# CSE 332: Data Structures & Parallelism

# Lecture 3: Priority Queues

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# *Today*

- Finish up Asymptotic Analysis
- New ADT! Priority Queues

# *Scenario*

What is the difference between waiting for service at a pharmacy versus an ER?

Pharmacies usually follow the rule
First Come, First Served

Emergency Rooms assign priorities
based on each individual's need

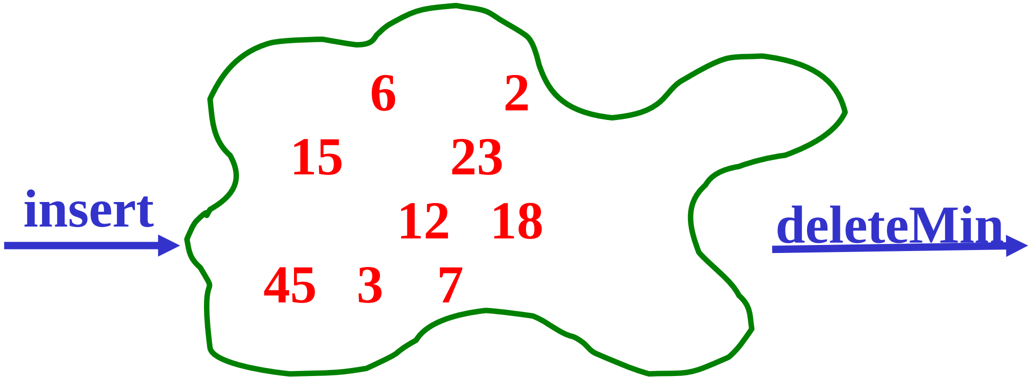
# *A new ADT: Priority Queue*

- Textbook Chapter 6
  - We will go back to binary search trees (ch4) and hash tables (ch5) later
  - Nice to see a new and surprising data structure first
- A **priority queue** holds *compare-able data*
  - Unlike stacks and queues need to *compare items*
    - Given $x$ and $y$, is $x$ less than, equal to, or greater than $y$
    - What this means can depend on your data
    - Much of course will require comparable data: e.g. sorting
  - Integers are comparable, so will use them in examples
    - But the priority queue ADT is much more general
    - Typically two fields, the *priority* and the *data*

# *Priority Queue ADT*

- Assume each item has a "priority"
  - The *lesser* item is the one with the *greater* priority
  - So "priority 1" is more important than "priority 4"
  - Just a convention, could also do a maximum priority

- Main Operations:
  - **`insert`**
  - **`deleteMin`**




- Key property: **`deleteMin`** returns and deletes from the queue the item with greatest priority (lowest priority value)
  - Can resolve ties arbitrarily

# *Aside: We will use ints as data and priority*

For simplicity in lecture, we'll often suppose items are just **`int`**s and the **`int`** is also the priority

- So an operation sequence could be

```
insert 6
insert 5
x = deleteMin  // Now x = 5.
```

– **`int`** priorities are common, but really just need comparable

- Not having "other data" is very rare
  - Example: print job has a priority *and* the file to print is the data

# *Priority Queue Example*

**To simplify our examples, we will just use the priority values from now on**

**`insert`** *a* with priority *5*

**`insert`** *b* with priority *3*

**`insert`** *c* with priority *4*

*w* = **`deleteMin`**

*x* = **`deleteMin`**

**`insert`** *d* with priority *2*

**`insert`** *e* with priority *6*

*y* = **`deleteMin`**

*z* = **`deleteMin`**

**`after execution:`**

**Analogy: `insert` is like `enqueue`, `deleteMin` is like `dequeue`**

**But the whole point is to use priorities instead of FIFO**

# *Priority Queue Example*

**To simplify our examples, we will just use the priority values from now on**

| | |
|---|---|
| `insert` *a* with priority *5* | **`after execution:`** |
| `insert` *b* with priority *3* | |
| `insert` *c* with priority *4* | *w* = *b* |
| *w* = `deleteMin` | *x* = *c* |
| *x* = `deleteMin` | *y* = *d* |
| `insert` *d* with priority *2* | *z* = *a* |
| `insert` *e* with priority *6* | |
| *y* = `deleteMin` | |
| *z* = `deleteMin` | |

**Analogy: `insert` is like `enqueue`, `deleteMin` is like `dequeue`**
**But the whole point is to use priorities instead of FIFO**

# *Applications*

Like all good ADTs, the priority queue arises often

- Sometimes "directly", sometimes less obvious

- Run multiple programs in the operating system
  - "critical" before "interactive" before "compute-intensive"
  - Maybe let users set priority level
- Treat hospital patients in order of severity (or triage)
- Select print jobs in order of decreasing length?
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (cf. CSE143)
- Sort: **`insert`** all, then repeatedly **`deleteMin`**

# *More applications*

- "Greedy" algorithms
  - Select the 'best-looking' choice at the moment
  - Will see an example when we study graphs in a few weeks
- Discrete event simulation (system modeling, virtual worlds, …)
  - Simulate how state changes when events fire
  - Each event *e* happens at some time t and generates new events *e1*, …, *en* at times *t+t1*, …, *t+tn*
  - Naïve approach: advance "clock" by 1 unit at a time and process any events that happen then
  - Better:
    - *Pending events* in a priority queue (priority = time happens)
    - Repeatedly: **`deleteMin`** and then **`insert`** new events
    - Effectively, "set clock ahead to next event"

# *Preliminary* Implementations of Priority Queue ADT

| | insert | deleteMin |
|---|---|---|
| Unsorted Array | | |
| Unsorted Linked-List | | |
| Sorted Circular Array | | |
| Sorted Linked-List | | |
| Binary Search Tree (BST) | | |

**Notes**: Worst case, Assume arrays have enough space

# *Aside: More on possibilities*

- Note: If priorities are inserted in random order, binary search tree will likely do better than $O(n)$
  - $O($**`log`** $n)$ **`insert`** and $O($**`log`** $n)$ **`deleteMin`** on average
  - Could get same performance from a *balanced* binary search tree (e.g. AVL tree we will study later)

- One more idea: if priorities are 0, 1, …, $k$ can use array of lists
  - **`insert`**: add to front of list at **`arr[priority]`**, $O(1)$
  - **`deleteMin`**: remove from lowest non-empty list $O(k)$

# *Our Data Structure: The Heap*

**The Heap:**

- Worst case: O(log n) for insert
- Worst case: O(log n) for deleteMin
- If items arrive in random order, then the average-case of insert is O(1)
- Very good constant factors

**Key idea:** Only pay for functionality needed

- We need something better than scanning unsorted items
- But we do not need to maintain a full sorted list

- We will *visualize* our heap as a tree, so we need to review some tree terminology

# *Q: Reviewing Some Tree Terminology*

***root*(T):**

***leaves*(T):**

***children*(B):**

***parent*(H):**

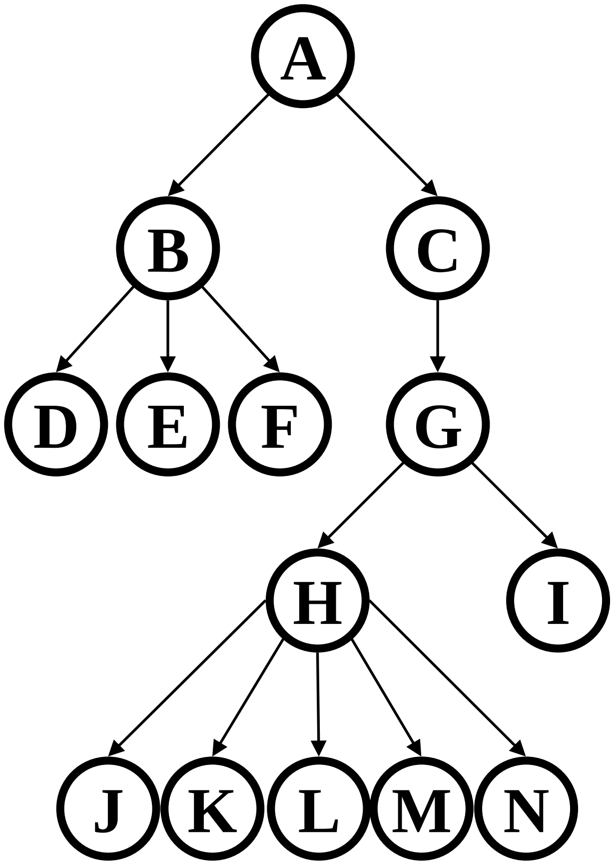
***siblings*(E):**

***ancestors*(F):**

***descendents*(G):**

***subtree*(G):**

**Tree T**



# *Q: Some More Tree Terminology*

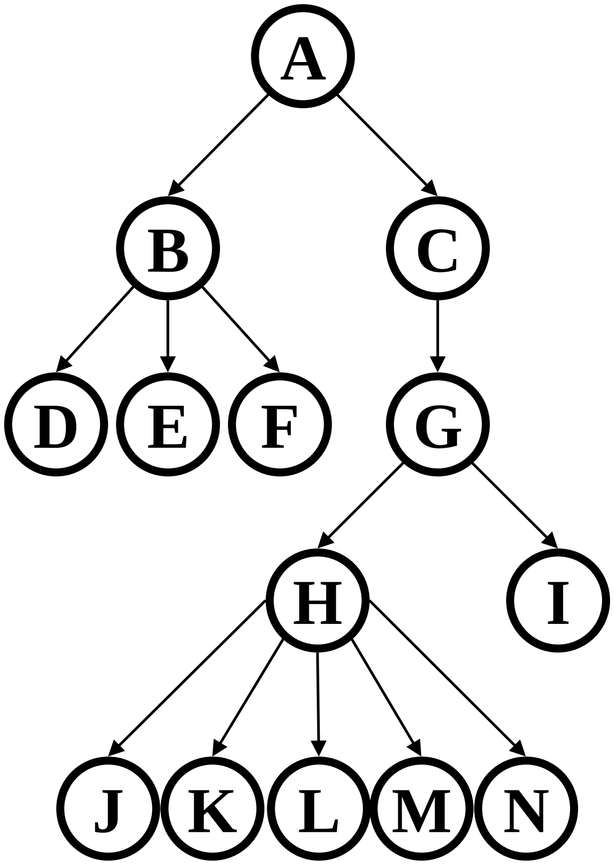
***depth*(B):**

***height*(G):**

***height*(T):**

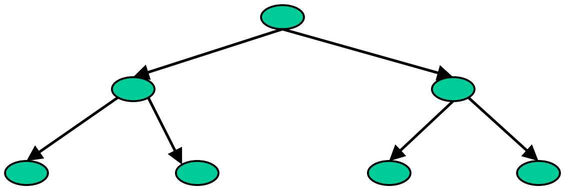
***degree*(B):**

***branching factor*(T):**

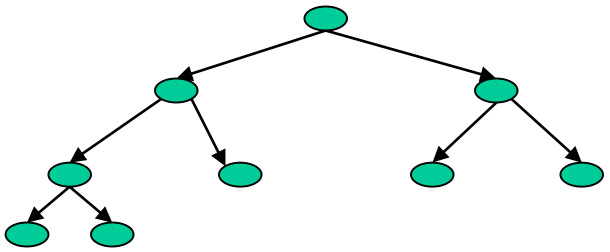


# *Types of Trees*

| | |
|---|---|
| Binary tree: | Every node has ≤2 children |
| n-ary tree: | Every node has ≤n children |
| Perfect tree: | Every row is completely full |
| Complete tree: | All rows except possibly the bottom are completely full, and it is filled from left to right |



**Perfect Tree**



**Complete Tree**

# *Some Basic Tree Properties*

*Nodes* in a perfect binary tree of height h?

*Leaf nodes* in a perfect binary tree of height h?

Height of a perfect binary tree with n nodes?

Height of a complete binary tree with n nodes?

# *Properties of a Binary Min-Heap*

More commonly known as a binary heap or simply a heap

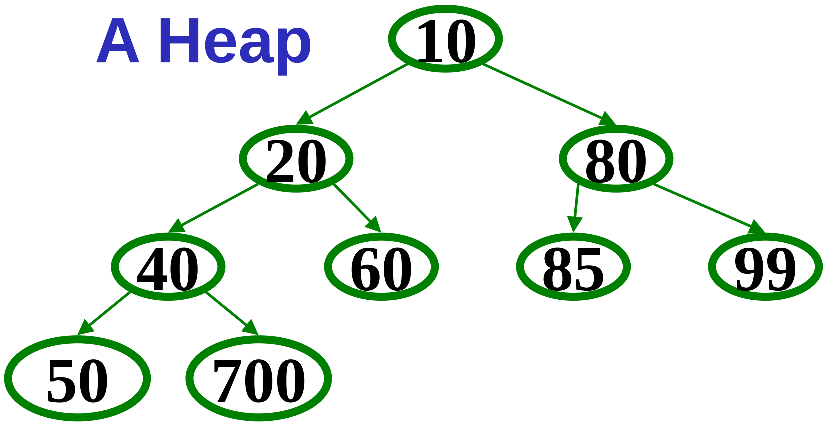
- Structure Property:
  A complete [binary] tree
- Heap Property:
  The priority of every non-root node is greater than (or possibly equal to) the priority of its parent

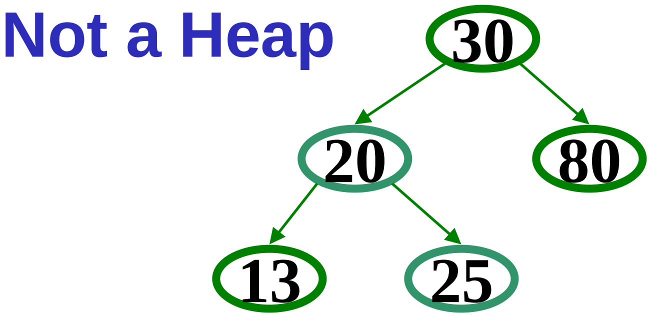
How is this different from a binary search tree?

# *Properties of a Binary Min-Heap*

More commonly known as a binary heap or simply a heap

- Structure Property:
  A complete [binary] tree
- Heap Order Property:
  The priority of every non-root node is greater than the priority of its parent

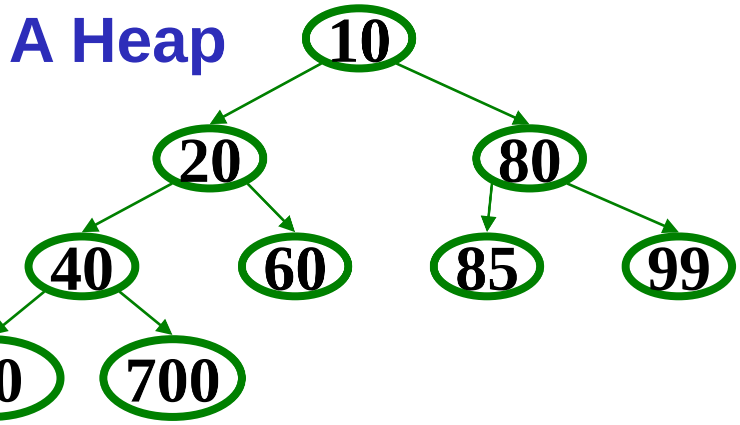


# *Properties of a Binary Min-Heap*

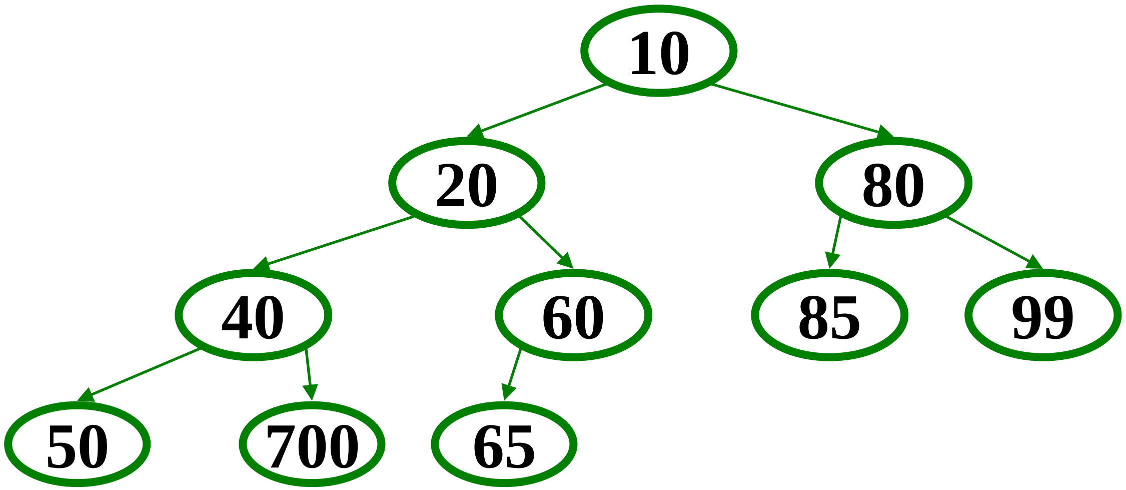
- Where is the minimum priority item?

- What is the height of a heap with n items?



# *Heap Operations*

- findMin:
- deleteMin: percolate down.
- insert(val): percolate up.



# Operations: basic idea

- **`findMin`**:

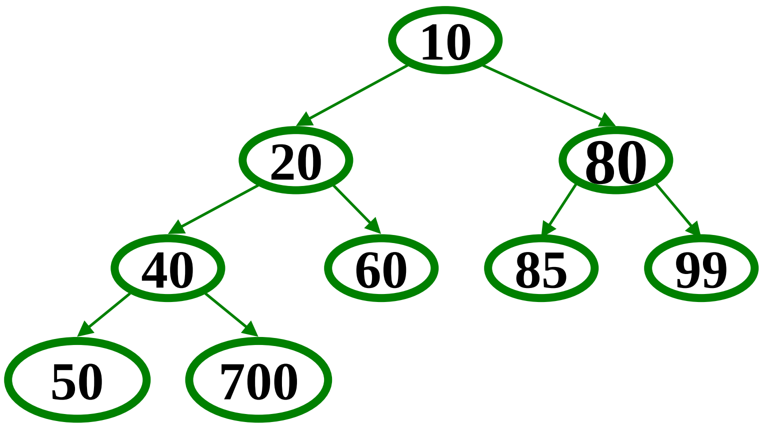
    return **`root.data`**

- **`deleteMin`**:

    1. **`answer = root.data`**
    2. Move right-most node in last row to root to restore structure property
    3. "Percolate down" to restore heap order property

- **`insert:`**

    1. Put new node in next position on bottom row to restore structure property
    2. "Percolate up" to restore heap order property



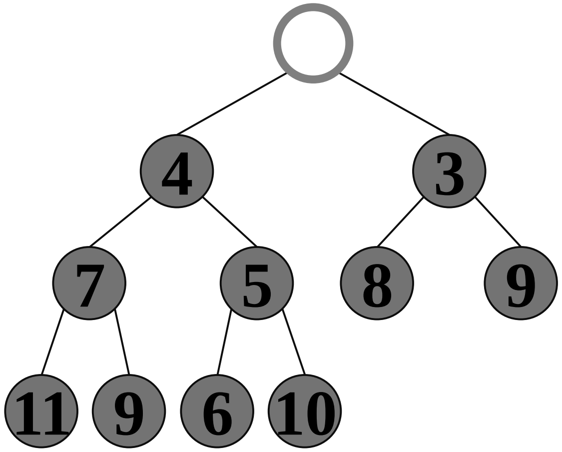

*Overall strategy:*

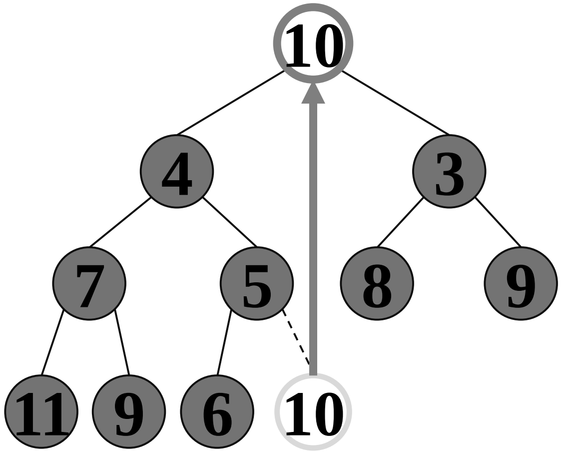
- *Preserve complete tree structure property*
- *This may break heap order property*
- *Percolate to restore heap order property*

# *DeleteMin Implementation*

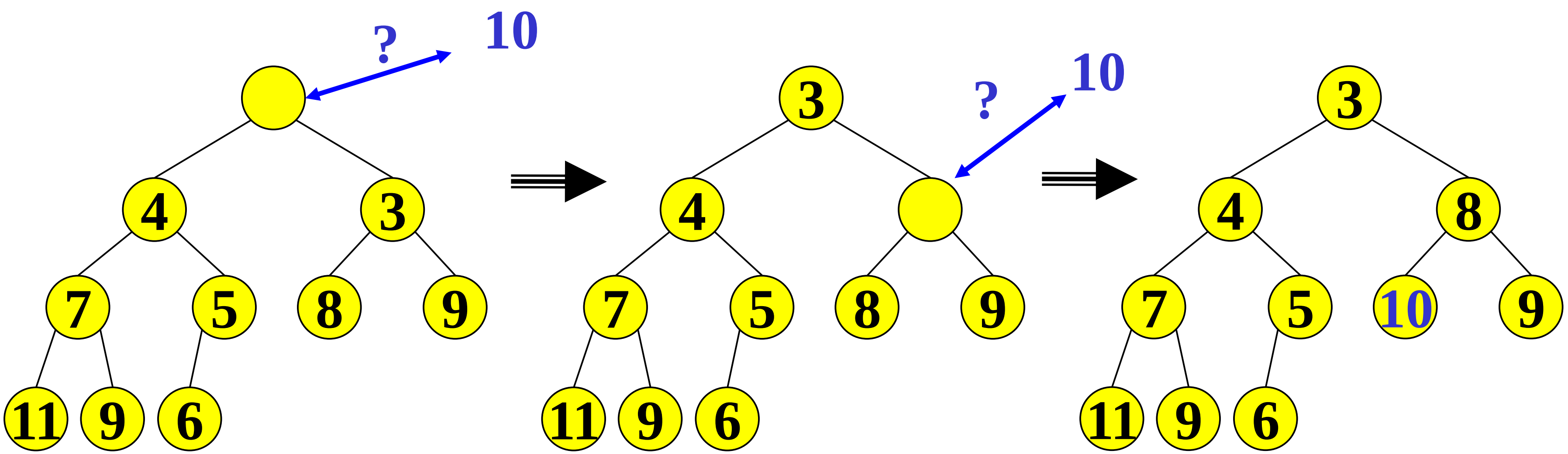
1. Delete value at root node (and store it for later return)
2. There is now a "hole" at the root. We must "fill" the hole with another value, must have a tree with one less node, and it must still be a complete tree
3. The "last" node is the is obvious choice, but now the heap order property is violated
4. We percolate down to fix the heap order:

```
While greater than either child
   Swap with smaller child
```



# *Percolate Down*




Percolate down:

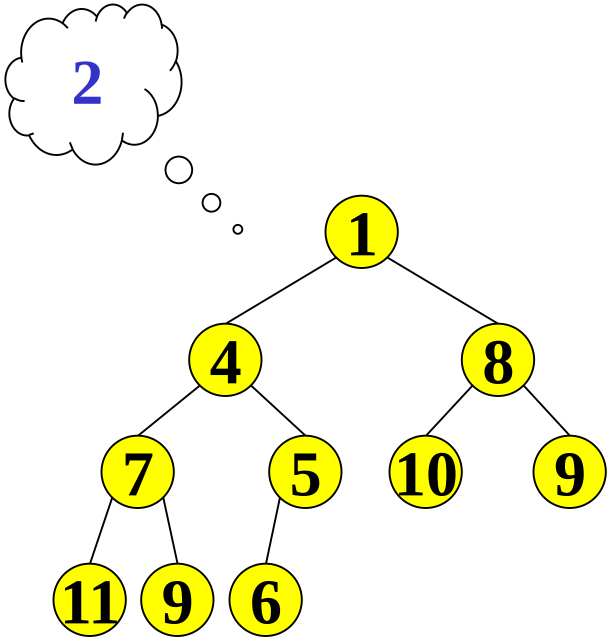
- Keep comparing with both children
- Move smaller child up and go down one level
- Done if both children are $\geq$ item or reached a leaf node
- Why does this work? What is the run time?

# *DeleteMin: Run Time Analysis*

- Run time is *O*(height of heap)
- A heap is a complete binary tree
- Height of a complete binary tree of *n* nodes?
  - height = $\lfloor \mathtt{log}_2(n) \rfloor$
- Run time of **`deleteMin`** is $O(\mathtt{log}\ n)$

# *Insert*

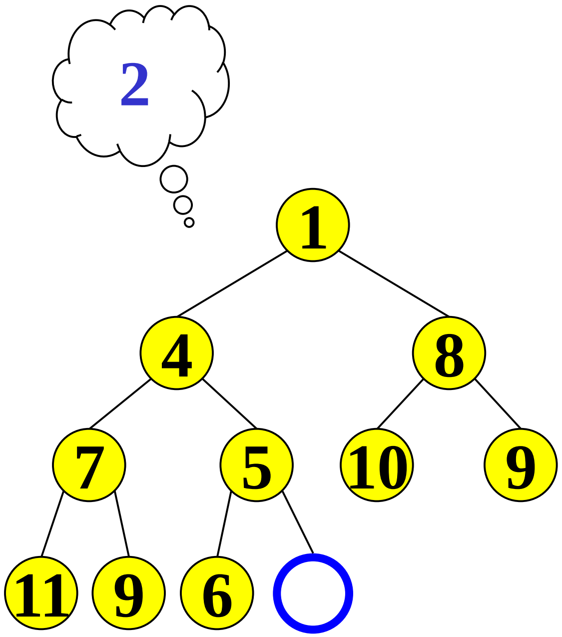
- Add a value to the tree
- Structure and heap order properties must still be correct afterwards



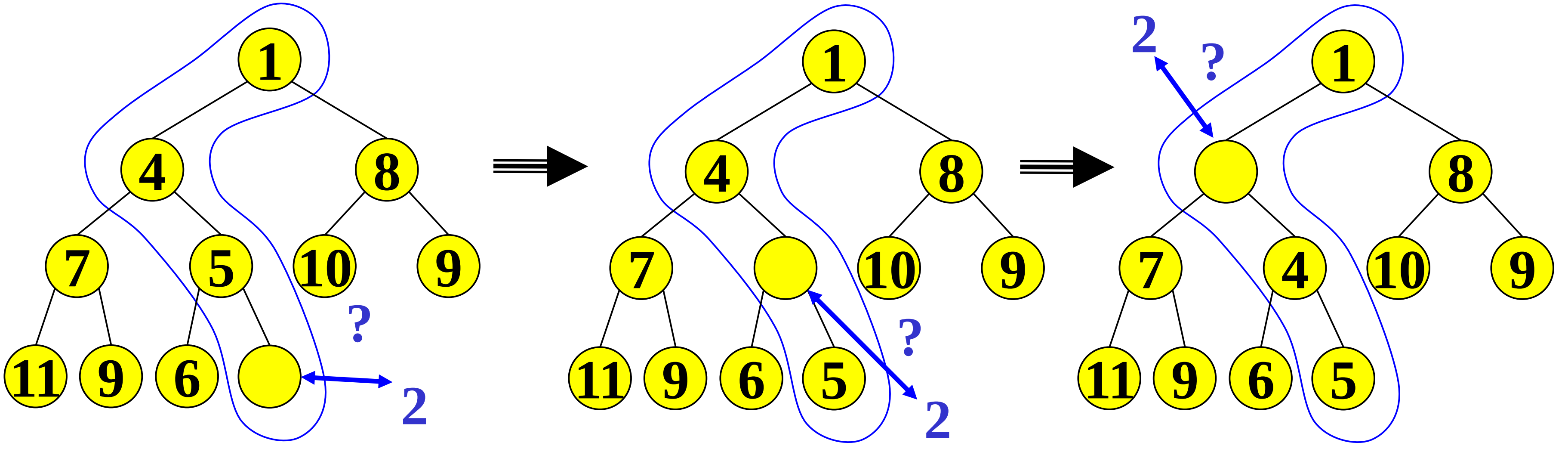
# *Insert: Maintain the Structure Property*

- There is only **one** valid tree shape after we add one more node!

- So put our new data there and then focus on restoring the heap order property



# *Maintain the heap order property*




Percolate up:

- Put new data in new location
- If parent larger, swap with parent, and continue
- Done if parent ≤ item or reached root
- Why does this work? What is the run time?

# *A Clever Trick for Storing the Heap…*

Clearly, insert and deleteMin are worst-case O(log n)

- But we promised average-case O(1) insert (how??)

Insert requires access to the “next to use” position in the tree

- Walking the tree from root to leaf requires O(log n) steps
- Insert and Deletemin would have to update the “next to use” reference each time: O(log n)

We should only pay for the functionality we need!!

- Why have we insisted the tree be complete? ☺

All complete trees of size n contain the same edges

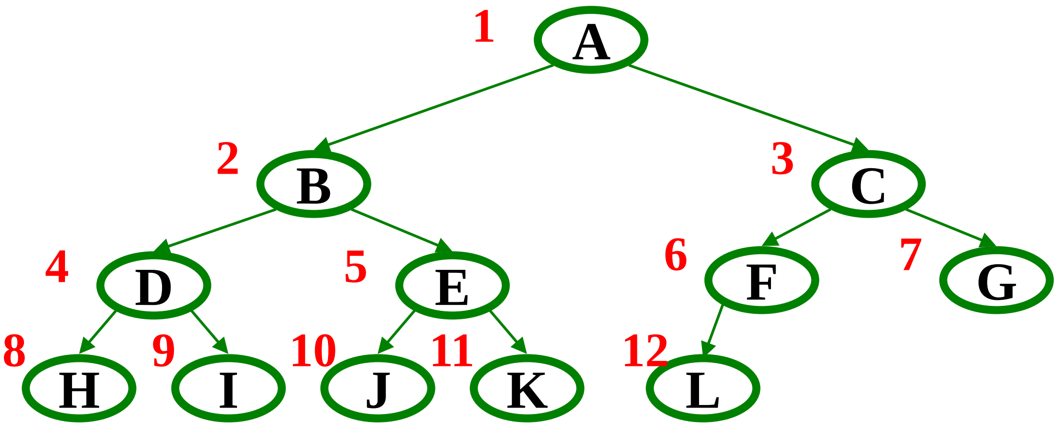
- So why are we even representing the edges?

**Here comes the really clever bit about implementing heaps!!!**

# *Array Representation of a Binary Heap*

From node i:

- left child:
- right child:
- parent:



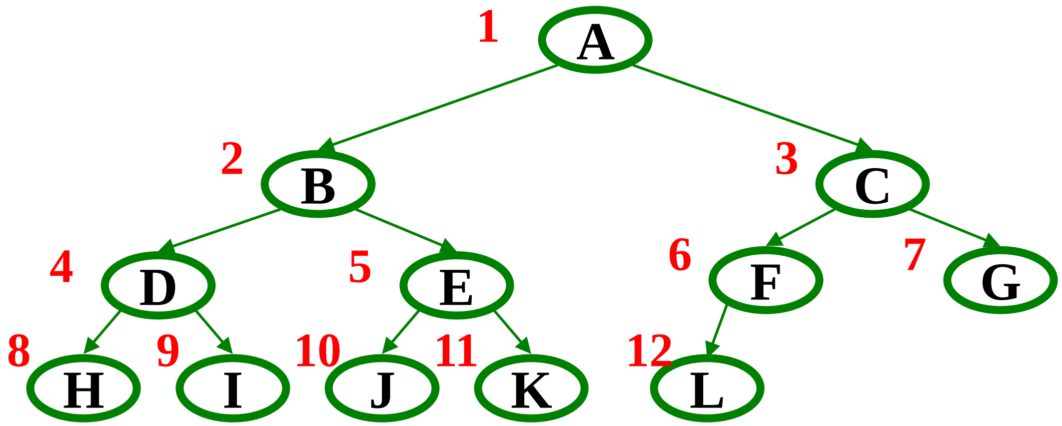

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K | L | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |

- We skip index 0 to make the math simpler
- Actually, it can be a good place to store the current size of the heap

# *Array Representation of a Binary Heap*

From node i:

- left child: 2i
- right child: 2i+1
- parent: i / 2




| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K | L | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |

- We skip index 0 to make the math simpler
- Actually, it can be a good place to store the current size of the heap