CSE 332

JULY 3RD – DICTIONARY ADT

TODAY'S SCHEDULE

- Dictionary ADT
- Binary Search Trees
- Height, Balance and the AVL property

DICTIONARY ADT

- New abstract data type

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  - Dictionary (aka Map)
  - Data – Keys and Values

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  - Example (Store inventory):

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  - Dictionary (aka Map)
  - Data – Key and Value pairs
    - Keys: must be comparable, used for lookup
    - Values: the actual data itself
  - Example (Store inventory):
    - Keys: IDs (barcodes)
    - Values: Product information

DICTIONARY ADT

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  - `insert(key, value):` inserts the key, value pair into the dictionary. Overwrites the value if the key is already in the dictionary
  - `find(key)`: returns the stored value for a particular key in the dictionary, returns null if not found.
  - `delete(key)`: removes the key and its corresponding value from the dictionary.

SET ADT

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- **Find, insert and delete have few differences**
- **Possible to implement other functions from sets**
    - Union, intersection, difference

APPLICATIONS

- **Store information in key, value pairs**
  - Very common usage pattern

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  - Very common usage pattern
    - Phone directories
    - Indexing
    - OS page tables
    - Databases

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- **Keys and Values should be stored together in some way**
  - Both objects in one node
  - Paired arrays (one stores keys and the other values)

IMPLEMENTATIONS

- Simple implementations

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- **Simple implementations**

	insert	find	delete
Unsorted linked-list			
Unsorted array			
Sorted linked list			
Sorted array			

IMPLEMENTATIONS

- **Simple implementations**

	`insert`	`find`	`delete`
Unsorted linked-list	*O(n)**	*O(n)*	*O(n)*
Unsorted array	*O(n)**	*O(n)*	*O(n)*
Sorted linked list	*O(n)*	*O(n)*	*O(n)*
Sorted array	*O(n)*	*O(log n)*	*O(n)*

* Because we need to check for duplicates

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    - All elements less than the root are in the left subtree and all elements larger than the root are in the right subtree
    - All, subtrees must also be binary search trees
  - With this property, all binary search trees have sorted in-order traversals

IMPLEMENTATIONS

- **Other implementations?**
  - Binary Search Tree (BST)
  - Sort based on keys (which have to be comparable)
  - How do we implement this?
  - What changes need to be made?

IMPLEMENTATIONS

- **BST Node:**
  - Before:

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- **BST Node:**
  - Before:
    - Node left
    - Node right
    - Value data

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  - Value is attached to the key
  - **Dictionary fact: All values have an associated key**
  - **For now, assume all keys are unique, i.e. each key only has one value**

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  - What is our time for the three functions?

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    - *Consider best and worst-case.*
    - *What are the inputs for best and worst-case?*

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      - O(height)

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    - Height = 1 + max(height(left),height(right))

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    - Generally, however: O(log n) when the tree is balanced

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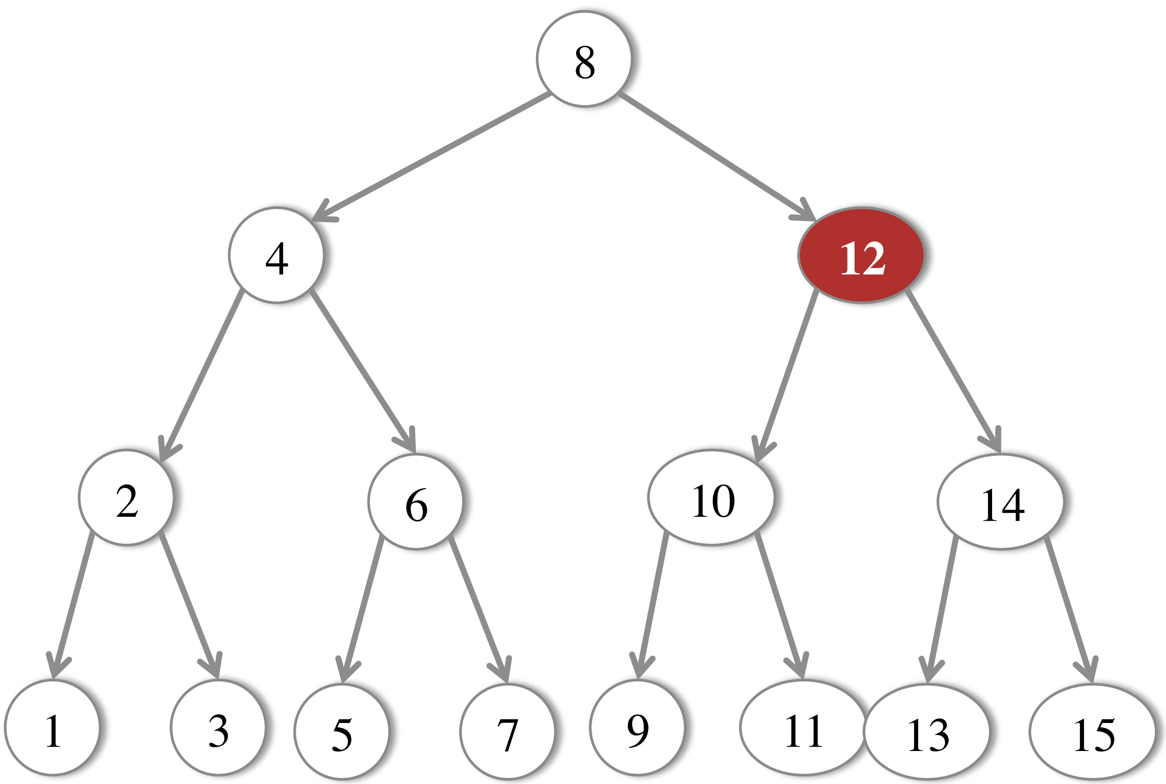
    - Case 3: The node has two children
      - *What are some possible strategies?*

IMPLEMENTATIONS

- **Deleting nodes with 2 children**
  - *How do we delete 12?*



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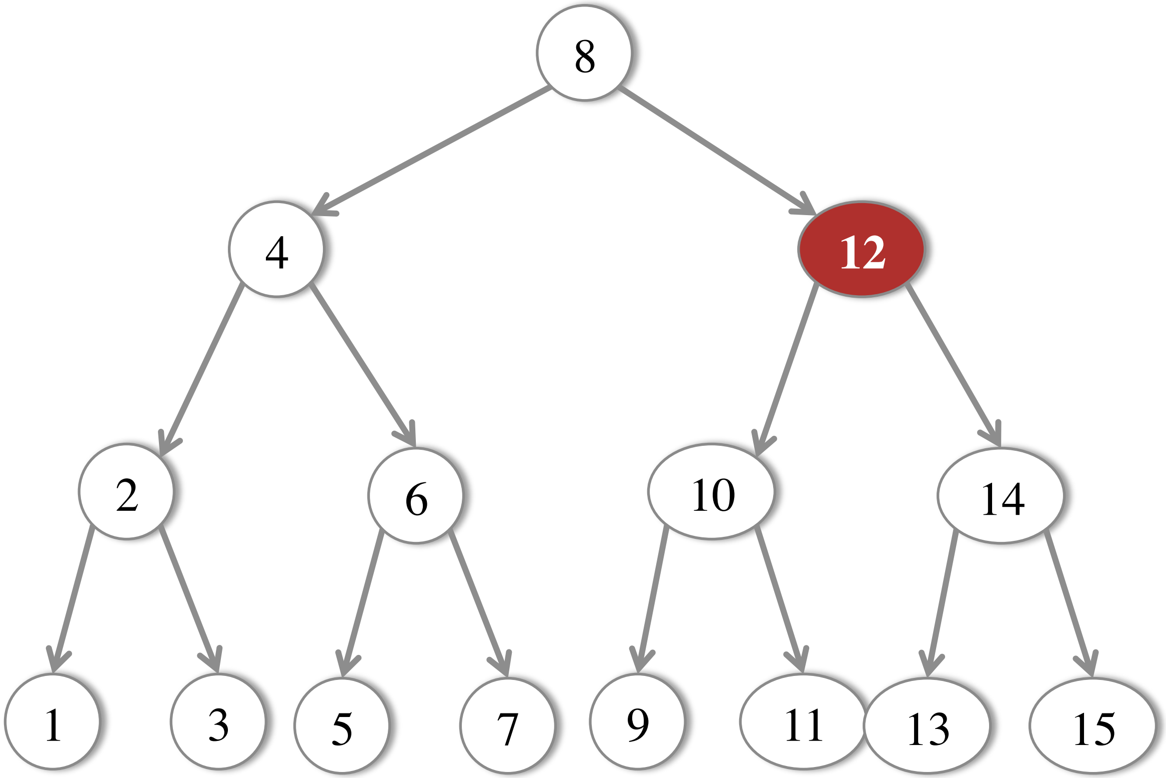

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- **Deleting nodes with 2 children**
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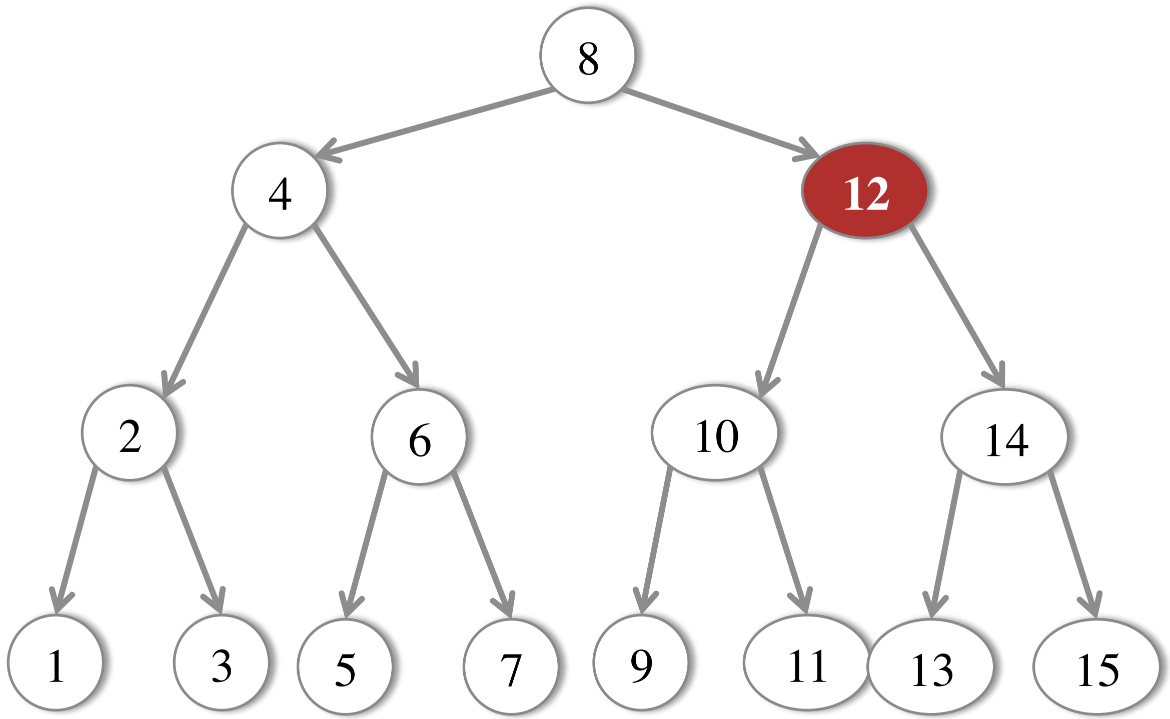
  - *Can we replace 12 with one of it's children?*
  - *Need to find candidate to replace 12*



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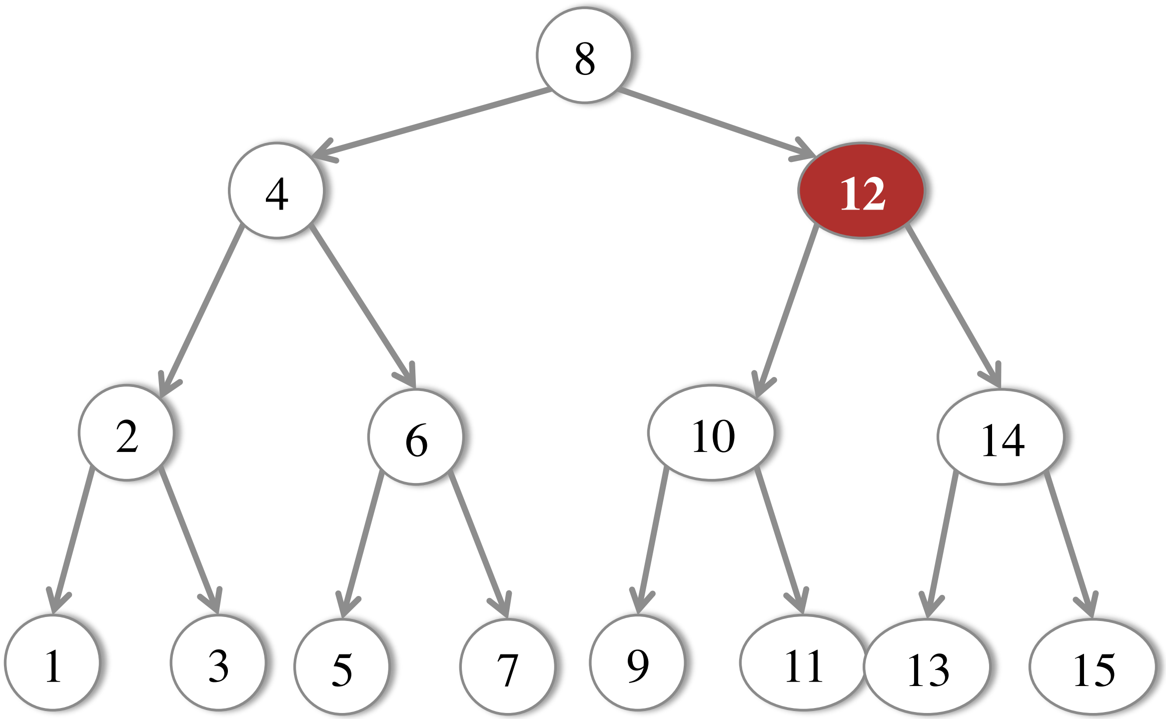
  - If a node has 2 children, then we can "delete" it by over writing the node with a different <key, value> pair
  - In order to avoid changing the shape and doing too much work, it must be either the predecessor (the element just before it in sorted order) or the successor (the element just after it in sorted order)

IMPLEMENTATIONS

- **Deleting nodes with 2 children**
  - *What are the predecessor and successor of 12?*



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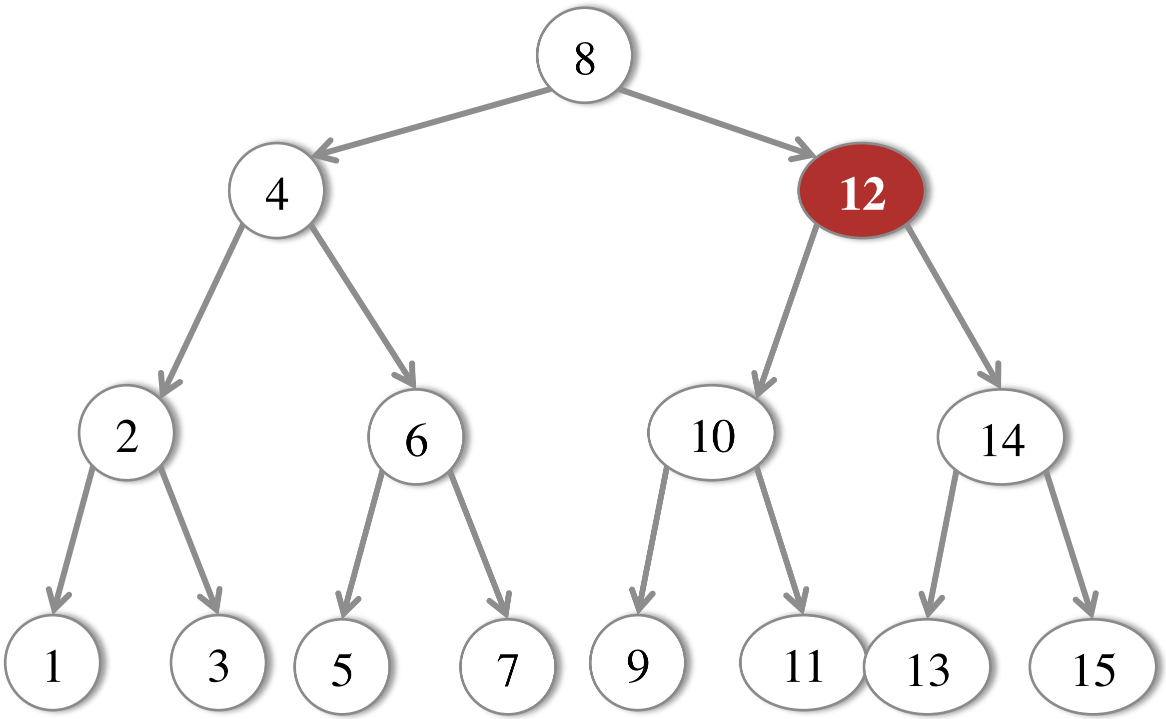

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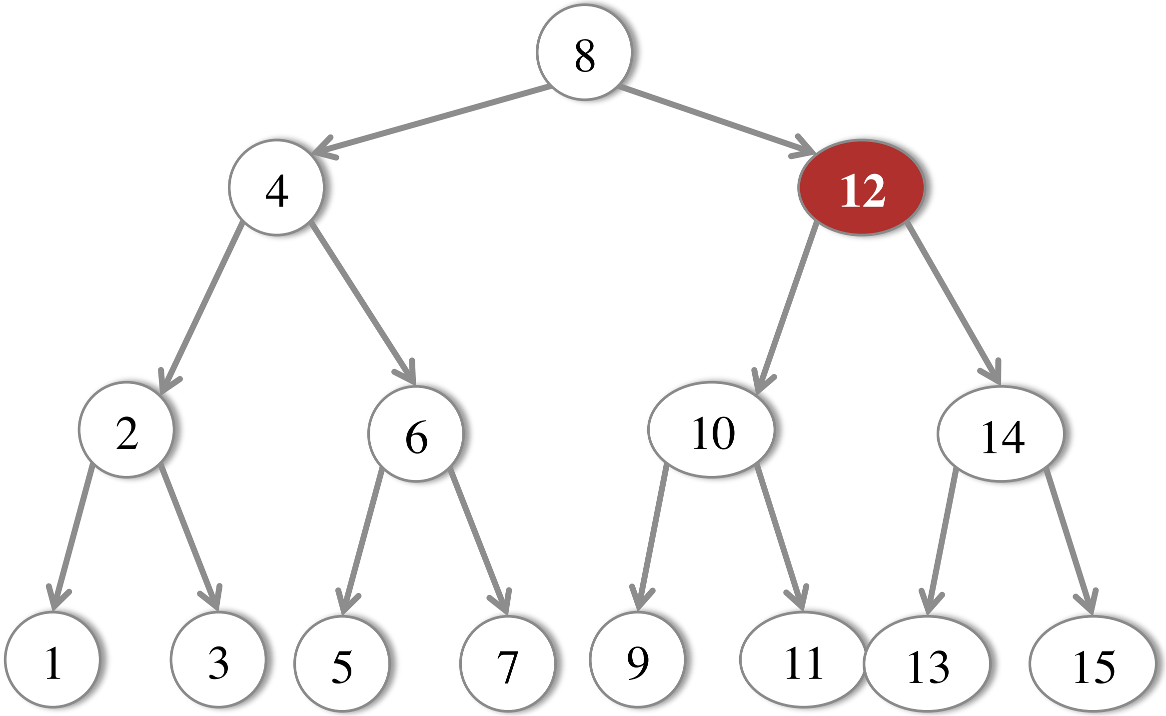

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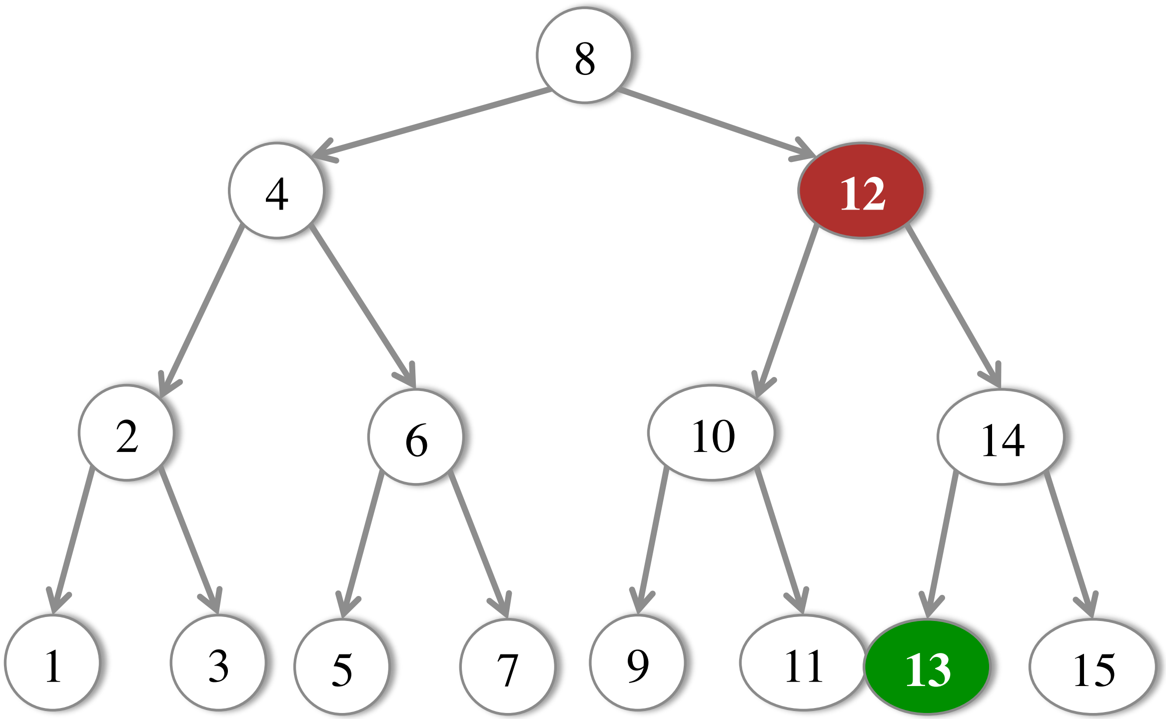

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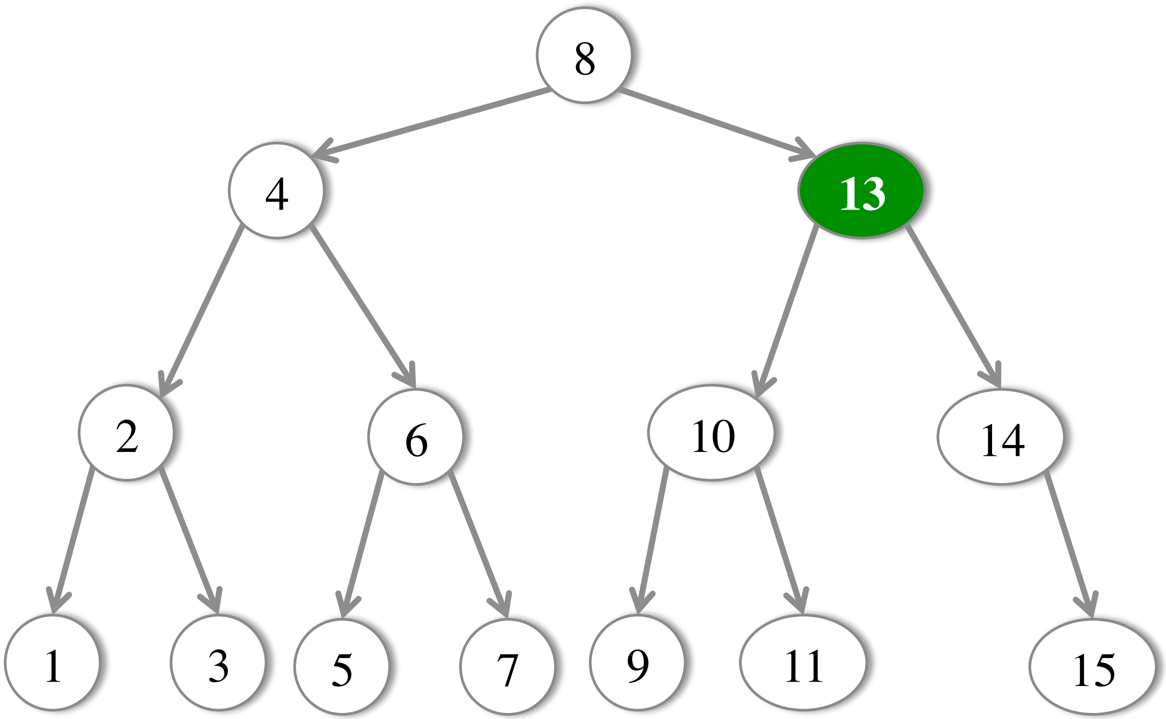

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  - Delete():
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    - Worst case(): O(n), finding the predecessor/successor takes time. *What is this case?*
    - Best case(): O(1) if we're deleting the root from a degenerate tree
      - "Degenerate" trees are those that are very unbalanced.

ANALYSIS

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  - Can we enforce this balance?
  - What are some possible balance conditions?
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    - Height of the left = height on the right?

ANALYSIS

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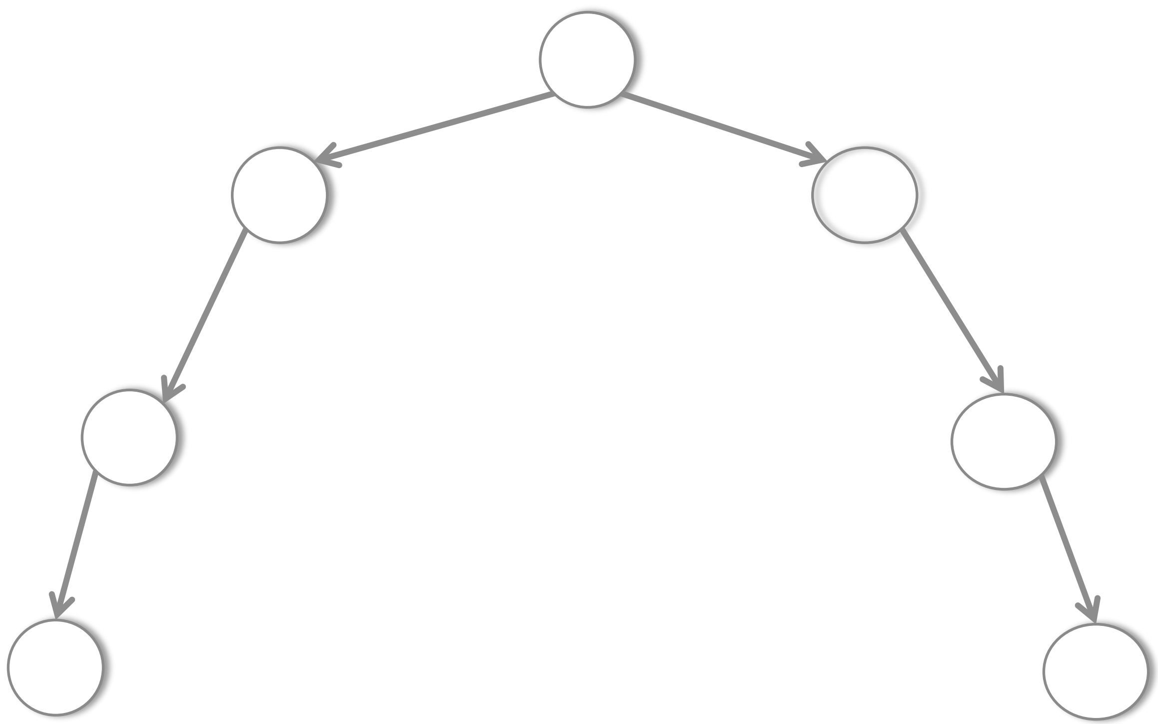
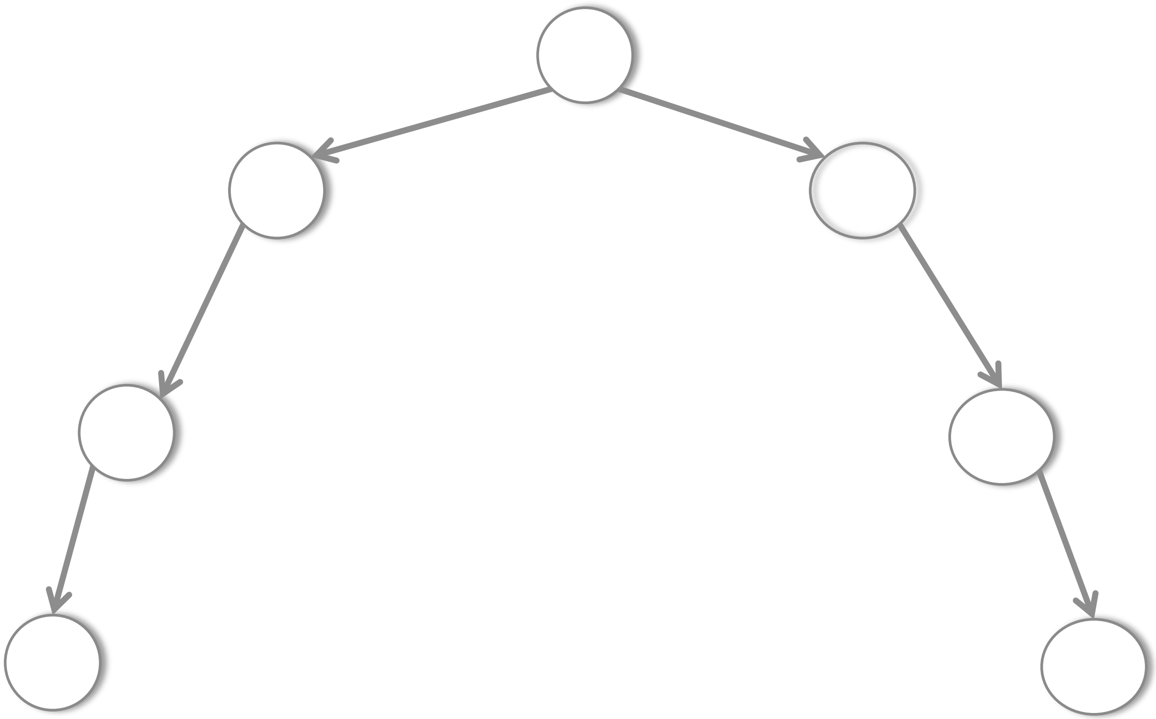


ANALYSIS

- **This doesn't help much**
  - Need the definition to be recursive
  - Let height(left) = height(right) for all nodes



ANALYSIS

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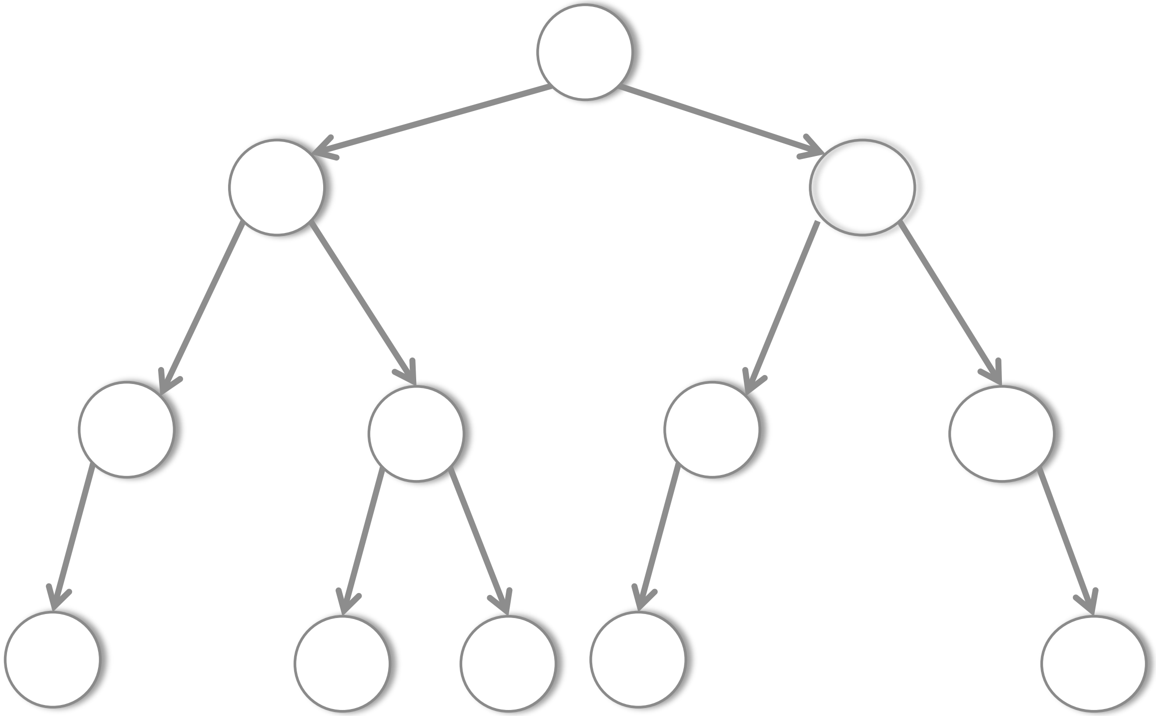
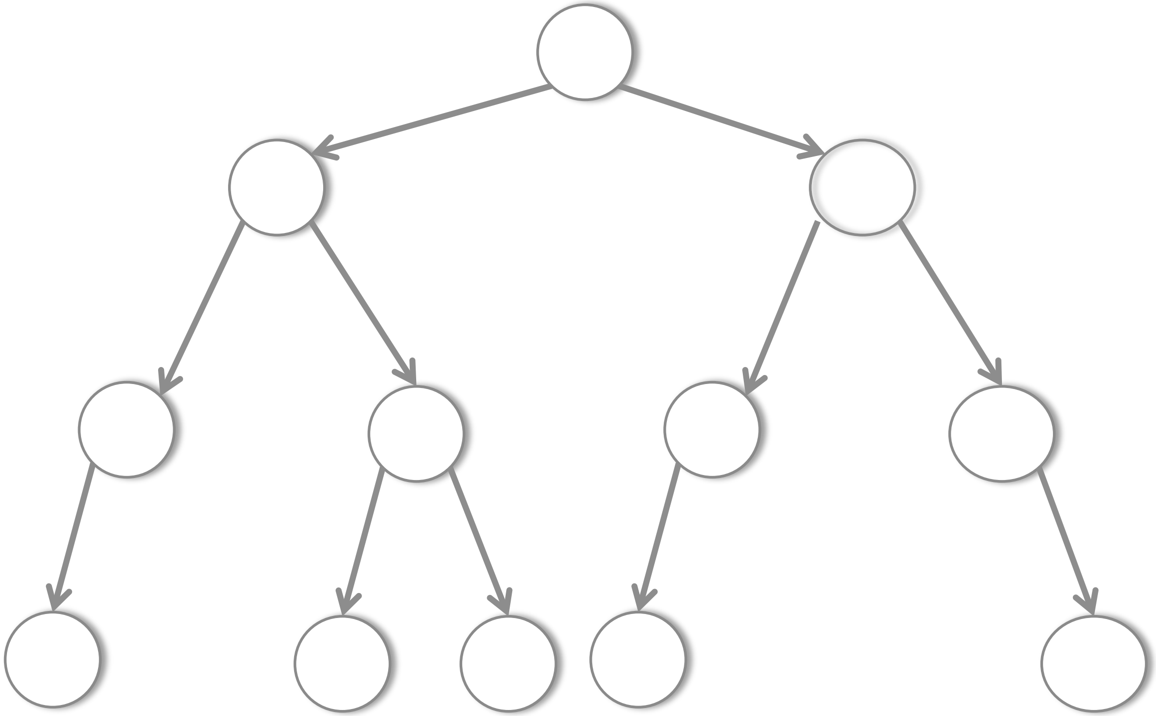


ANALYSIS

- **Now what's wrong?**
  - Only perfect trees (with 2^k children) can exist



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- For each node in the tree, the height of its left and right subtrees can differ by at most one
- $|(\text{height}(\text{left}) - \text{height}(\text{right})| \leq 1$
- This is the AVL property, and we can use it to create self balancing trees

NEXT CLASS

- AVL Trees and implementation