Section 7: Parallel Prefix_Sum

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Problem: Scan

Scan

Suppose we have an array `a`:

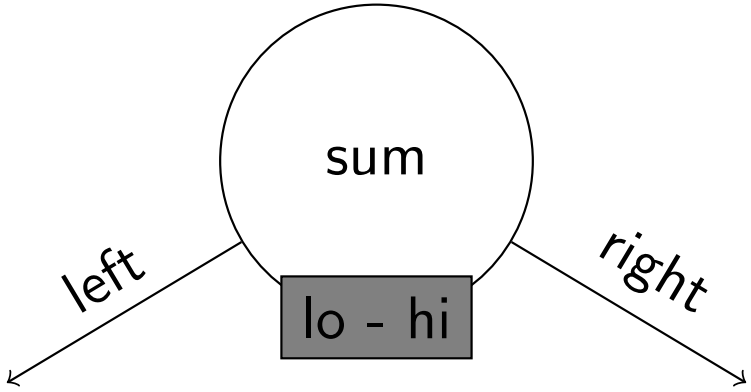
0	1	2	3	4	5
`a[0]`	`a[1]`	`a[2]`	`a[3]`	`a[4]`	`a[5]`

Return an array `b` where `b[i] = a[0] + a[1] + ... + a[i]`

0	1	3	6	10	15
`b[0]`	`b[1]`	`b[2]`	`b[3]`	`b[4]`	`b[5]`

We solve this in parallel by building a tree!

PST: Prefix Sum Tree

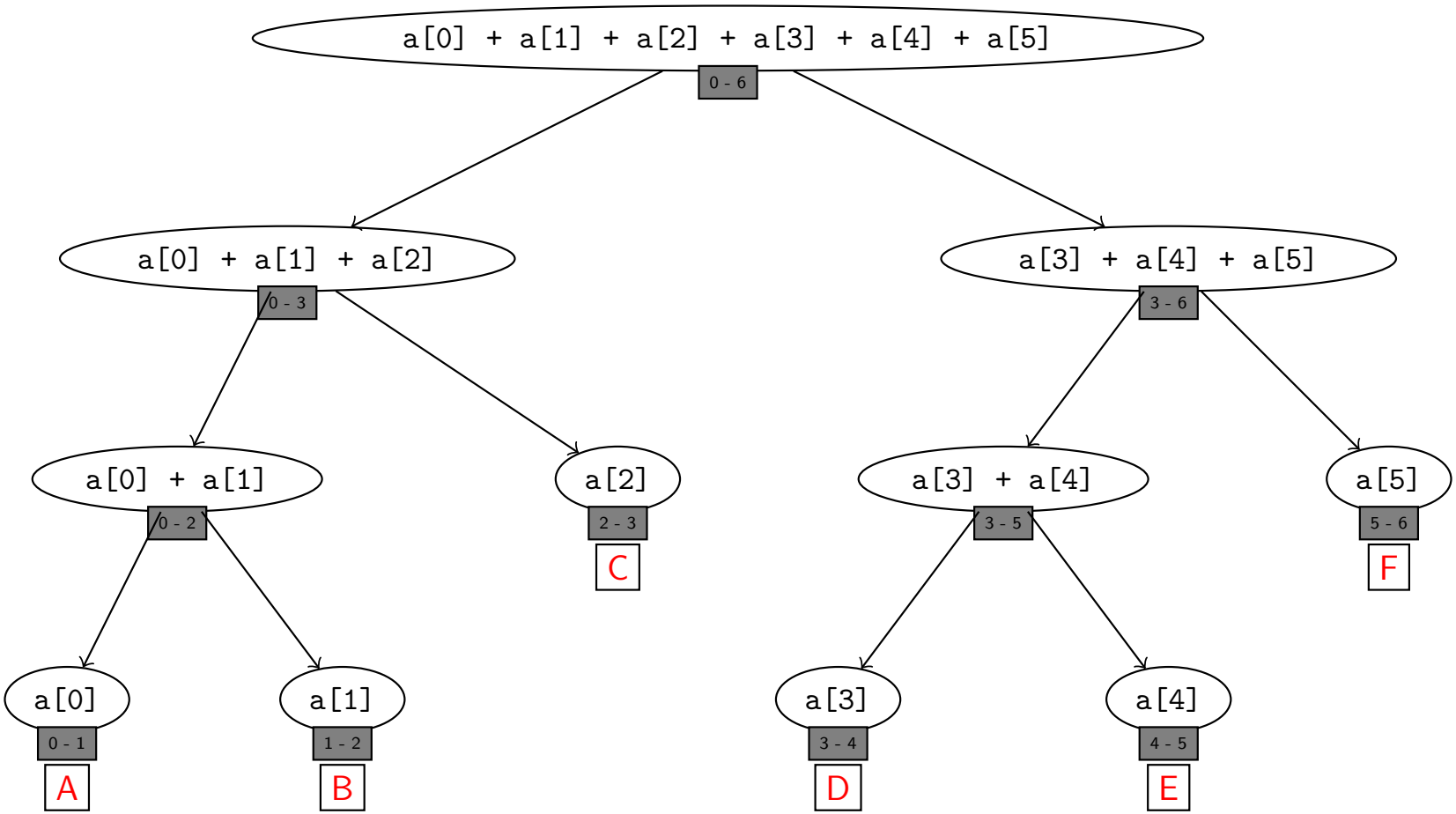


PSTNode

```
PSTNode {
  int lo; // include
  int hi; // exclude
  // sum(arr[lo], arr[lo + 1], ..., arr[hi - 1])
  int sum;
  PSTNode left, right;
}
```

Build the tree (parse input)

```
PSTNode buildTree(int[] arr, int lo, int hi) {
  if (hi - lo == 1) {
    // sum is simply arr[lo]
    return new PSTNode(lo, hi, arr[lo])
  } else {
    mid = lo + (hi - lo) / 2;
    PSTNode left = buildTree(arr, lo, mid);
    PSTNode right = buildTree(arr, mid, hi);
    return new PSTNode(lo, hi, left.sum + right.sum,
        left, right);
  }
}
```


a[0] + a[1] + a[2] + a[3] + a[4] + a[5]
0 - 6
a[0] + a[1] + a[2]
0 - 3
a[3] + a[4] + a[5]
3 - 6
a[0] + a[1]
0 - 2
a[2]
2 - 3
C
a[3] + a[4]
3 - 5
a[5]
5 - 6
F
a[0]
0 - 1
A
a[1]
1 - 2
B
a[3]
3 - 4
D
a[4]
4 - 5
E

Build output

We want one output per leaf!

- A: I'm good!
- B: I need `a[0]`
- C: I need `a[0] + a[1]`
- D: I need `a[0] + a[1] + a[2]`
- E: I need `a[0] + a[1] + a[2] + a[3]`
- F: I need `a[0] + a[1] + a[2] + a[3] + a[4]`

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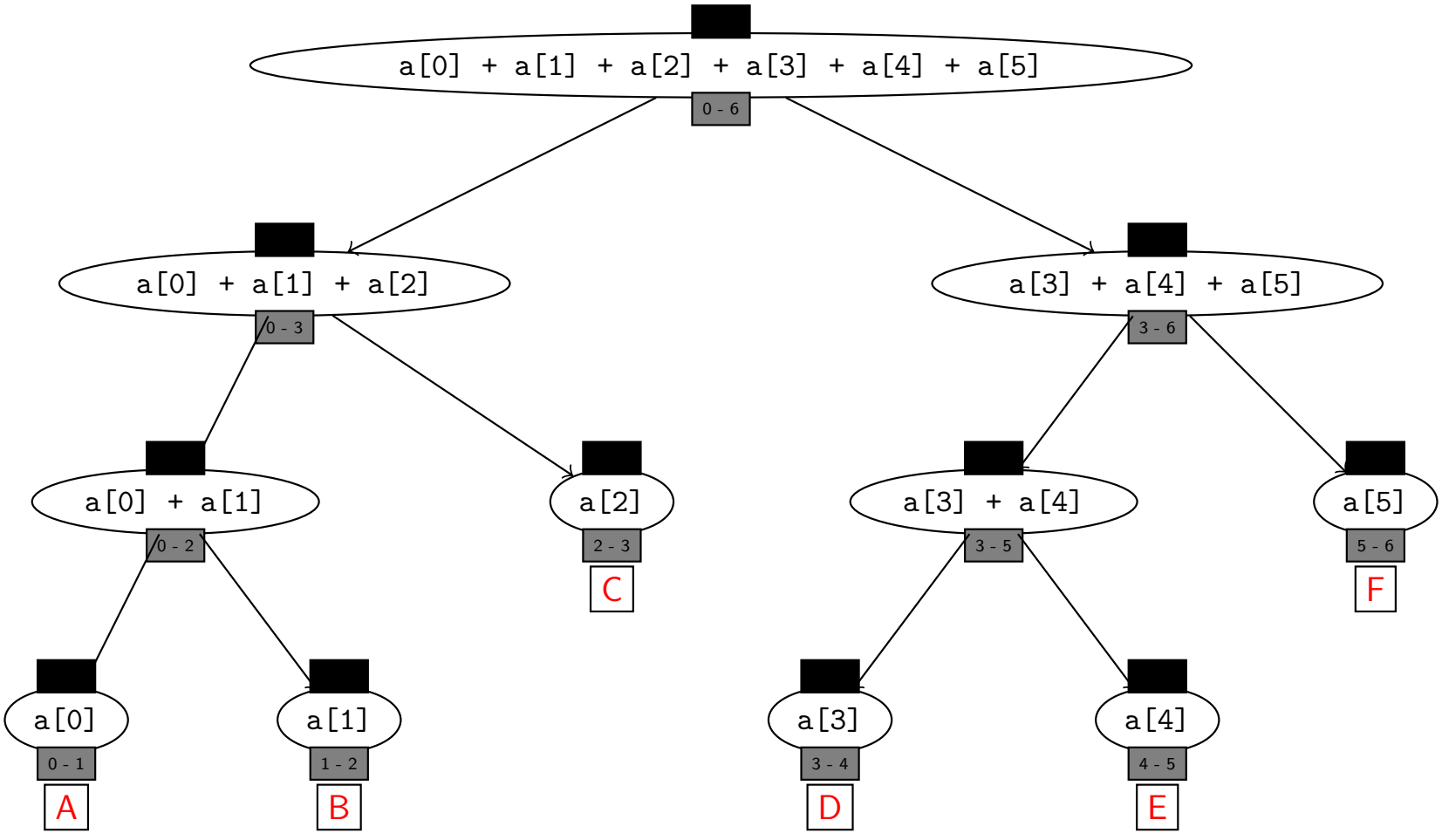
This is why we need FromLeft

How do we store FromLeft?

- For left node, inherit from parent node
- For right node, inherit parent node + your left neighbor

How do we store FromLeft?



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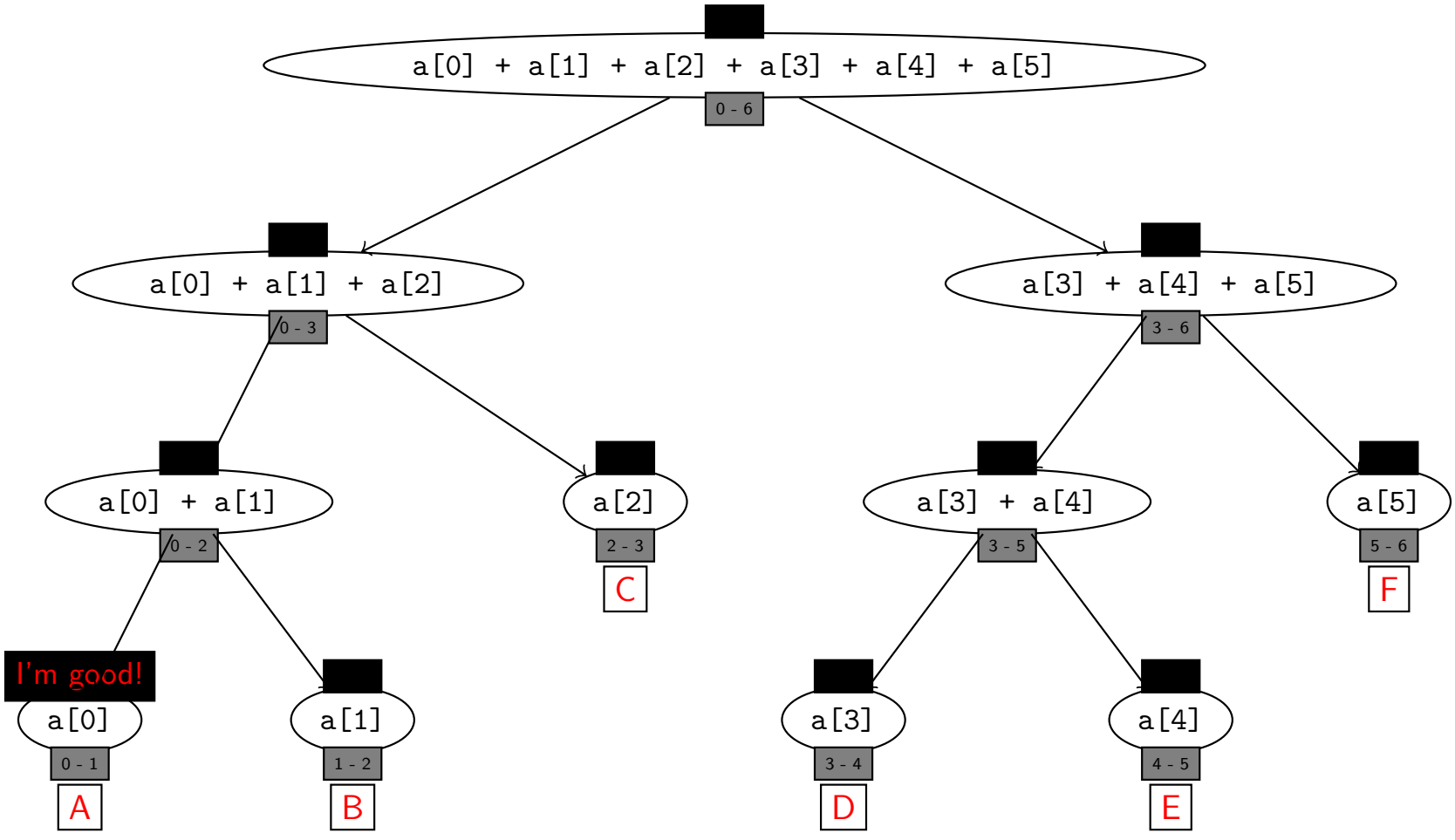

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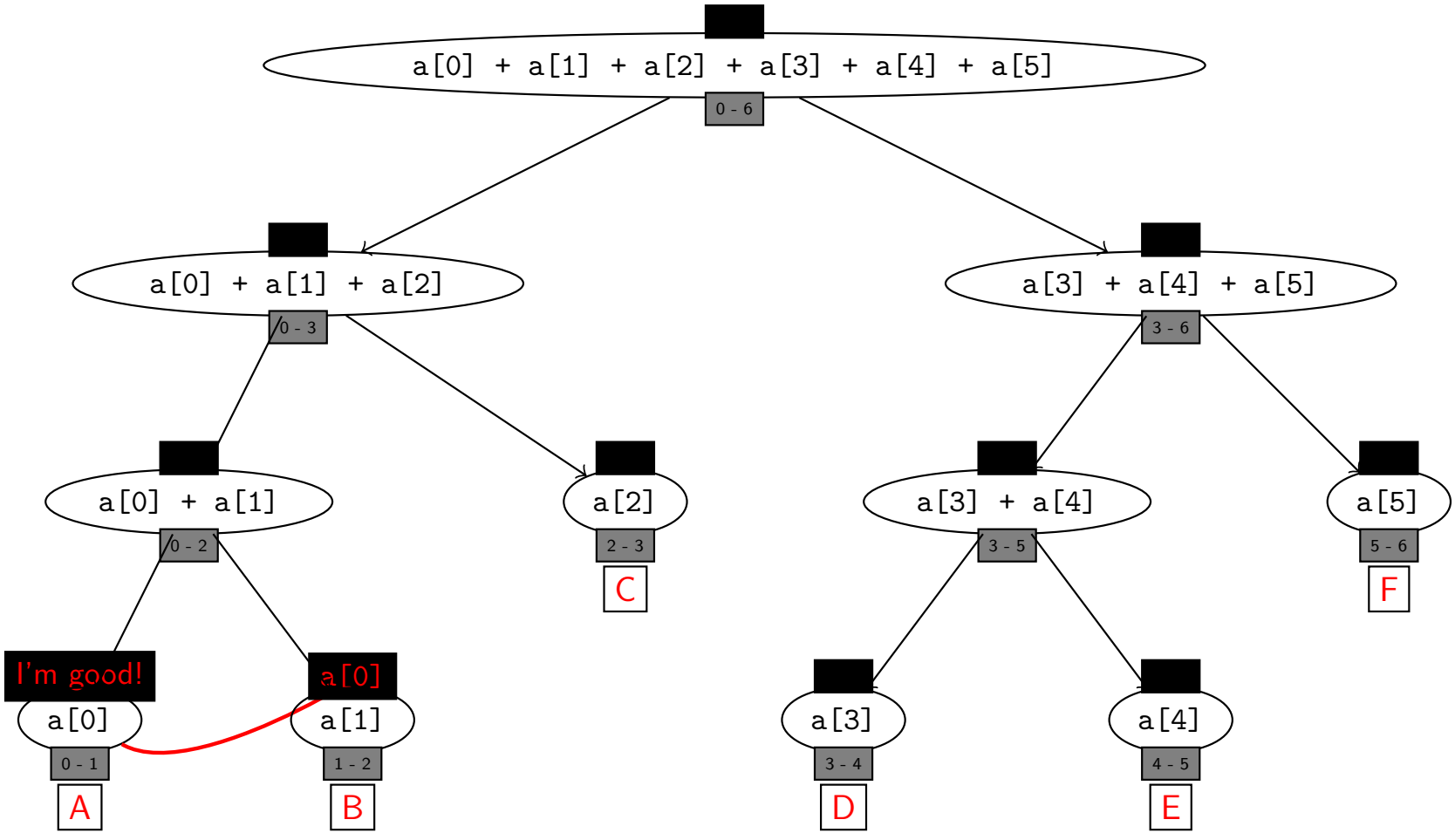
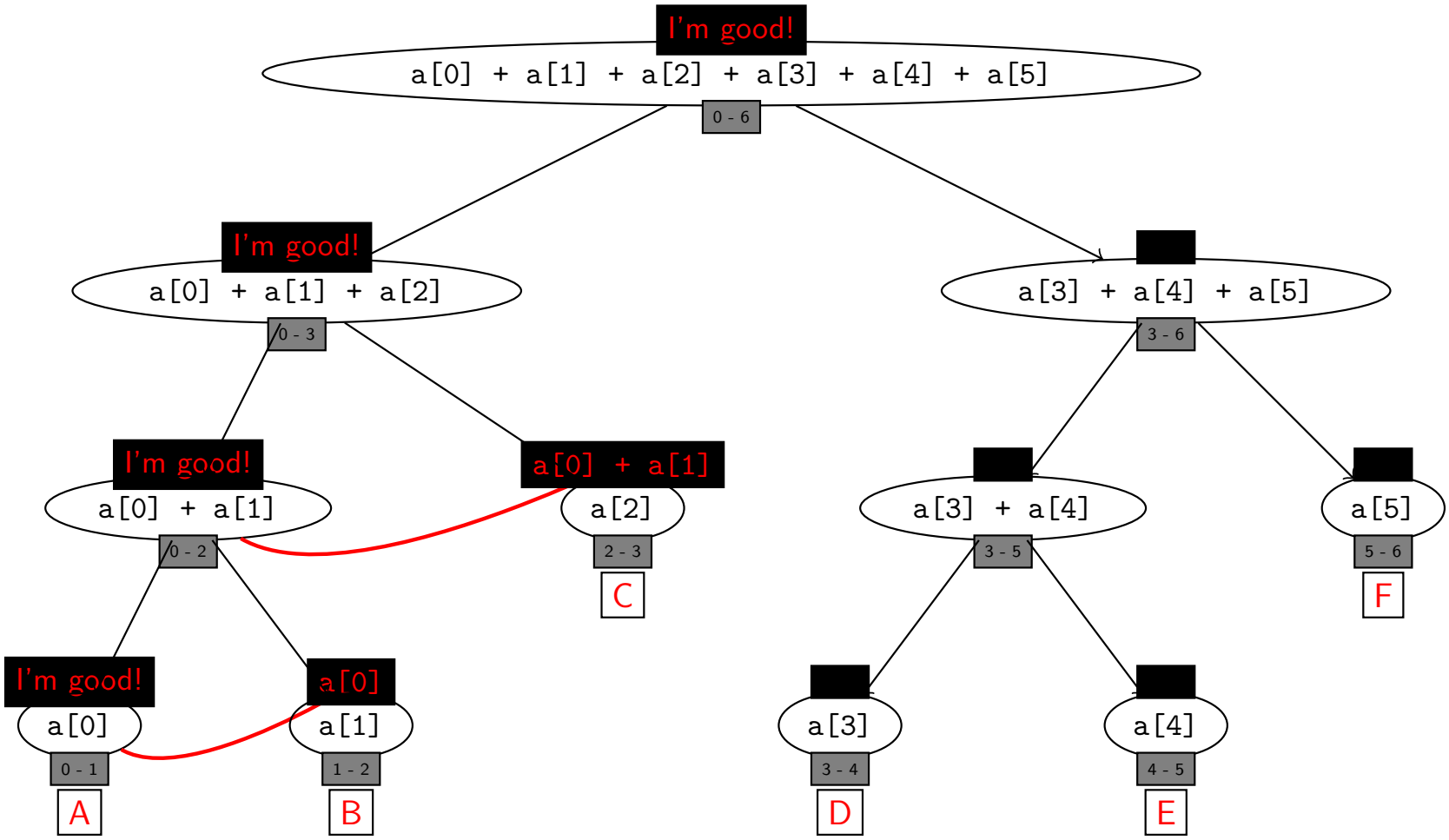


How do we store FromLeft?



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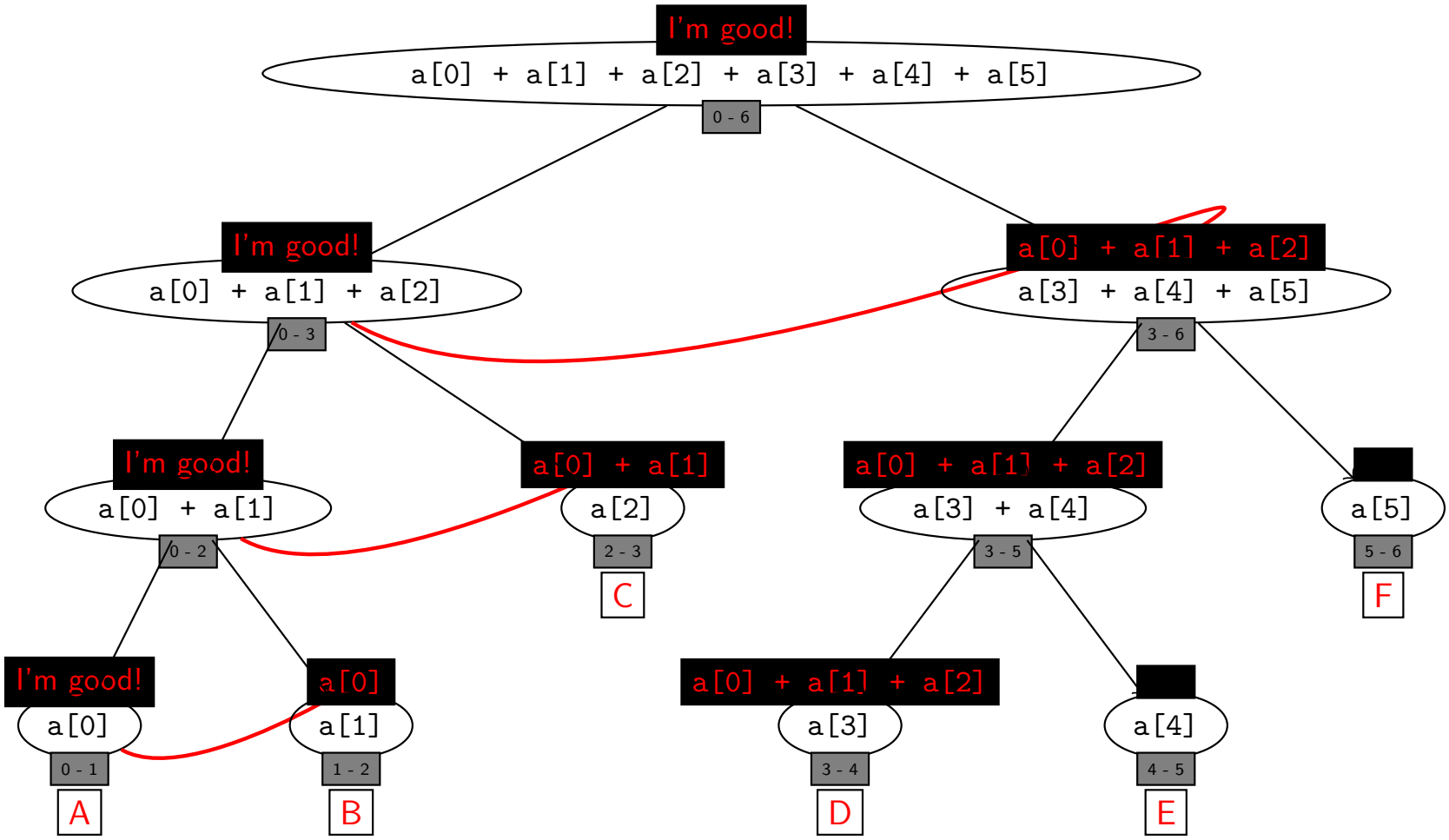

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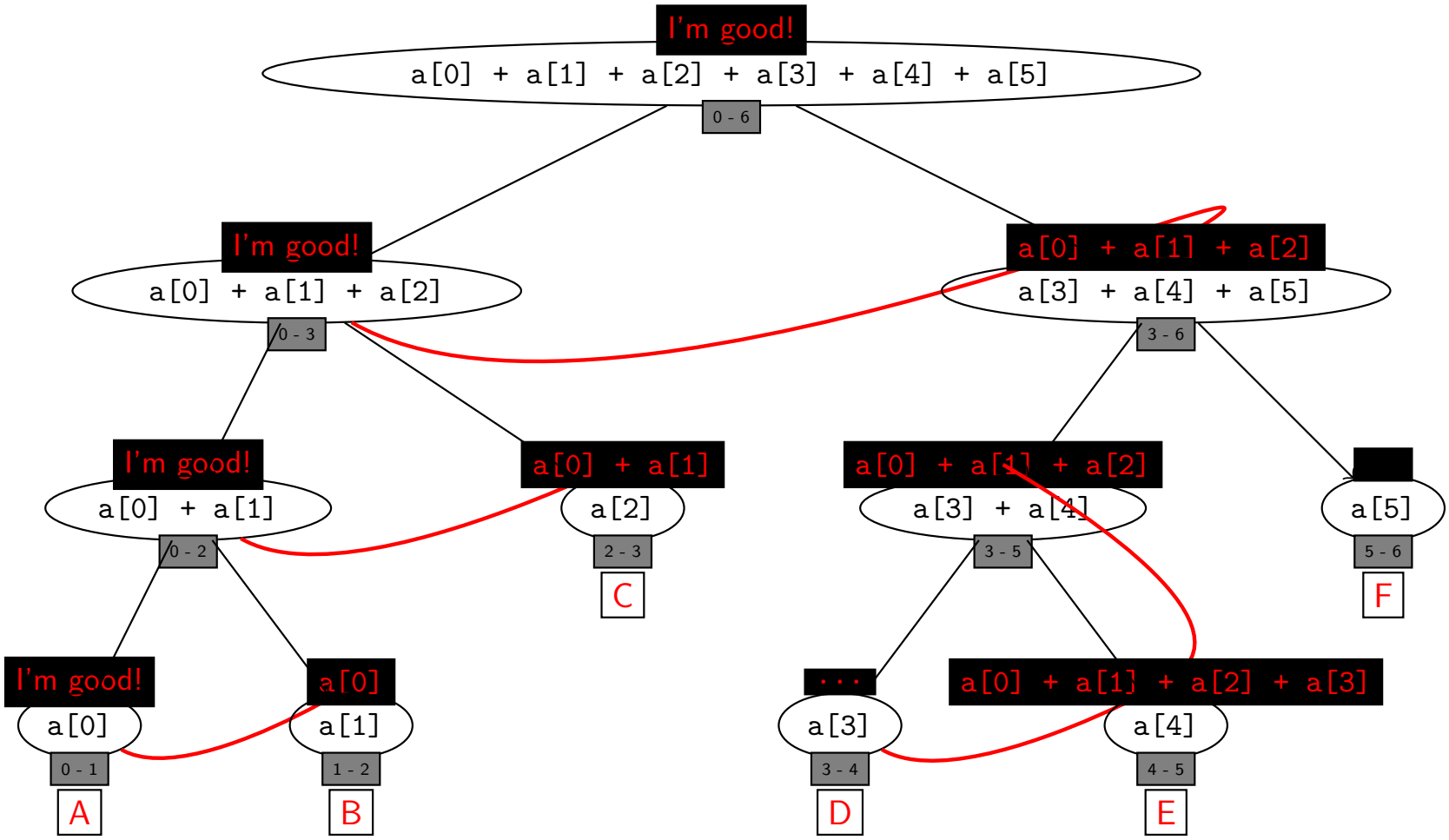

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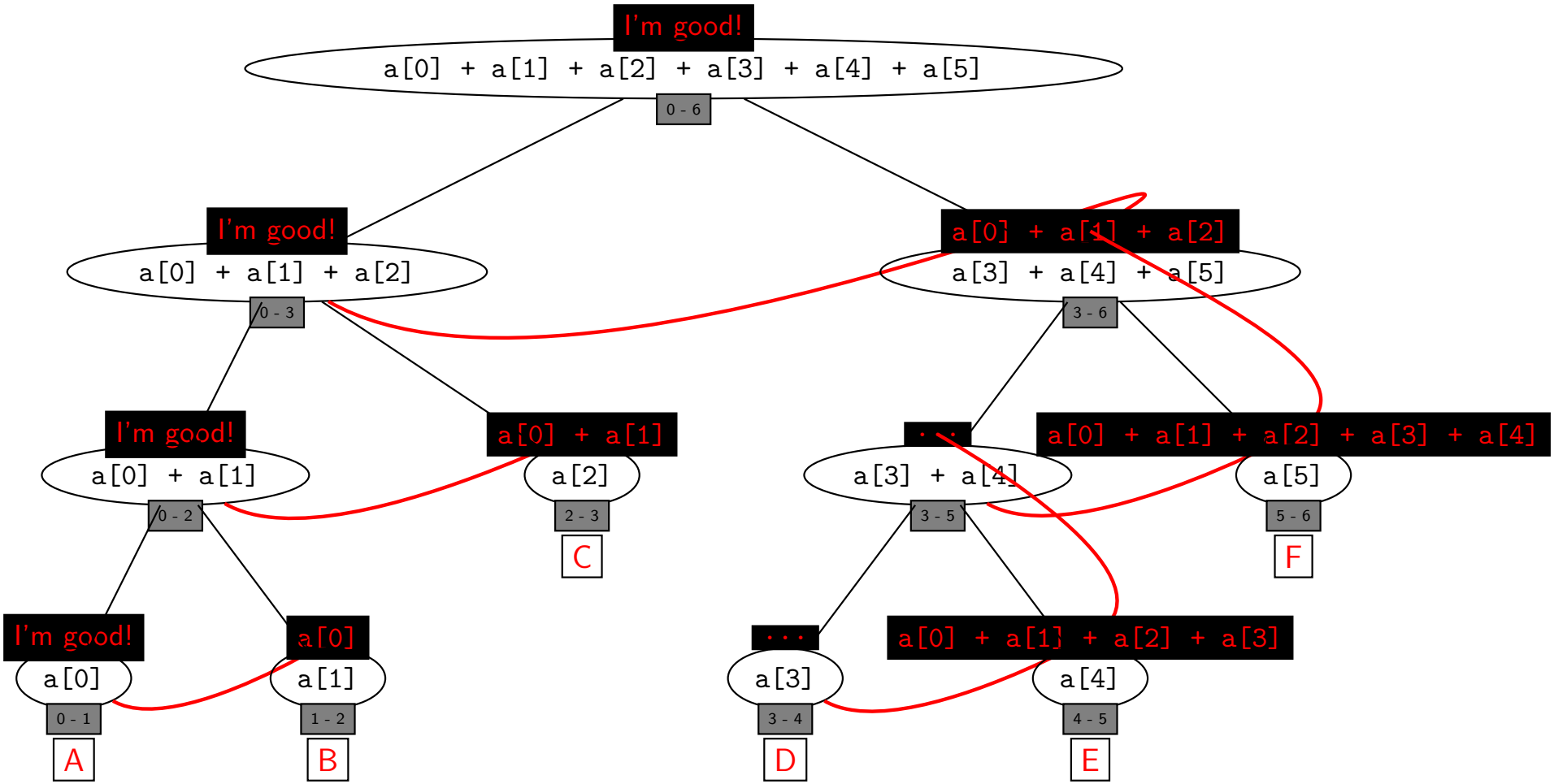


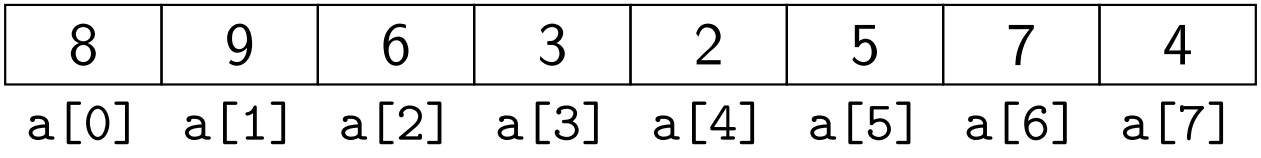
Now that we are done...

For every leaf node, PSTNode.sum + FromLeft is the result!

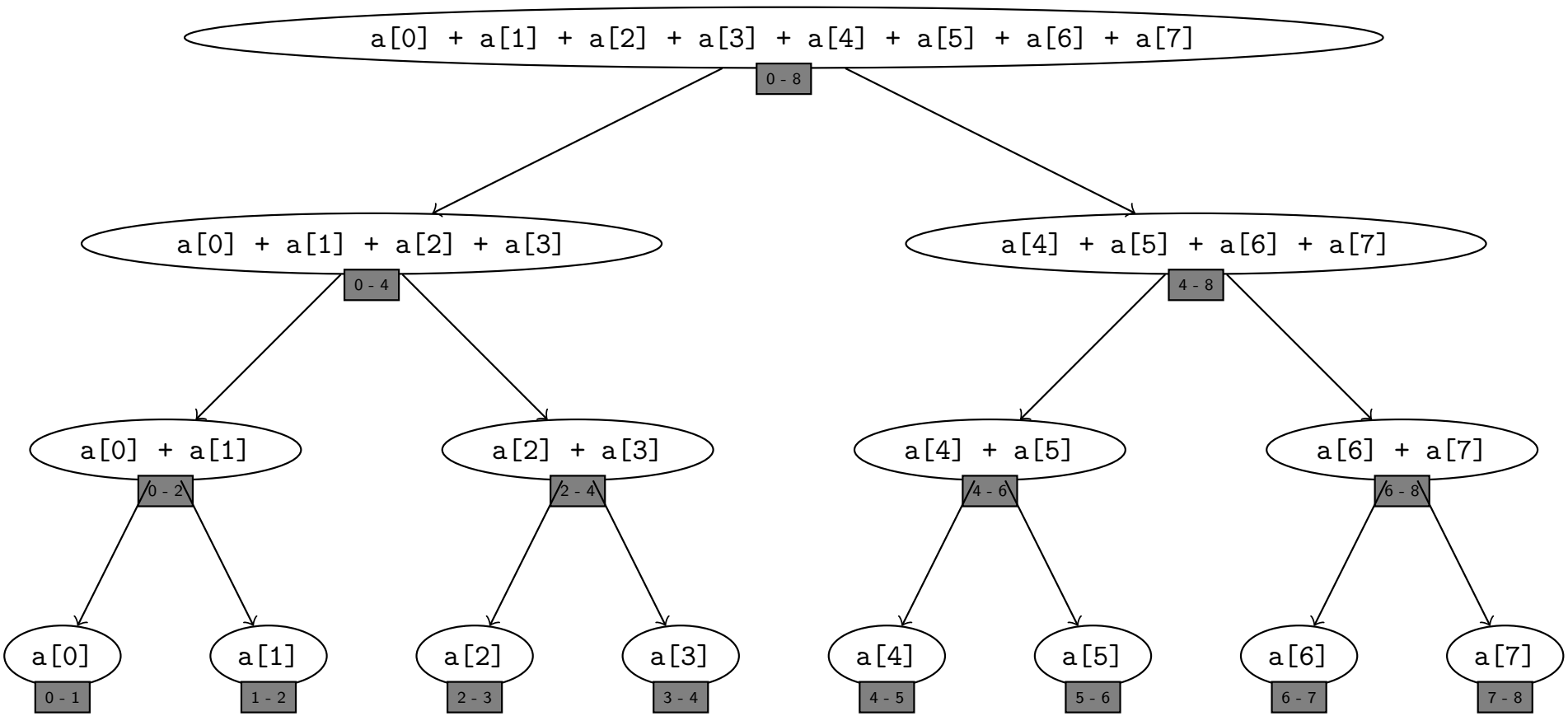
Code here, if you are interested

```
// our output
int[] output;
// prefix sum
void prefix_sum(int[] output, PSTNode curr_node, int
    fromLeft) {
  if (curr_node.left == null && curr_node.right ==
      null) { // leaf
    output[curr_node.lo] = fromLeft + curr_node.sum;
  } else {
    prefix_sum(output, curr_node.left, fromLeft);
    prefix_sum(output, curr_node.right, fromLeft +
        curr_node.left.sum);
  }
}
```

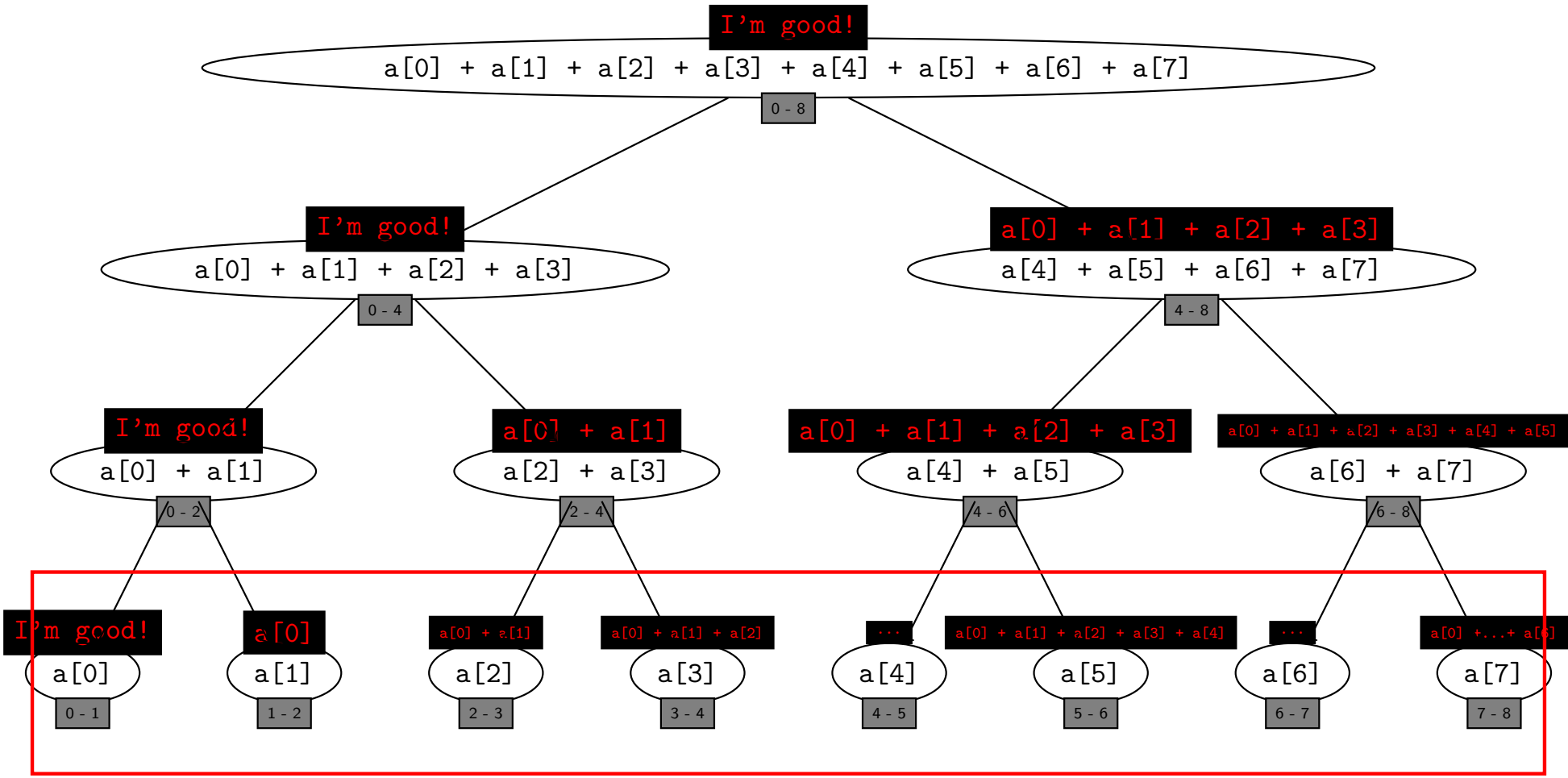
Handout solution 1: prefix sum



Step1: Build Tree



Step 2: Build FromLeft



Handout solution 2: prefix FindMin

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`lo, hi, min, left, right`

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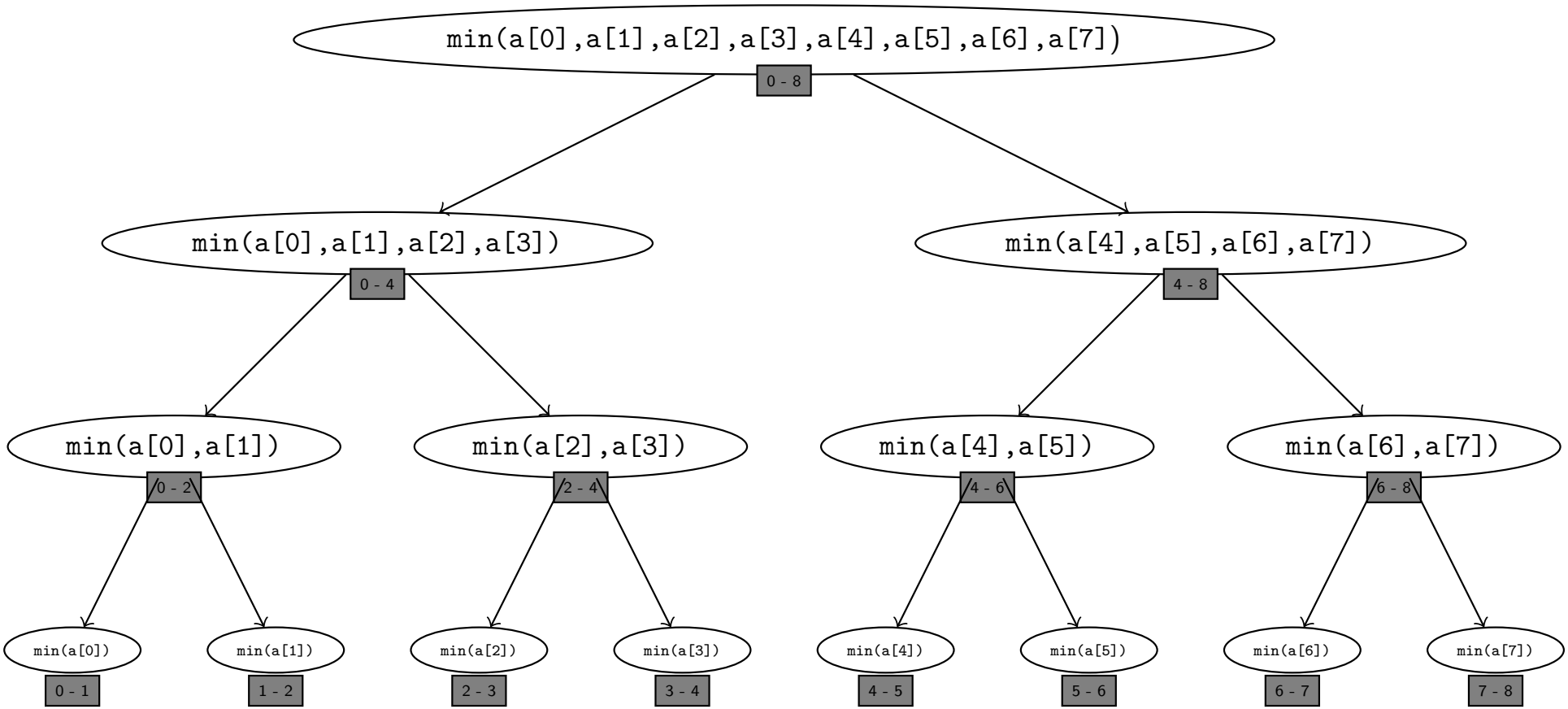
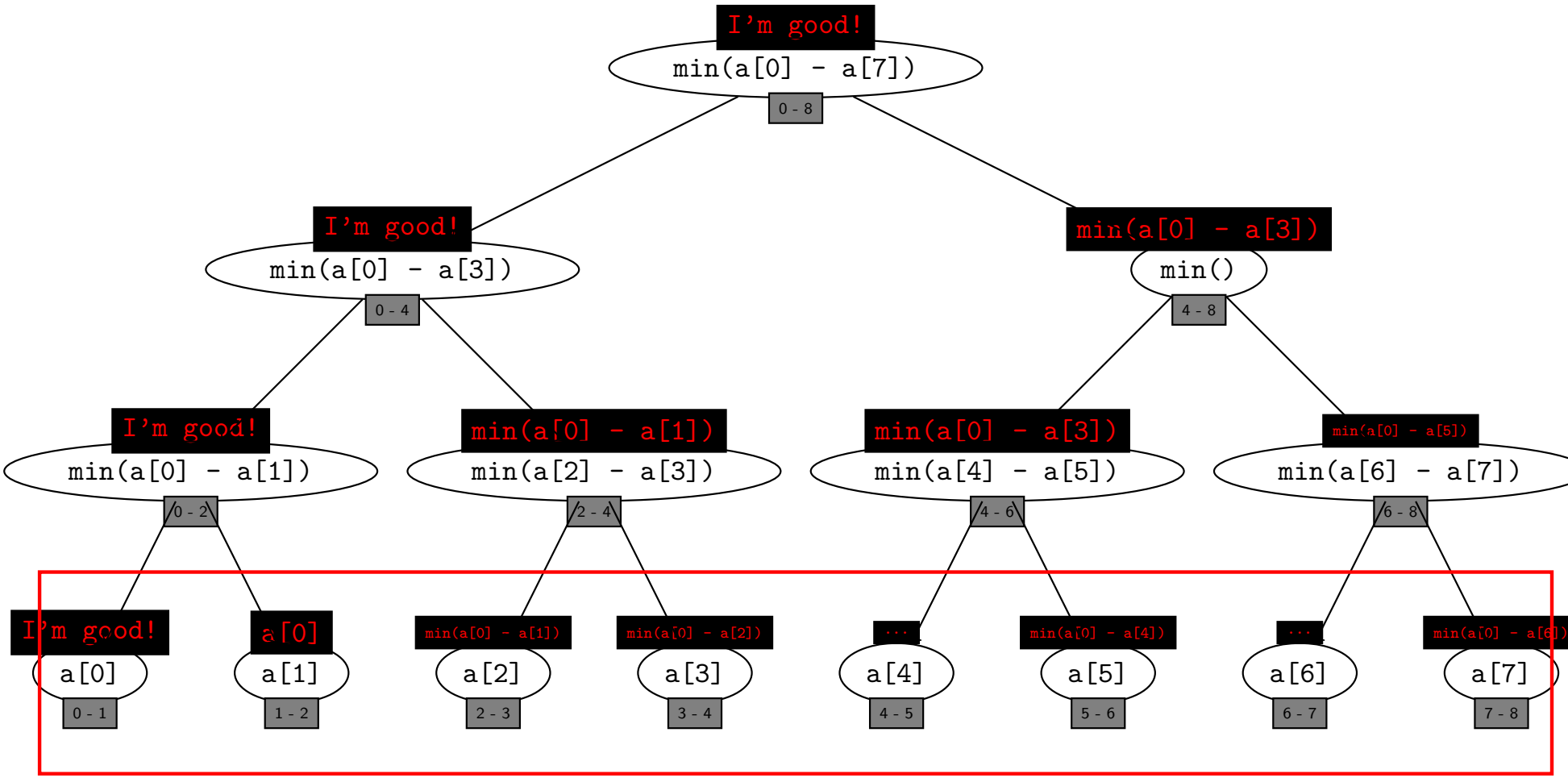
How should buildTree work now?

```
PSTNode buildTree(int[] arr, int lo, int hi) {
  if (hi - lo == 1) {
    // sum is simply arr[lo]
    return new PSTNode(lo, hi, arr[lo])
  } else {
    mid = lo + (hi - lo) / 2;
    PSTNode left = buildTree(arr, lo, mid);
    PSTNode right = buildTree(arr, mid, hi);
    // changed here!
    return new PSTNode(lo, hi, min(left.min, right.min),
        left, right);
  }
}
```

Step 1: Build Tree



Step 2: Build FromLeft



Finally, new code for findMin

```
// our output
int[] output;
// findMin
void findMin(int[] output, PSTNode curr_node, int
    fromLeft) {
  if (curr_node.left == null && curr_node.right ==
      null) { // leaf
    output[curr_node.lo] = min(fromLeft, curr_node.min);
  } else {
    findMin(output, curr_node.left, fromLeft);
    findMin(output, curr_node.right,
        min(fromLeft, curr_node.left.min));
  }
}
```