CSE 332

JULY 10TH – HASHING

- Practice exam after class today
- Topics:
 - Stacks and Queues
 - BigO Notation and runtime Analysis
 - Heaps
 - Trees (BST and AVL)
 - Design Tradeoffs

• Format

- Format
 - No note sheet

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- One section of short answer

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- 4-5 Technical Questions

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- 1 Design Decision Question

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- 4-5 Technical Questions
- 1 Design Decision Question
- Less than 10 minutes per problem

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- Any questions?

• Hashing

- Hashing
 - Basic Concept

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 - Hash functions

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 - Collision Resolution

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 - Collision Resolution
 - Runtimes

- Introduction
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- Any data we might want to store is a member of this set. For example, M might be the set of all strings
- There is a set of data that we actually care about storing **D**, where **D** << **M**
- For an English Dictionary, D might be the set of English words

• What is our ideal data structure?

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 - Accesses should be as fast as possible

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- Need to know the size of **D** in advance or lose memory to pointer overhead
- Hard to go from **M** -> **D** in O(1) time

• Memory: The Hash Table

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Consider an array of size c * D

• Memory: The Hash Table

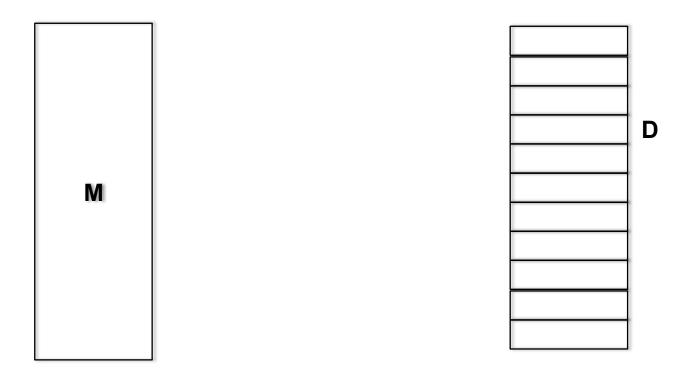
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- Each index in the array corresponds to some element in M that we want to store.

• Memory: The Hash Table

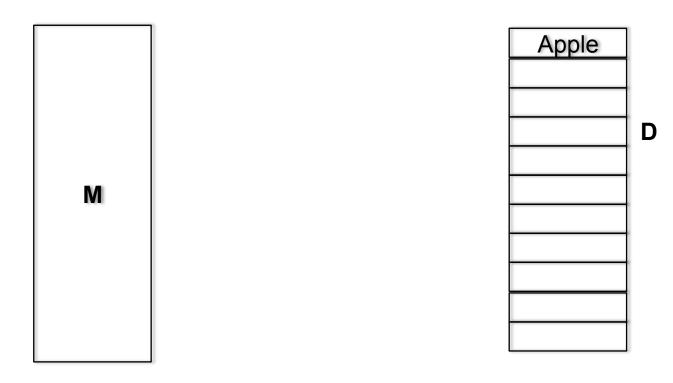
- Consider an array of size c * D
- Each index in the array corresponds to some element in M that we want to store.
- The data in **D** does not need any particular ordering.

THE HASH TABLE

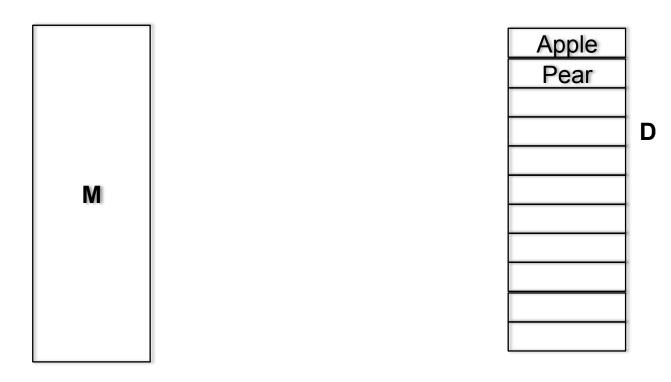
How can we do this?



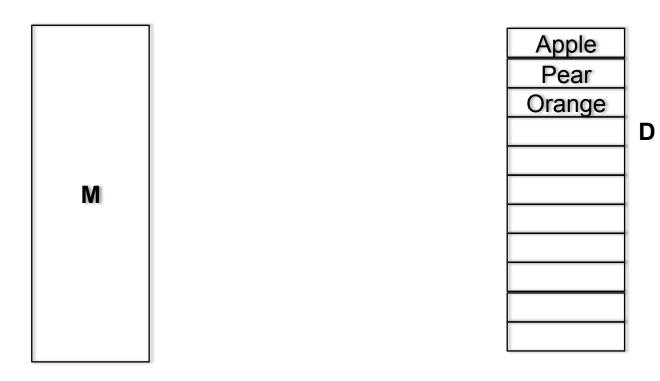
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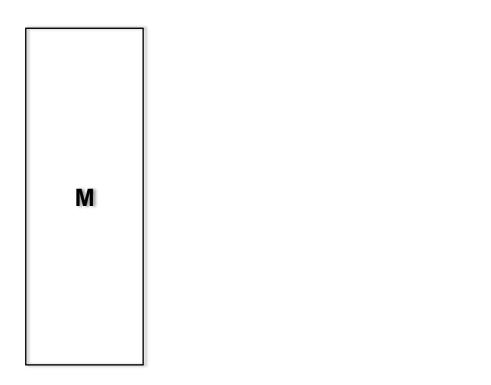


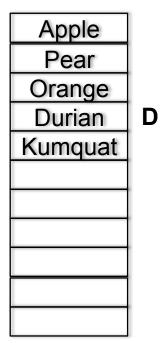
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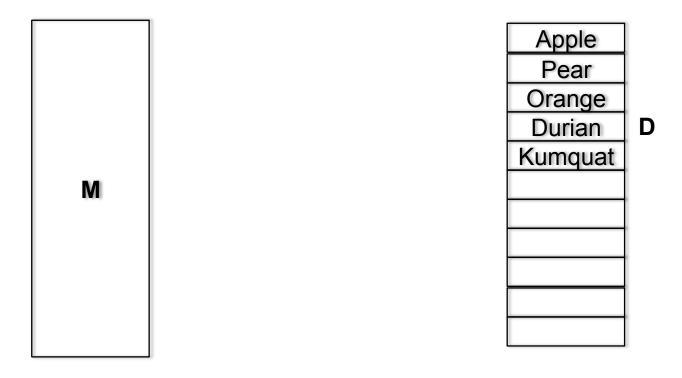
D

- How can we do this?
 - Unsorted Array



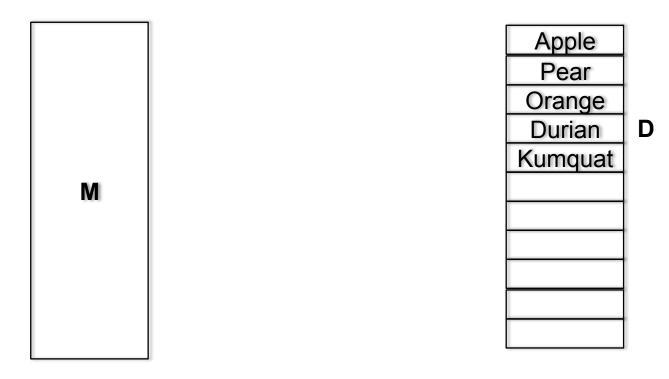


• What is the problem here?



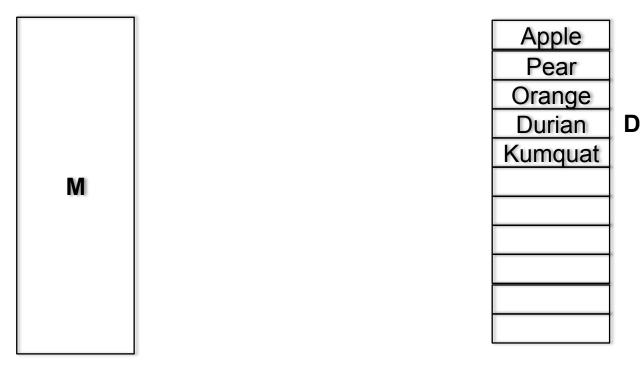
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• Takes O(D) time to find the word in the list!

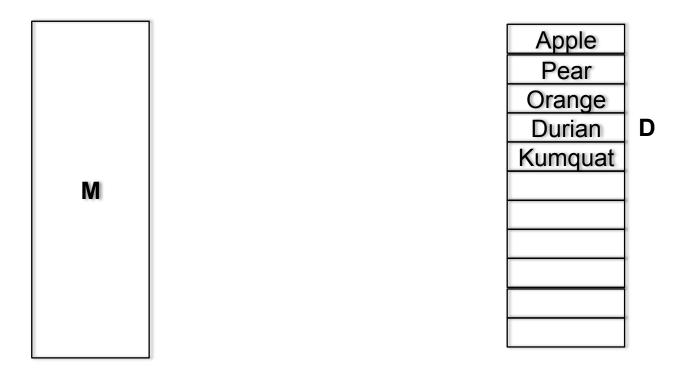


• What is the problem here?

- Takes O(D) time to find the word in the list
- Same problem with sorted arrays!

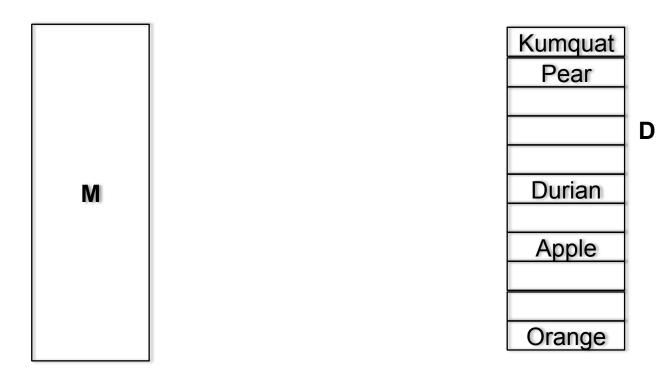


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Random mapping



• What's the problem here?

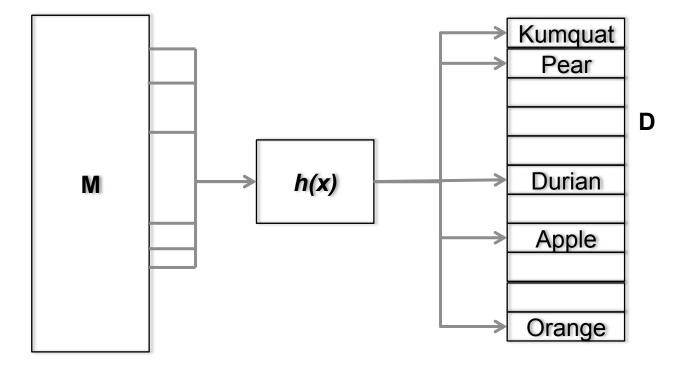


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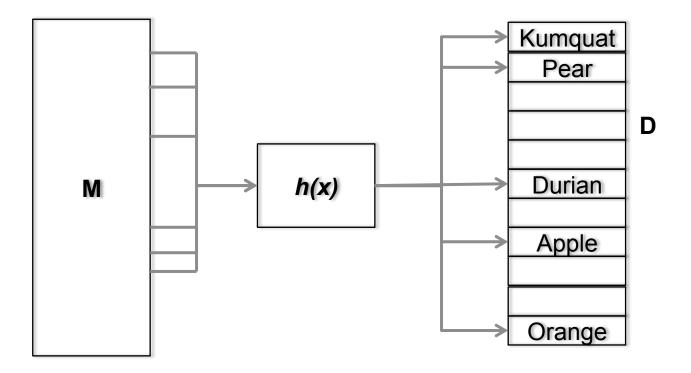
• Can't retrieve the random variable, O(D) search!



What about a pseudo-random mapping?



- What about a pseudo-random mapping?
 - This is "the hash function"



• The Hash Function maps the large space M to our target space D.

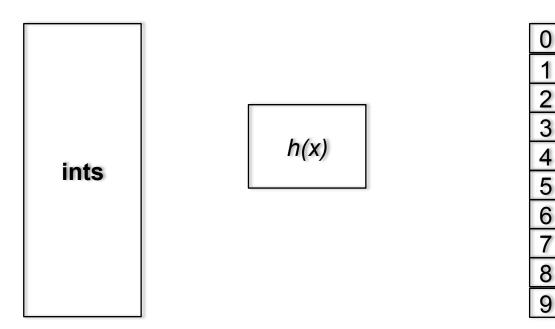
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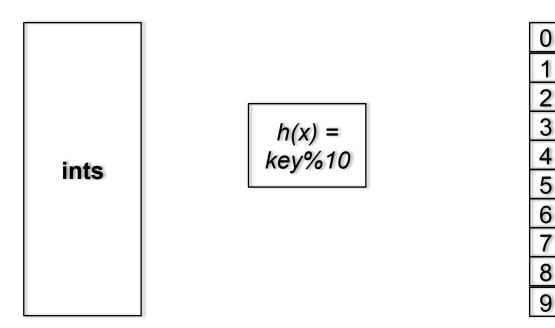
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- We want our hash function to do the following:
 - Be repeatable: H(x) = H(x) every run
 - Be equally distributed: For all y,z in D,
 P(H(y)) = P(H(z))
 - Run in constant time: H(x) = O(1)

 Let's consider an example. We want to save 10 numbers from all possible Java ints

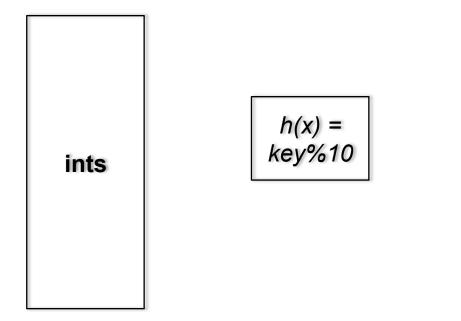
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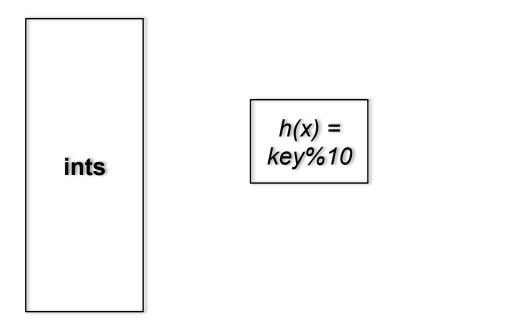


• Let's insert(519) table



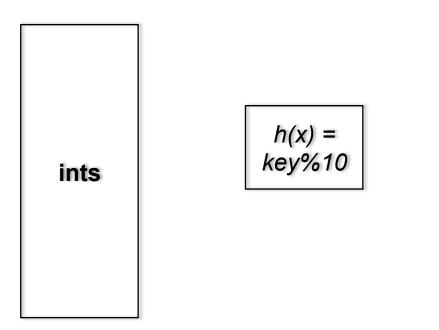
0	
1	
2	
2 3 4 5 6	
4	
5	
6	
7	
8	
9	

- Let's insert(519) table
 - Where does it go?



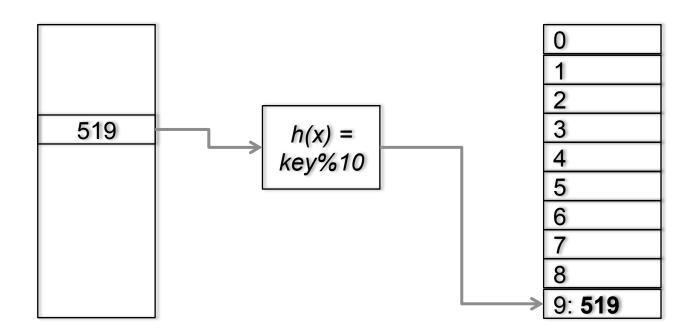
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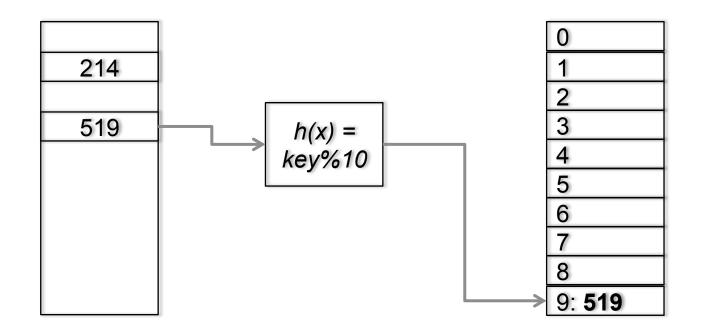


0
1
2
3
4
5
6
7
2 3 4 5 6 7 8 9
9

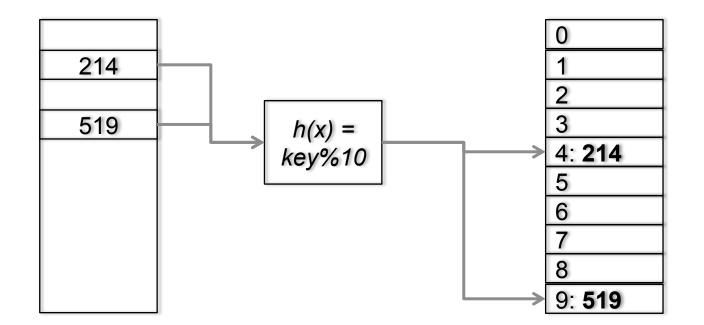
- Let's insert(519) table
 - Where does it go?
 - 519%10 = 9



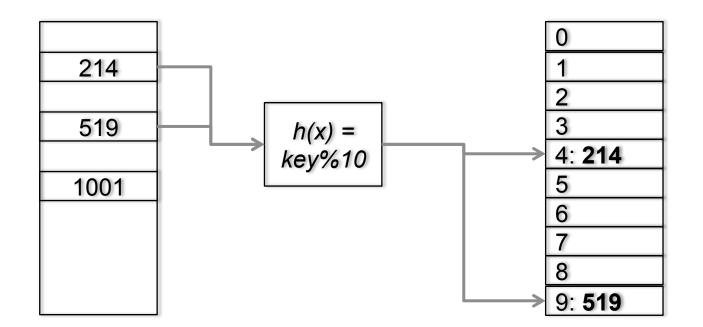
• Insert(214)



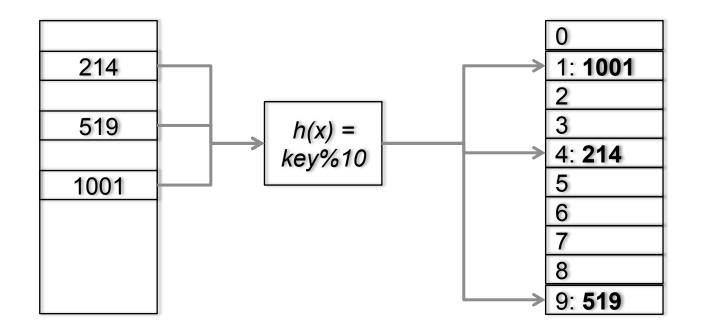
• Insert(214)



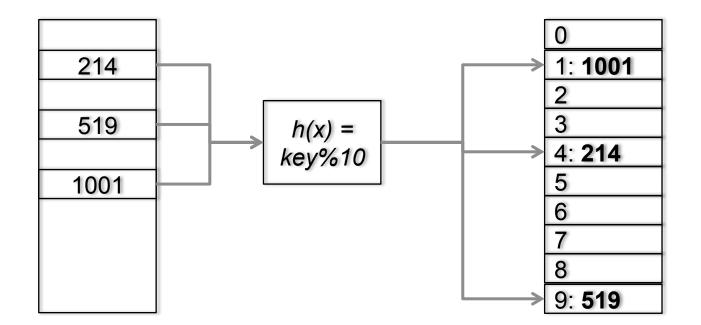
• insert(1001)



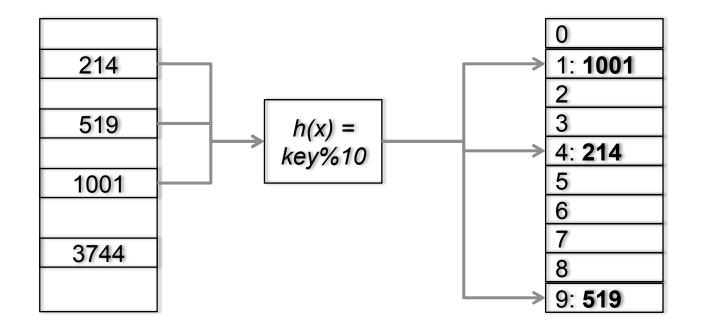
• insert(1001)



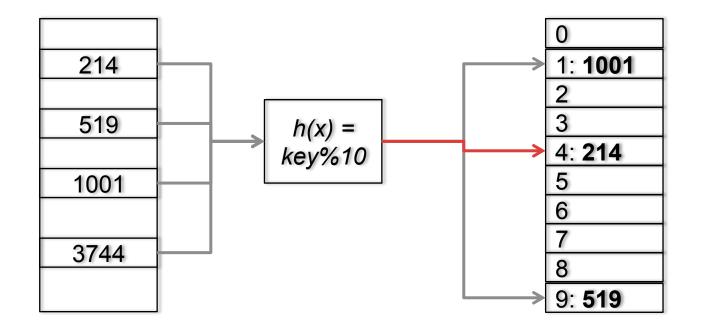
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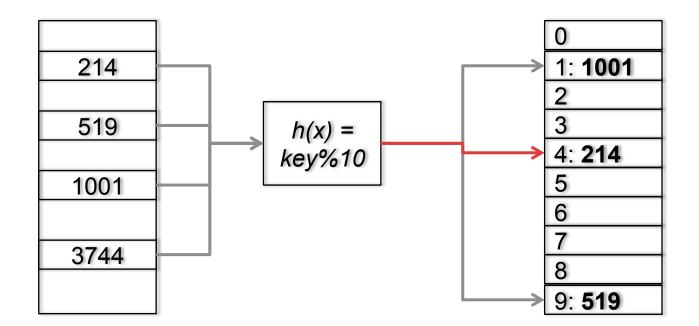
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- Is there a problem here?
 - insert(3744)
 - This is called a collision!



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 - We want a hash that distributes our data evenly throughout the space
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 - Needs to incorporate all the data in the keys

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- These hash functions are chosen in advance, you should not pick a hash function relative to your data

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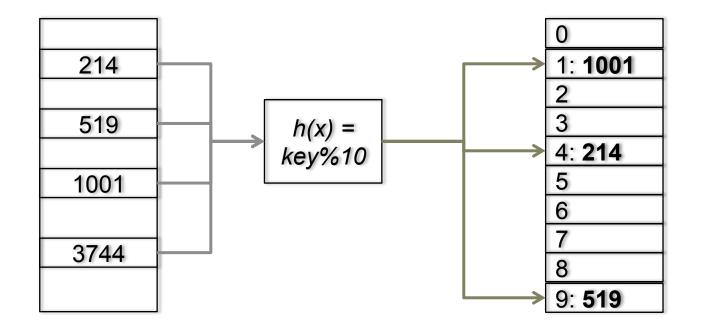
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 - Store both in the same space
 - Try a different hash
 - Resize the array

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 - This is called **clustering**

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- What are the negative effects of clustering?
 - If the cluster becomes too large, two things happen:
 - The chances of colliding with the cluster increase
 - The time it takes to find something in the cluster increases. This isn't O(1) time!

• How can we solve this problem?

• Resize the array

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 - We'll discuss it later



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 - Certain tables can cause secondary clustering
 - Can fail to insert if the table is over half full

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 - If two keys collide in the hash table, then a secondary hash indicates the probing size
 - Need to be careful, possible for infinite loops with a very empty array

Chaining

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- Some data structure is necessary here
- Commonly a linked list, AVL tree or secondary hash table.
- Resizing isn't necessary, but if you don't, you will get O(n) runtime.

Array sizes

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- Why?

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- This is because for any number we pick, so long as it is not a multiple of our table size, they must be coprime
- Two numbers x and y are **coprime** if they do not share any common factors.
- If the hash table size and the secondary hash value are coprime, then the search will succeed if there is space available
- However, many primes cause secondary clustering when used with quadratic probing

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 - We've discussed this a bit implicitly before
 - What are good load-factor (λ) values for each of our collision techniques?

- Linear Probing?
- Quadratic Probing?
- Secondary Hashing?
- Chaining?

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 - Access times?

- Linear Probing? $0.25 < \lambda < 0.5$
- Quadratic Probing?
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- **Quadratic Probing?** $0.10 < \lambda < 0.30$
- Secondary Hashing?
- Chaining?

- Linear Probing? $0.25 < \lambda < 0.5$
- Quadratic Probing? $0.10 < \lambda < 0.30$
 - If it gets to 0.5, then there is a chance of failure, and a high chance of O(n) runtime
- Secondary Hashing?
- Chaining?

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 - But we've eliminated primary clustering
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- **Chaining?** $3.0 < \lambda < 10$

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 - Because we allow multiple items in each space, we can increase memory efficiency by taking advantage
 - As long as there are a constant number in each space, we get O(1) runtimes.

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 - Here, these resizes are often for performance, rather than failure.
 - Hash table maintenance is important
 - Resizing is costly (but still O(n)) because you have to resize the array and rehash every element into the new table.

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 - Can be memory inefficient
 - Probing can fail, and delete with probing mechanisms is difficult
 - Chaining can be a good balance, but there is a lot of overhead maintaining all those data structures

 Understand these tradeoffs and how these implementations work

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- Section tomorrow will provide practice problems for each of these hash table methods

Take-aways for the midterm

• Hashtables should provide O(1) dictionary operations

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- The array should be relative to the size of the data you want to keep
- The hash function should run in constant time and should distribute among the indices in the target array
- Linear probing is a solution for collisions, but only works when there is lots of free space
- Resizing is very costly

NEXT CLASS

- Hash Tables
 - Examples, examples, examples
 - Finish discussion
- Exam review