CSE 332

JULY 28TH – ALPHA BETA

EXAMPLE PROBLEMS

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  - What does the recursive task need to return?
  - How do we break up the data?

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  - What if I told you to program a computer to play checkers?

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    - Zero sum (in the final board, the two players receive either (-1,1) or (1,-1). The outcomes for both players always sum to one. You cannot win unless your opponent fails

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    - Ultimatum game
    - Derivatives

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  - Assuming the game is symmetrical (I am playing the same game as my opponent, which is not always true) I calculate what would be the best possible move for my opponent in that scenario (which is the worst move for me)

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  - This is n! options! (This assumes we can calculate a board in constant time)

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  - Depends on how many moves it takes for us to complete the game, we’re computing 10^t situations, where t is the number of turns

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  - The average branching factor is 35
  - The average number of moves in a game is 40
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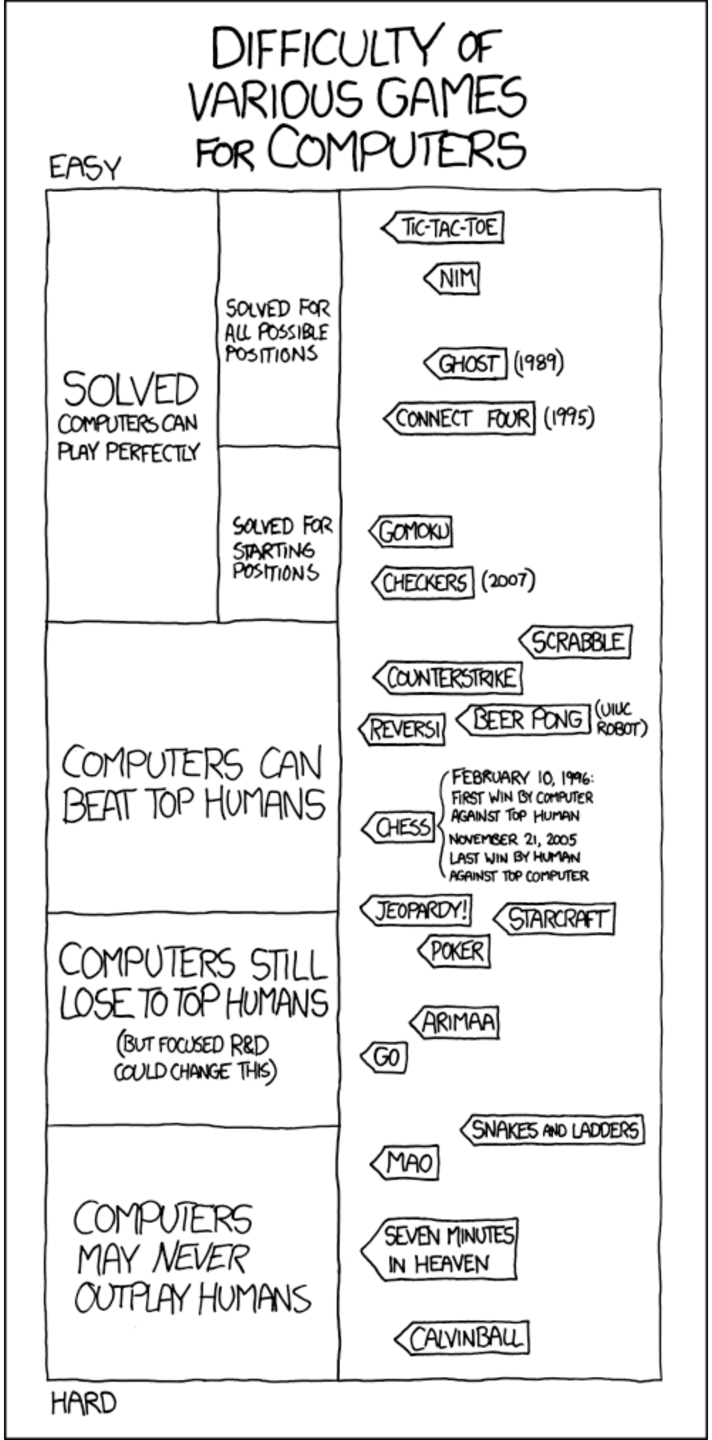
  - The universe is ~10^{10} old


DIFFICULTY OF VARIOUS GAMES FOR COMPUTERS
EASY
SOLVED
COMPUTERS CAN PLAY PERFECTLY
SOLVED FOR ALL POSSIBLE POSITIONS
SOLVED FOR STARTING POSITIONS
COMPUTERS CAN BEAT TOP HUMANS
COMPUTERS STILL LOSE TO TOP HUMANS
(BUT FOCUSED R&D COULD CHANGE THIS)
COMPUTERS MAY NEVER OUTPLAY HUMANS
HARD
TIC-TAC-TOE
NIM
GHOST (1989)
CONNECT FOUR (1995)
GOMOKU
CHECKERS (2007)
SCRABBLE
COUNTERSTRIKE
REVERSI
BEER PONG (UIUC ROBOT)
CHESS
FEBRUARY 10, 1996: FIRST WIN BY COMPUTER AGAINST TOP HUMAN
NOVEMBER 21, 2005 LAST WIN BY HUMAN AGAINST TOP COMPUTER
JEOPARDY!
STARCRAFT
POKER
ARIMAA
GO
SNAKES AND LADDERS
MAO
SEVEN MINUTES IN HEAVEN
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  - Remember, since this is a zero sum game, our accessment can (and should be a partial assignment)
  - For example, we can access a board to be (-.85,.85) to indicate that we estimate player 2 has an 85% chance of winning

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  - In tic-tac-toe, once we recognize that our move lets the opponent win, do we need to consider any other opposing moves?

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  - If we consider moves in order (and what that order is actually ends up being important), if our best move is the last one we consider, and our opponents best move is the last one we consider from that, we could actually avoid pruning all together

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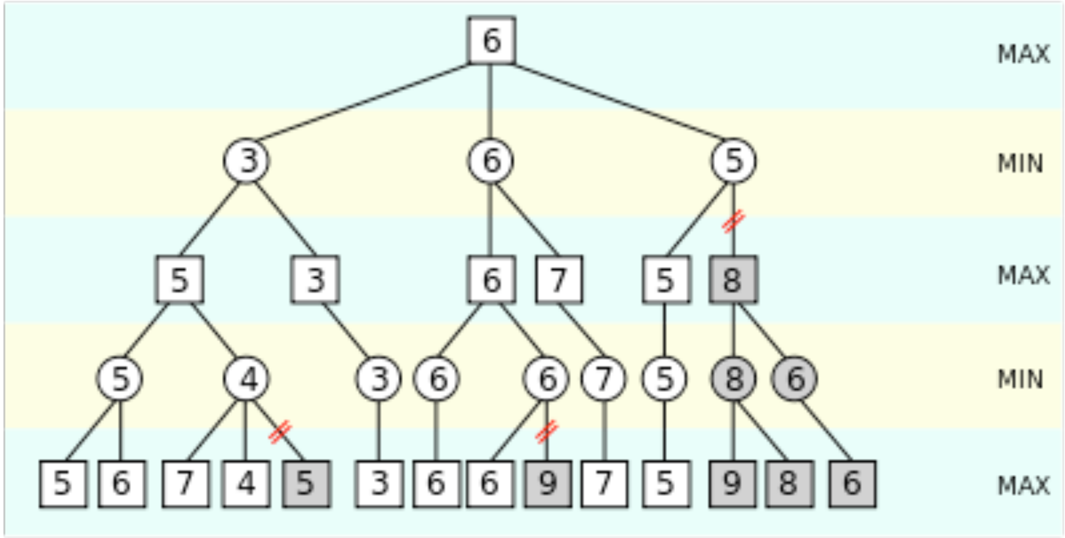

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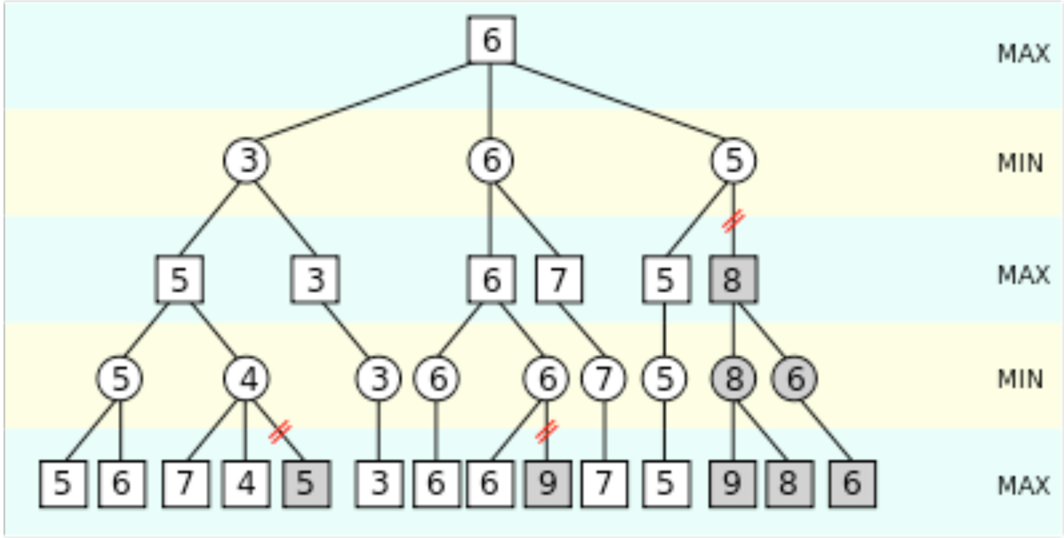

- The “max” player wants to maximize the score, whereas the “min” player wants to minimize the score. This arrangement can be reduced to zero sum format (there is no “all-winners” scenario.

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  - Poker is a turn-based, zero-sum game, do we advise minimax?

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- **Minimax works on all these types of games, the interchangeable part comes from the board comparator, we need to know something about what's good/bad**

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  - The first turn is when it costs the most to determine which move is best, but it also has the biggest impact on how the game will play out
  - Select a restricted starting sequence to simplify (all chess boards start the same way)
  - Not a coincidence that chess strategy books start with classic openings, humans don’t think that differently from computers.

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  - Is sorting really a map or a reduction?

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  - 2 simple ones will go out tonight and 2 more complicated ones will go out next week