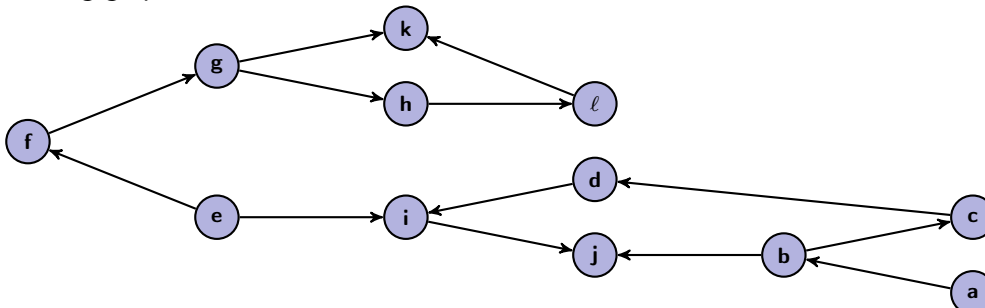


Section 9: Graphs

0. It Rhymes with Flopological Sort

Consider the following graph:



- (a) Does this graph have a topological sort? Explain why or why not. If you answered that it does not, remove the **MINIMUM** number of edges from the graph necessary for there to be a topological sort and carefully mark the edge(s) you are removing. Otherwise, just move on to the next part.

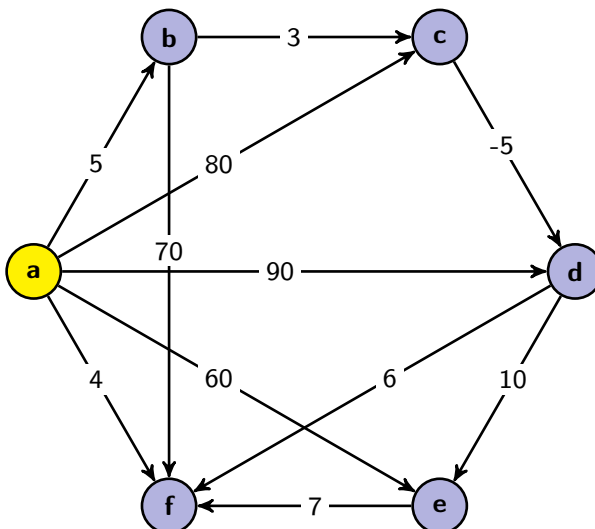
For the remaining parts, work with this (potentially) new version of the graph.

- (b) Find a topological sort of the graph. Do not bother showing intermediary work.
- (c) If this graph represented various tasks in a ForkJoin algorithm, what would the work of the algorithm be assuming each individual task is $\Theta(1)$.

Hint: There are 12 tasks...

1. Better Find the Shortest Path Before It Catches You!

Consider the following graph:

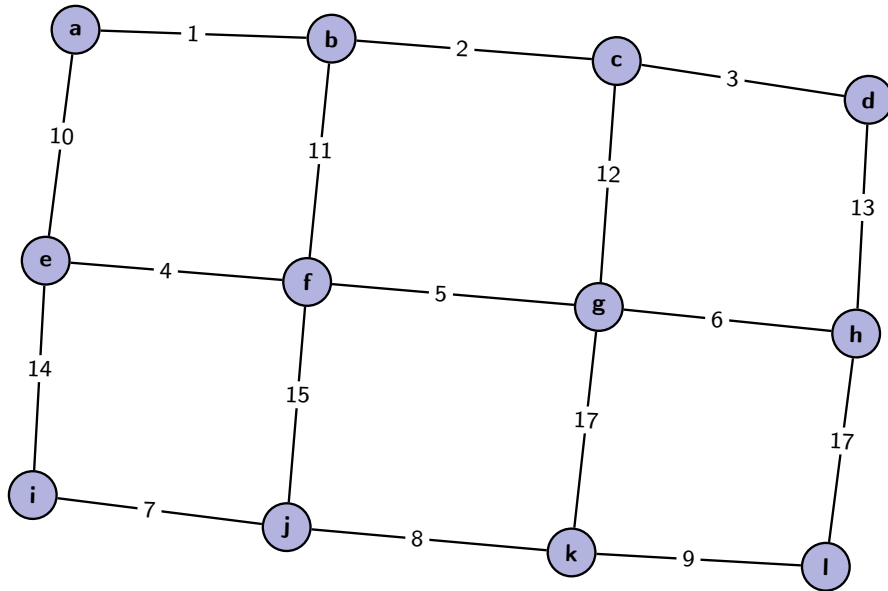


- (a) Use Dijkstra's Algorithm to find the **lengths** of the shortest paths from **a** to each of the other vertices. For full credit, you must show the worklist at every step, but how you show it is up to you.
- (b) Are any of the lengths you computed using Dijkstra's Algorithm in part (a) incorrect? Why or why not?

(c) Explain how you would use Dijkstra's Algorithm to recover the actual paths (rather than just the lengths).

2. LMNST!

Consider the following graph:



- (a) Find an MST of this graph using both of the two algorithms we've discussed in lecture. Make sure you say which algorithm you're using and show all iterations of the worklist.
- (b) Using just the graph, how can you determine if *it's possible* that there are multiple MSTs of the graph? Does this graph have multiple MSTs?
- (c) What is the asymptotic runtime of the algorithms that you used to compute the MSTs?