`arr[i]`** *

***skip step 4 if pivot ends up being least element**

EXAMPLE

Step one: pick pivot as median of 3

- **`lo`** = 0, **`hi`** = 10

0	1	2	3	4	5	6	7	8	9
8	1	4	9	**0**	3	5	2	7	**6**

- Step two: move pivot to the `lo` position

0	1	2	3	4	5	6	7	8	9
6	1	4	9	**0**	3	5	2	7	**8**

EXAMPLE

Often have more than one swap during partition – this is a short example

Now partition in place

6	1	4	9	**0**	3	5	2	7	**8**

Move cursors

6	1	4	9	**0**	3	5	2	7	**8**

Swap

6	1	4	2	**0**	3	5	9	7	**8**

Move cursors

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Move pivot

5	1	4	2	**0**	3	6	9	7	**8**