

CSE 332

**AUGUST 14TH – EFFICIENT
REDUCTIONS**

ADMINISTRIVIA

- **P2 should be back and graded**

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- **P1 ClassNotFound fixed tonight**

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- **P1 ClassNotFoundException fixed tonight**
- **EX11 graded and back tonight**

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- **P1 ClassNotFound fixed tonight**
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- **Parallelism exercises**
 - If you passed the tests on your home computer, you will get full marks (provided you actually used parallelism)

ADMINISTRIVIA

- **P2 should be back and graded**
- **P1 ClassNotFoundException fixed tonight**
- **EX11 graded and back tonight**
- **Parallelism exercises**
 - If you passed the tests on your home computer, you will get full marks (provided you actually used parallelism)
- **P3 due tonight at midnight**

EXERCISE TOKENS

- **Redoing exercises with tokens**

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 - Fill out token form

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 - This should unlock the assignment so that you can resubmit

EXERCISE TOKENS

- **Redoing exercises with tokens**
 - Fill out token form
 - This should unlock the assignment so that you can resubmit
 - Additionally, for a brief paragraph, explain the mistakes that you made and how you learned from them (even if you just didn't submit at all)

COURSE EVALUATIONS

- **Course evaluations are out, please take 5 or 10 minutes to fill out the evaluations**
 - <https://uw.iasystem.org/survey/179903>

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- **Course evaluations are out, please take 5 or 10 minutes to fill out the evaluations**
 - <https://uw.iasystem.org/survey/179903>
- **These are very important, not just for me but for the department**
- **Summer quarter: what went well and what was difficult**

COURSE EVALUATIONS

- **Course evaluations are out, please take 5 or 10 minutes to fill out the evaluations**
 - <https://uw.iasystem.org/survey/179903>
- **These are very important, not just for me but for the department**
- **Summer quarter: what went well and what was difficult**
 - Prereq course, want to balance preparing you and not overworking you

EXAM REVIEW

- **Tuesday 1:30 PM 045**

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 - Friday: Graphs and Remaining

EXAM REVIEW

- **Tuesday 1:30 PM 045**
 - Topics list will go out soon
- **Exam will be a two-period exam**
 - Section: Sorting and Parallelism
 - Friday: Graphs and Remaining
 - Material from before the midterm is fair game for both days

TODAY'S LECTURE

- **Graph algorithm review**

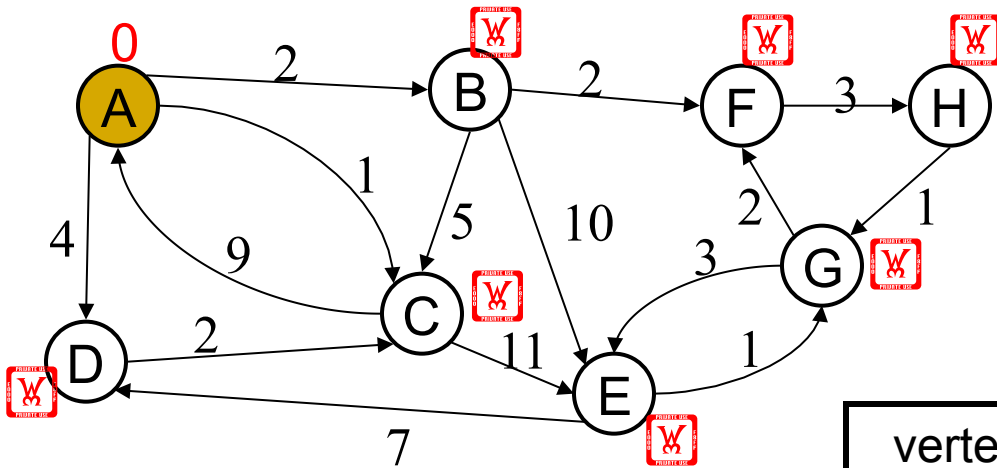
TODAY'S LECTURE

- **Graph algorithm review**
- **Efficient reductions**

DIJKSTRAS ALGORITHM

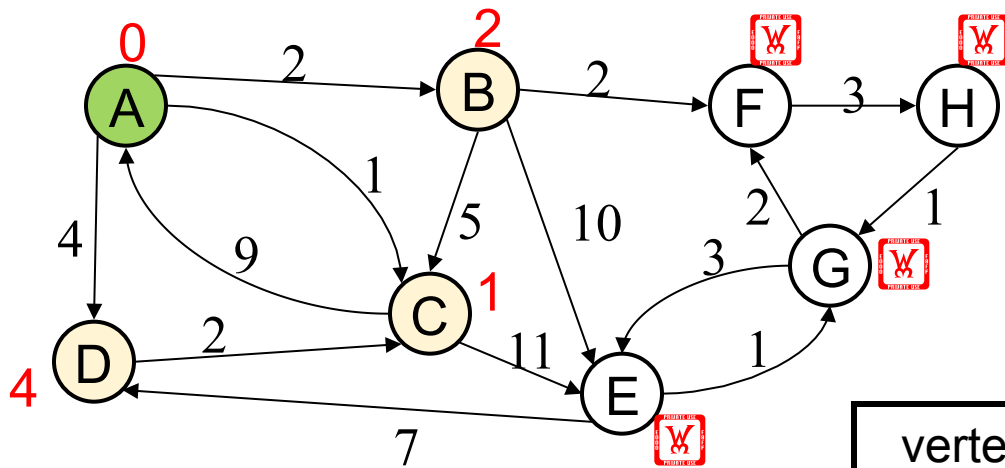
1. For each node v , set $v.cost = \infty$ **and** $v.known = false$
2. Set $source.cost = 0$
3. While there are unknown nodes in the graph
 - a) Select the unknown node v with lowest cost
 - b) Mark v as known
 - c) For each edge (v, u) with weight w ,

```
c1 = v.cost + w // cost of best path through v to u
c2 = u.cost // cost of best path to u previously known
if (c1 < c2) { // if the path through v is better
    u.cost = c1
    u.path = v // for computing actual paths
}
```



| vertex | known? | cost | path |
|--------|--------|------|------|
| A | | 0 | |
| B | | ?? | |
| C | | ?? | |
| D | | ?? | |
| E | | ?? | |
| F | | ?? | |
| G | | ?? | |
| H | | ?? | |

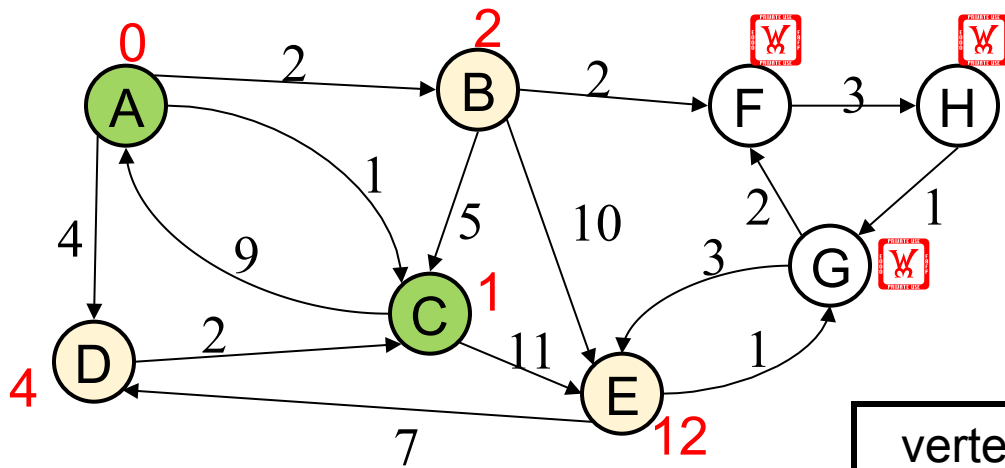
Order Added to Known Set:



| vertex | known? | cost | path |
|--------|--------|----------|------|
| A | Y | 0 | |
| B | | ≤ 2 | A |
| C | | ≤ 1 | A |
| D | | ≤ 4 | A |
| E | | ?? | |
| F | | ?? | |
| G | | ?? | |
| H | | ?? | |

Order Added to Known Set:

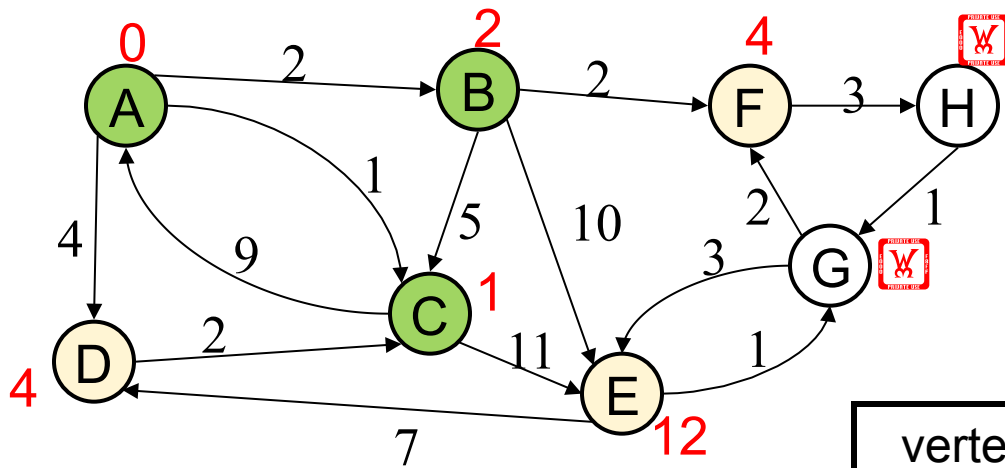
A



| vertex | known? | cost | path |
|--------|--------|-----------|------|
| A | Y | 0 | |
| B | | ≤ 2 | A |
| C | Y | 1 | A |
| D | | ≤ 4 | A |
| E | | ≤ 12 | C |
| F | | ?? | |
| G | | ?? | |
| H | | ?? | |

Order Added to Known Set:

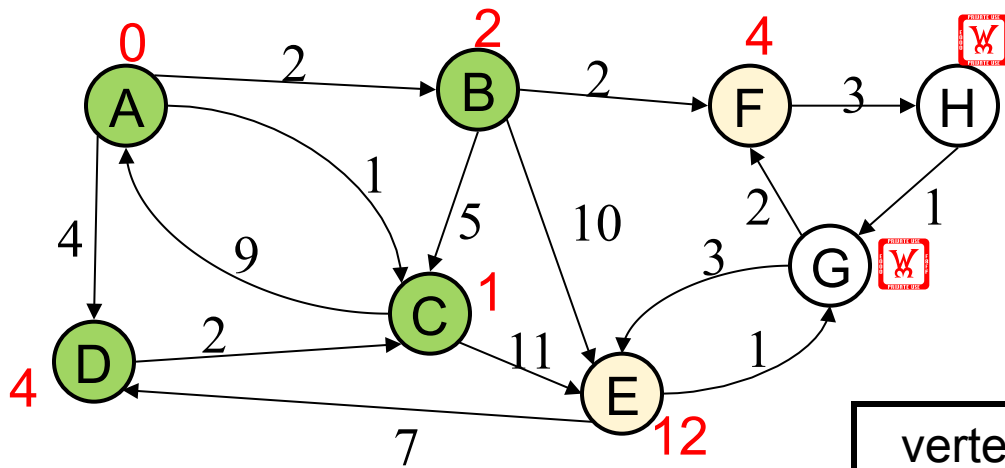
A, C



| vertex | known? | cost | path |
|--------|--------|-----------|------|
| A | Y | 0 | |
| B | Y | 2 | A |
| C | Y | 1 | A |
| D | | ≤ 4 | A |
| E | | ≤ 12 | C |
| F | | ≤ 4 | B |
| G | | ?? | |
| H | | ?? | |

Order Added to Known Set:

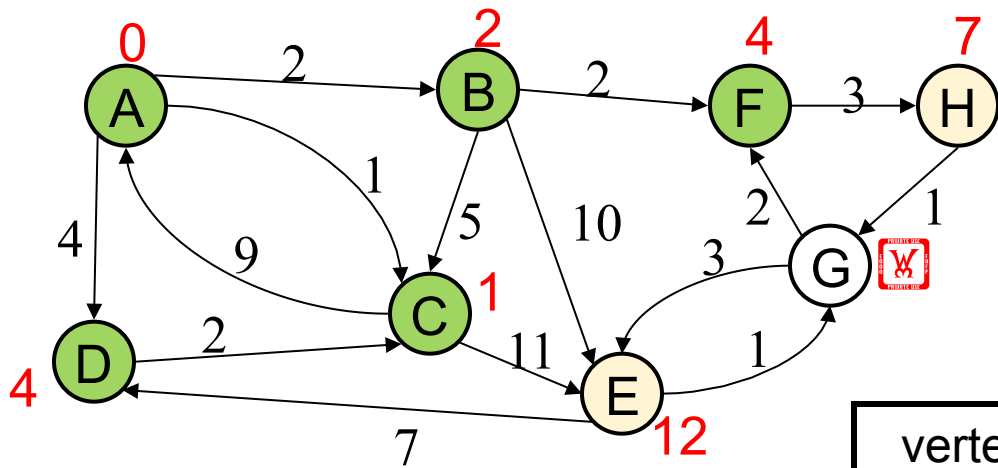
A, C, B



| vertex | known? | cost | path |
|--------|--------|-----------|------|
| A | Y | 0 | |
| B | Y | 2 | A |
| C | Y | 1 | A |
| D | Y | 4 | A |
| E | | ≤ 12 | C |
| F | | ≤ 4 | B |
| G | | ?? | |
| H | | ?? | |

Order Added to Known Set:

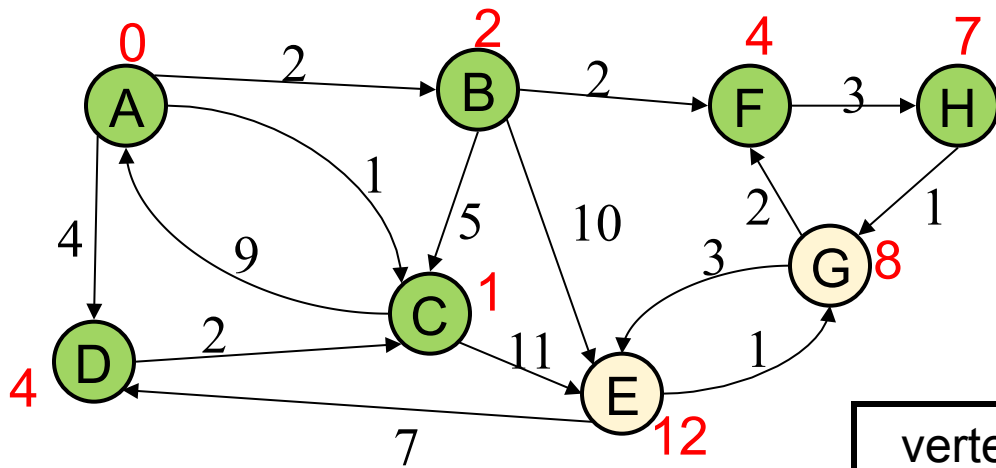
A, C, B, D



| vertex | known? | cost | path |
|--------|--------|-----------|------|
| A | Y | 0 | |
| B | Y | 2 | A |
| C | Y | 1 | A |
| D | Y | 4 | A |
| E | | ≤ 12 | C |
| F | Y | 4 | B |
| G | | ?? | |
| H | | ≤ 7 | F |

Order Added to Known Set:

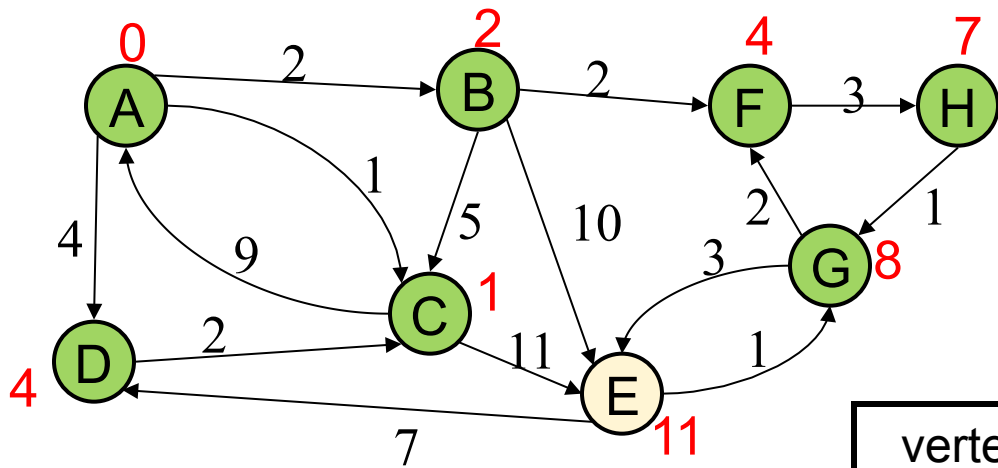
A, C, B, D, F



| vertex | known? | cost | path |
|--------|--------|-----------|------|
| A | Y | 0 | |
| B | Y | 2 | A |
| C | Y | 1 | A |
| D | Y | 4 | A |
| E | | ≤ 12 | C |
| F | Y | 4 | B |
| G | | ≤ 8 | H |
| H | Y | 7 | F |

Order Added to Known Set:

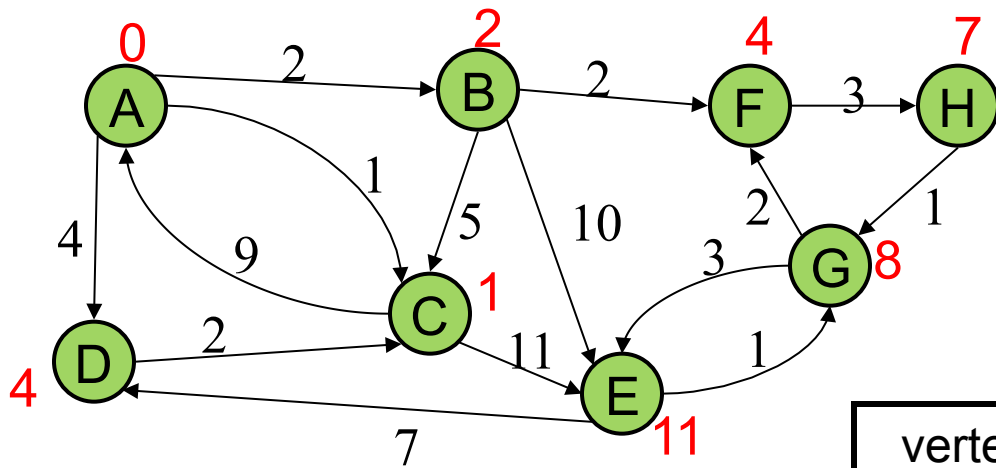
A, C, B, D, F, H



| vertex | known? | cost | path |
|--------|--------|-----------|------|
| A | Y | 0 | |
| B | Y | 2 | A |
| C | Y | 1 | A |
| D | Y | 4 | A |
| E | | ≤ 11 | G |
| F | Y | 4 | B |
| G | Y | 8 | H |
| H | Y | 7 | F |

Order Added to Known Set:

A, C, B, D, F, H, G



| vertex | known? | cost | path |
|--------|--------|------|------|
| A | Y | 0 | |
| B | Y | 2 | A |
| C | Y | 1 | A |
| D | Y | 4 | A |
| E | Y | 11 | G |
| F | Y | 4 | B |
| G | Y | 8 | H |
| H | Y | 7 | F |

Order Added to Known Set:

A, C, B, D, F, H, G, E

PRIM'S ALGORITHM

- A traversal

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- **A traversal**
 - Pick a start node

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 - Keep track of all of the vertices you can reach

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- **A traversal**
 - Pick a start node
 - Keep track of all of the vertices you can reach
 - Add the vertex that is closest (has the edge with smallest weight) to the current spanning tree.

PRIM'S ALGORITHM

- **A traversal**
 - Pick a start node
 - Keep track of all of the vertices you can reach
 - Add the vertex that is closest (has the edge with smallest weight) to the current spanning tree.
- **Is this similar to something we've seen before?**

PRIM'S ALGORITHM

- **Modify Dijkstra's algorithm**

PRIM'S ALGORITHM

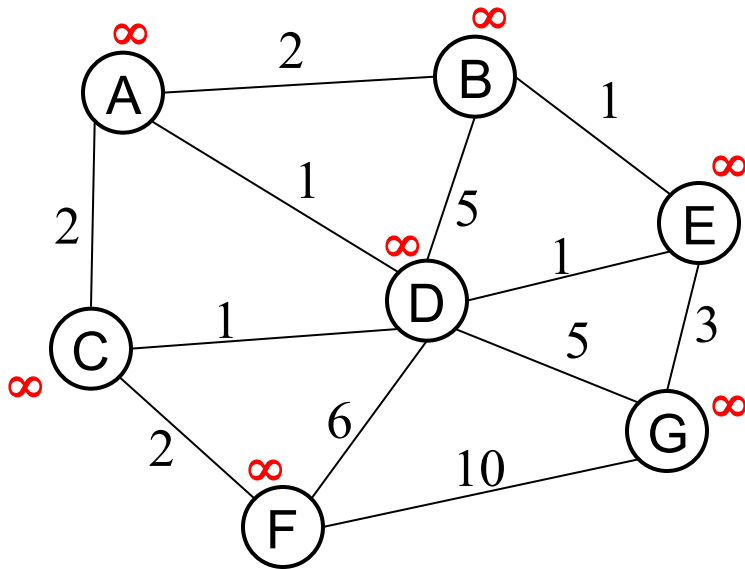
- **Modify Dijkstra's algorithm**
 - Instead of measuring the total length from start to the new vertex, now we only care about the edge from our current spanning tree to new nodes

THE ALGORITHM

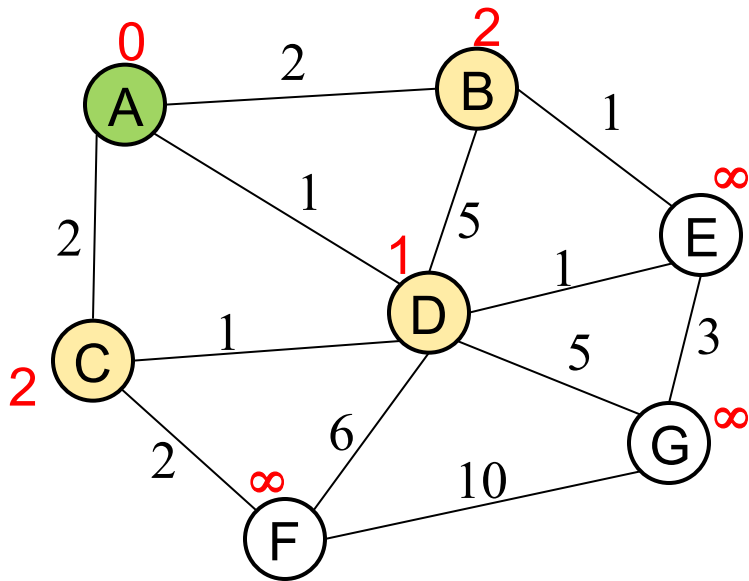
1. For each node v , set $v.cost = \infty$ and $v.known = false$
2. Choose any node v
 - a) Mark v as known
 - b) For each edge (v,u) with weight w , set $u.cost=w$ and $u.prev=v$
3. While there are unknown nodes in the graph
 - a) Select the unknown node v with lowest cost
 - b) Mark v as known and add $(v, v.prev)$ to output
 - c) For each edge (v,u) with weight w ,

```
        if( $w < u.cost$ ) {  
             $u.cost = w$ ;  
             $u.prev = v$ ;  
        }
```

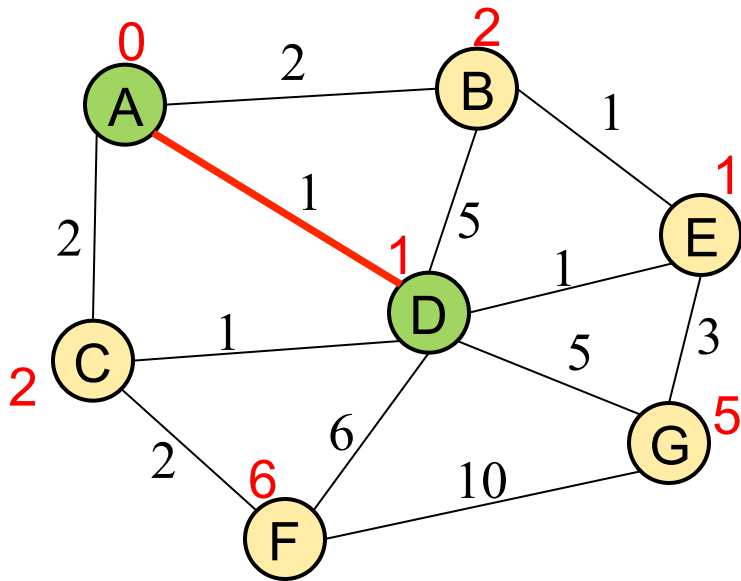

EXAMPLE



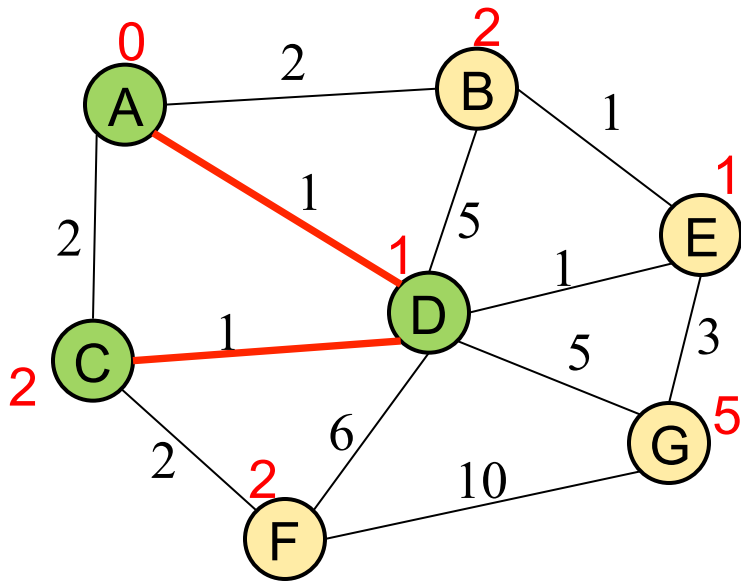
| vertex | known? | cost | prev |
|--------|--------|----------|------|
| A | | ∞ | |
| B | | ∞ | |
| C | | ∞ | |
| D | | ∞ | |
| E | | ∞ | |
| F | | ∞ | |
| G | | ∞ | |



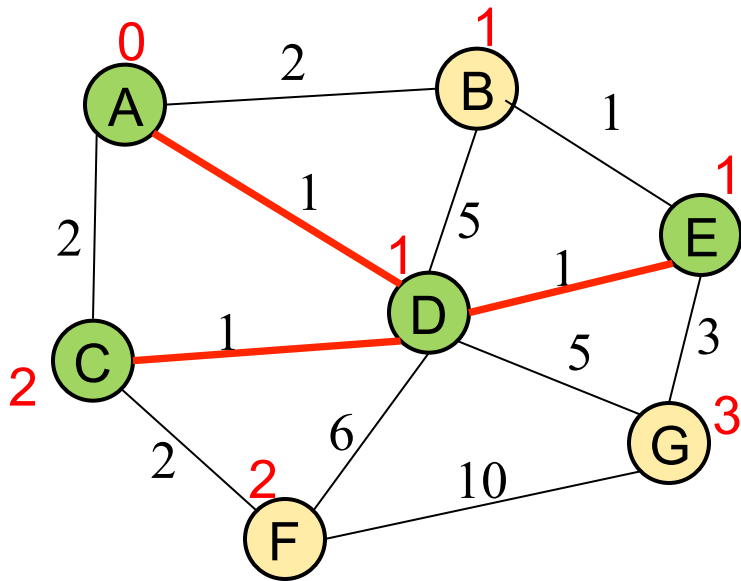
| vertex | known? | cost | prev |
|--------|--------|----------|------|
| A | Y | 0 | |
| B | | 2 | A |
| C | | 2 | A |
| D | | 1 | A |
| E | | ∞ | |
| F | | ∞ | |
| G | | ∞ | |



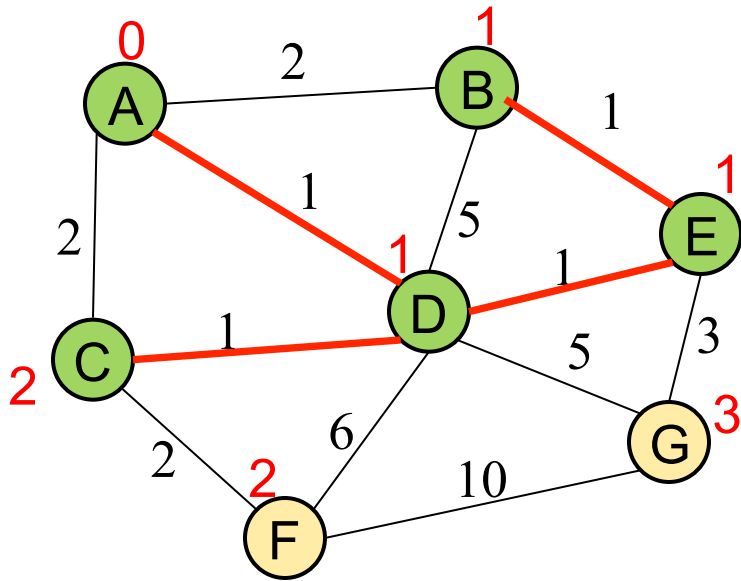
| vertex | known? | cost | prev |
|--------|--------|------|------|
| A | Y | 0 | |
| B | | 2 | A |
| C | | 1 | D |
| D | Y | 1 | A |
| E | | 1 | D |
| F | | 6 | D |
| G | | 5 | D |



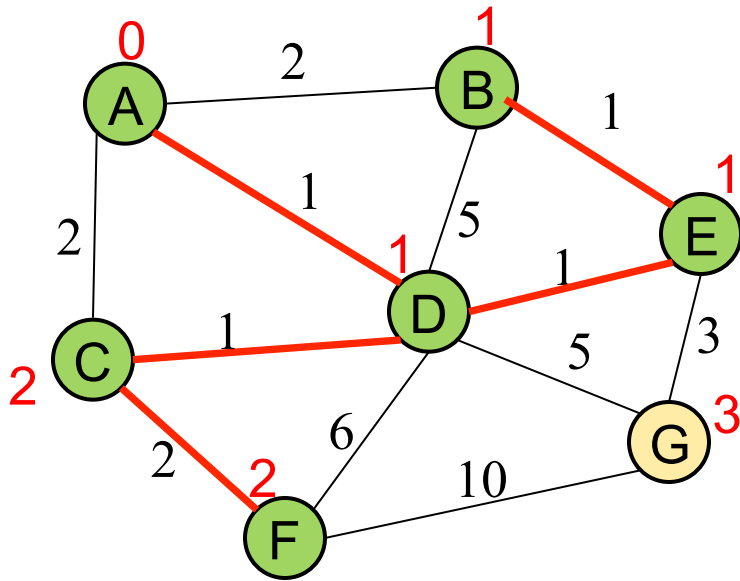
| vertex | known? | cost | prev |
|--------|--------|------|------|
| A | Y | 0 | |
| B | | 2 | A |
| C | Y | 1 | D |
| D | Y | 1 | A |
| E | | 1 | D |
| F | | 2 | C |
| G | | 5 | D |



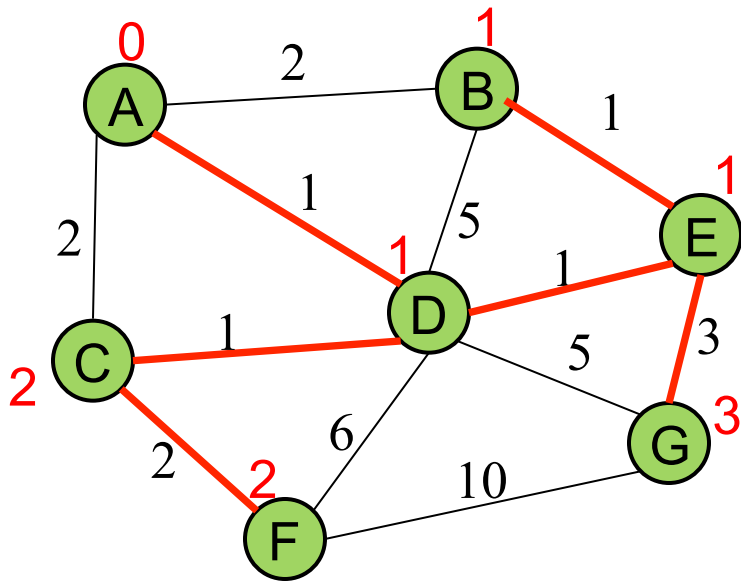
| vertex | known? | cost | prev |
|--------|--------|------|------|
| A | Y | 0 | |
| B | | 1 | E |
| C | Y | 1 | D |
| D | Y | 1 | A |
| E | Y | 1 | D |
| F | | 2 | C |
| G | | 3 | E |



| vertex | known? | cost | prev |
|--------|--------|------|------|
| A | Y | 0 | |
| B | Y | 1 | E |
| C | Y | 1 | D |
| D | Y | 1 | A |
| E | Y | 1 | D |
| F | | 2 | C |
| G | | 3 | E |



| vertex | known? | cost | prev |
|--------|--------|------|------|
| A | Y | 0 | |
| B | Y | 1 | E |
| C | Y | 1 | D |
| D | Y | 1 | A |
| E | Y | 1 | D |
| F | Y | 2 | C |
| G | | 3 | E |



| vertex | known? | cost | prev |
|--------|--------|------|------|
| A | Y | 0 | |
| B | Y | 1 | E |
| C | Y | 1 | D |
| D | Y | 1 | A |
| E | Y | 1 | D |
| F | Y | 2 | C |
| G | Y | 3 | E |

KRUSKAL'S ALGORITHM

- Pseudocode:

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- **Pseudocode:**
 - Sort the edges (or place them into a heap)
 - Create a union-find data structure with all separate vertices

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- **Pseudocode:**
 - Sort the edges (or place them into a heap)
 - Create a union-find data structure with all separate vertices
 - For each edge, add it to the minimum spanning tree if it does not form a cycle

KRUSKAL'S ALGORITHM

- **Pseudocode:**
 - Sort the edges (or place them into a heap)
 - Create a union-find data structure with all separate vertices
 - For each edge, add it to the minimum spanning tree if the two vertices don't have the same representative in the union find

KRUSKAL'S ALGORITHM

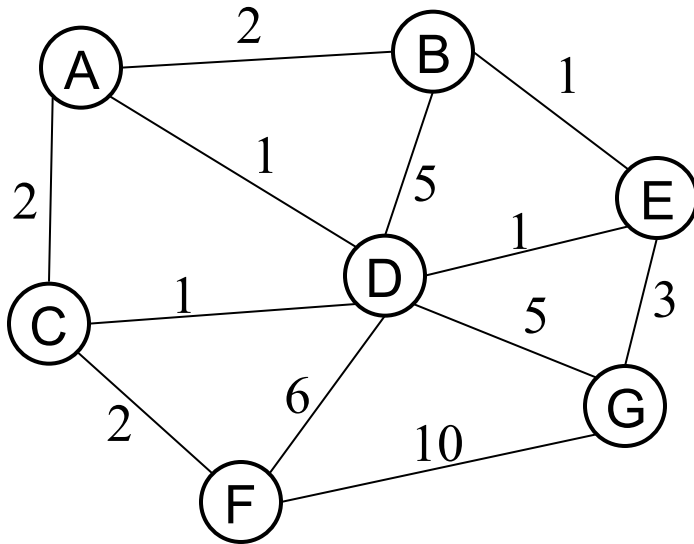
- **Pseudocode:**
 - Sort the edges (or place them into a heap)
 - Create a union-find data structure with all separate vertices
 - For each edge, add it to the minimum spanning tree if the two vertices don't have the same representative in the union find
 - Union the two vertices in the union find

KRUSKAL'S ALGORITHM

- **Pseudocode:**

- Sort the edges (or place them into a heap)
- Create a union-find data structure with all separate vertices
- For each edge, add it to the minimum spanning tree if the two vertices don't have the same representative in the union find
- Union the two vertices in the union find
- Stop after you've added $|V|-1$ edges

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

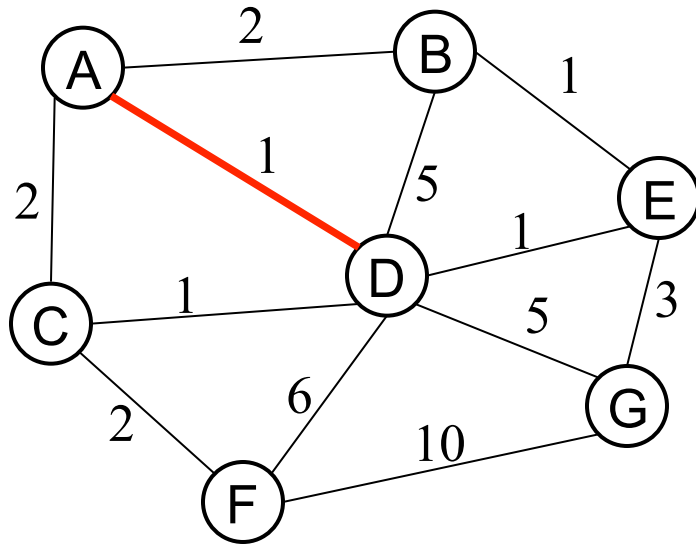
6: (D,F)

10: (F,G)

Output:

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

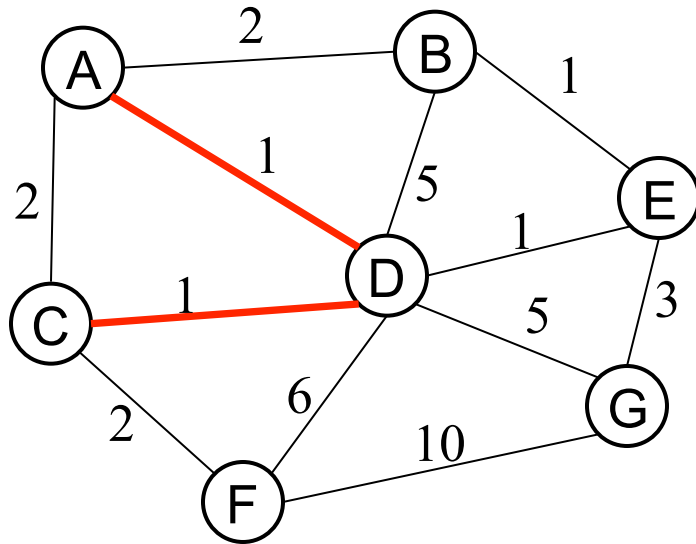
6: (D,F)

10: (F,G)

Output: (A,D)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

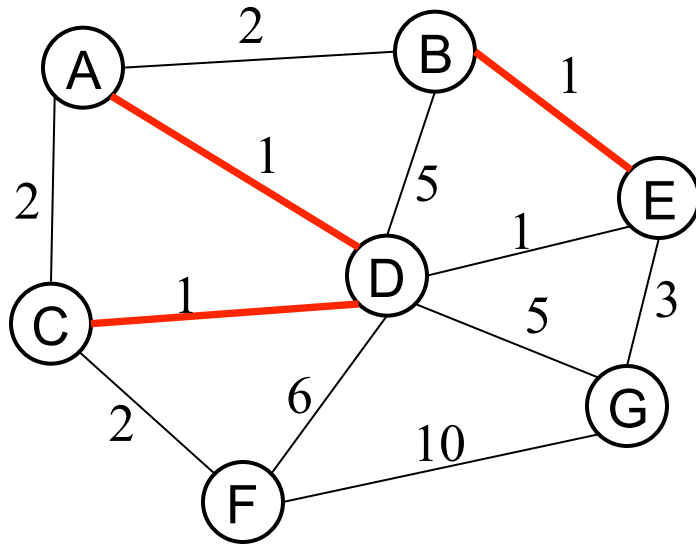
6: (D,F)

10: (F,G)

Output: (A,D), (C,D)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

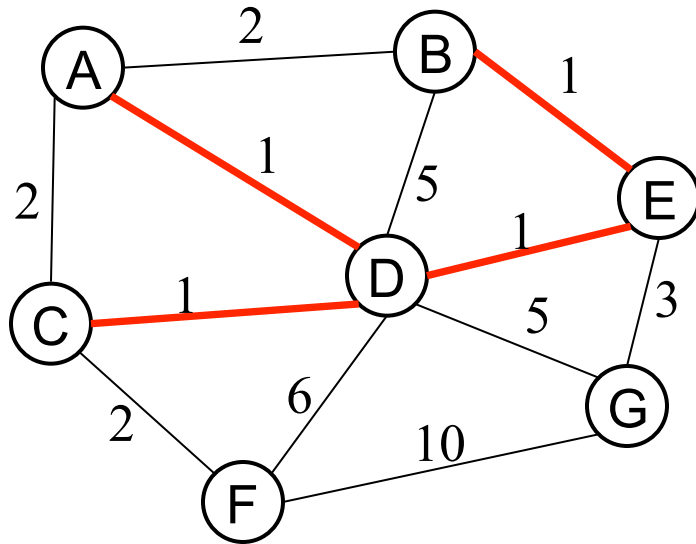
6: (D,F)

10: (F,G)

Output: (A,D), (C,D), (B,E)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

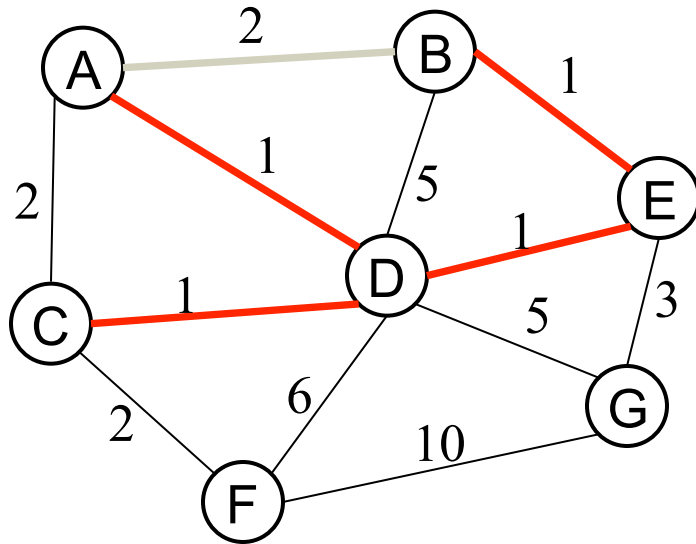
6: (D,F)

10: (F,G)

Output: (A,D), (C,D), (B,E), (D,E)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

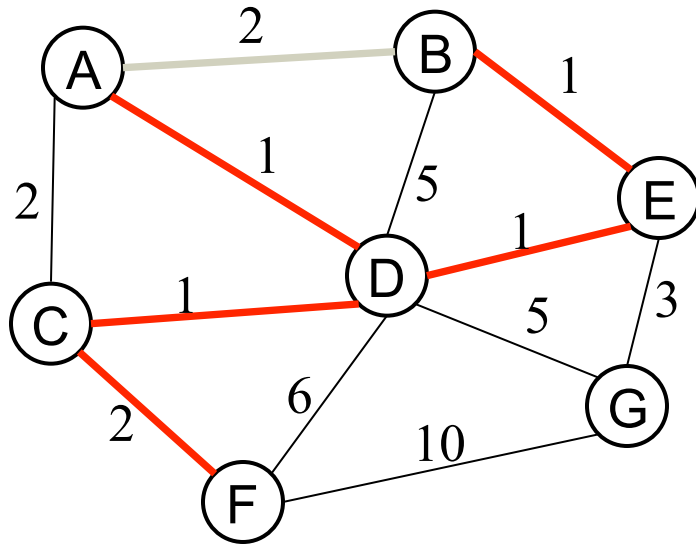
6: (D,F)

10: (F,G)

Output: (A,D), (C,D), (B,E), (D,E)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

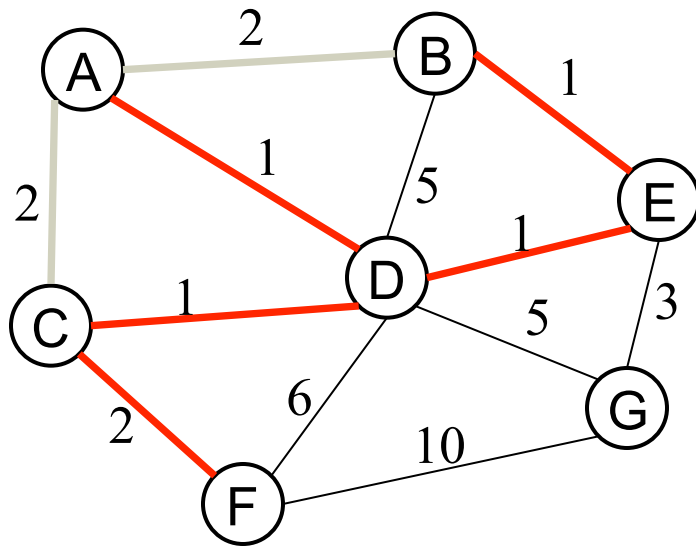
6: (D,F)

10: (F,G)

Output: (A,D), (C,D), (B,E), (D,E), (C,F)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

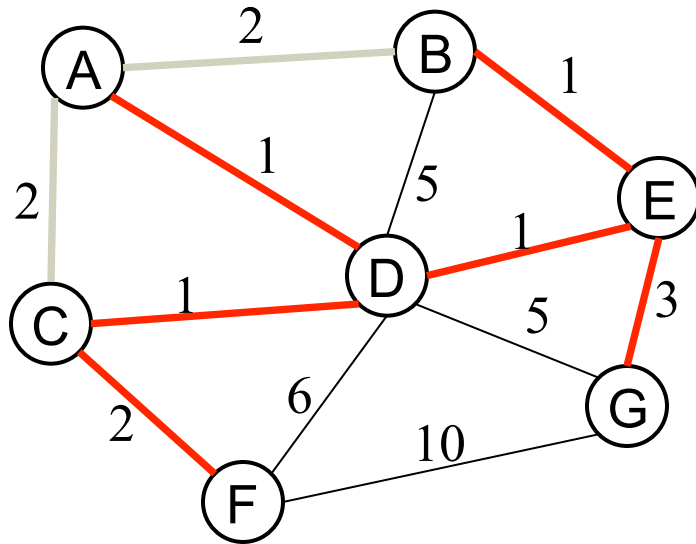
6: (D,F)

10: (F,G)

Output: (A,D), (C,D), (B,E), (D,E), (C,F)

Note: At each step, the union/find sets are the trees in the forest

EXAMPLE



Edges in sorted order:

1: (A,D), (C,D), (B,E), (D,E)

2: (A,B), (C,F), (A,C)

3: (E,G)

5: (D,G), (B,D)

6: (D,F)

10: (F,G)

Output: (A,D), (C,D), (B,E), (D,E), (C,F), (E,G)

Note: At each step, the union/find sets are the trees in the forest

EFFICIENT REDUCTIONS

- <https://courses.cs.washington.edu/courses/cse332/17wi/lectures/p-np-1/efficient-reductions.pdf>
- <https://courses.cs.washington.edu/courses/cse332/17wi/lectures/p-np-2/p-np.pdf>

NEXT CLASS

- **Randomization and Approximation**

NEXT CLASS

- **Randomization and Approximation**
- **Exam review**