Adam Blank Lecture 24 Winter 2016

CSE 332

Data Abstractions

P vs. NP: Efficient Reductions Between Problems

More Graph Problems

Let's consider the **longest path** problem on a graph.

Remember, we were able to do **shortest paths** using Dijkstra's.

Take a few minutes to try to solve the **longest path** problem.

Decision Problems

Definition (Decision Problem)

A **decision problem** (or **language**) is a set of strings ($L \subseteq \Sigma^*$).
An algorithm (from Σ^* to `boolean`) solves a decision problem when it outputs `true` iff the input is in the set.

PRIMES	
Input(s):	Number x
Output:	`true` iff x is prime

An Algorithm that solves PRIMES

```
isPrime(x) {
   for (i = 2; i < x; i++) {
      if (x % i == 0) {
         return false;
      }
   }
   return true;
}
```

Efficient?

In this lecture, we'll be talking about **efficient reductions**. So, naturally, we have to answer two questions:

- What is an efficient algorithm?
- What is a reduction?

Efficient Algorithm

We say an algorithm is **efficient** if the worst-case analysis is a **polynomial**. Okay, but...

- $n^{10000000\ldots}$ is polynomial
- $3000000000000000n^3$ is polynomial

Are those really efficient?
Well, no, but, in practice...

when a polynomial algorithm is found the constants are actually low

Polynomial runtime is a **very** low bar, if we can't even get that...

Reductions

This lecture is about exposing **hidden** similarities between problems.

We will show that problems that are **cosmetically different** are **substantially the same**!

Our main tool to do this is called a **reduction**:

Reductions

We have two **decision problems**, **A** and **B**. To show that **A** is "at least as hard as" **B**, we

- Suppose we can solve **A**
- Create an algorithm, which calls **A** as a method, to solve **B**

To show they're the same, we have to do both directions.

Longest Paths and HAM! 5

Two New Computational Problems

LONG-PATH
Input(s): Unweighted Graph G; Number k
Output: `true` iff G has a path with k edges

HAM-PATH
Input(s): Unweighted Graph G
Output: `true` iff G has a path using all vertices

Suppose we could solve **LONG-PATH**...

"Algorithm"

```
HAM-PATH(G) {
   return LONG-PATH(G, |V| - 1)
}
```

Suppose we could solve **HAM-PATH**...

"Algorithm"

```
LONG-PATH(G, k) {
   for (G' = (v1, v2, ..., vk) in G) {
      if (HAM-PATH(G')) {
         return true;
      }
   }
   return false;
}
```

A 2-CRAYOLA Question 6

Definition (k-coloring)
A k-**coloring** of a graph G is an assignment of k colors to vertices such that no two adjacent vertices have the same color.

2-COLOR
Input(s): Graph G
Output: `true` iff G has a valid 2-coloring

Can we solve this?

Algorithm For 2-COLOR
Try all 2^n possible colorings of the input graph!

Can we solve this efficiently?

Efficient Algorithm For 2-COLOR
Do a `dfs` on the graph! Every time we hit a vertex, assign it the opposite color from the vertex we just visited. If there's a color conflict, output `false`. If we finish with no color conflict, output `true`.

A 3-CRAYOLA Question 7

Definition (k-coloring)
A k-**coloring** of a graph G is an assignment of k colors to vertices such that no two adjacent vertices have the same color.

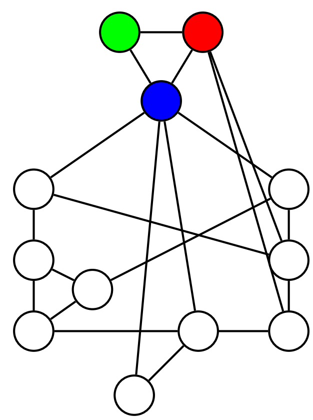
3-COLOR
Input(s): Graph G
Output: `true` iff G has a valid 3-coloring

Inefficient Algorithm For 3-COLOR
Try all 3^n possible colorings of the input graph!

Efficient Algorithm For 3-COLOR
UNKNOWN

A Graph Called "Gadget" 8

Find a valid 3-coloring of this graph. To orient ourselves, I've started it:



Another Decision Problem! 9

CIRCUITSAT
Input(s): n-Input/1-Output Circuit C
Output: `true` iff C has a satisfying assignment

Inefficient Algorithm For CIRCUITSAT
Try all 2^n possible assignments of variables

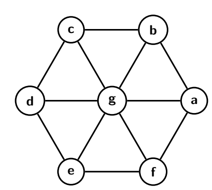
Efficient Algorithm For CIRCUITSAT
UNKNOWN

Suspicious... 10

CIRCUITSAT

X Y Z
NOT
OR
OR
OUT

3-COLOR



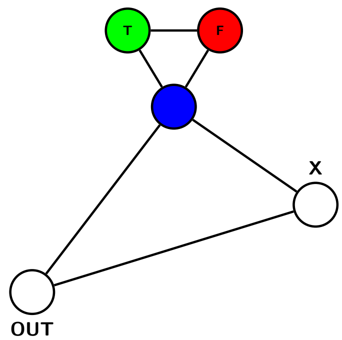
We don't know how to solve either of these problems...

Could they be the same problem in disguise?

Not Gadget with Labels

X

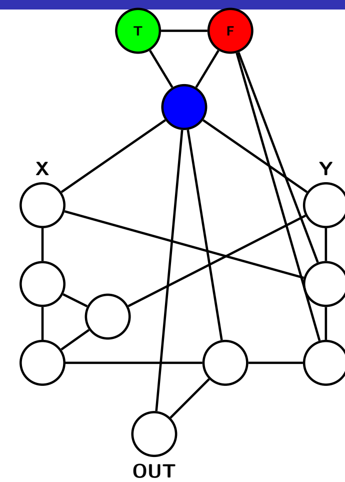
OUT



Or Gadget with Labels

X Y

OUT



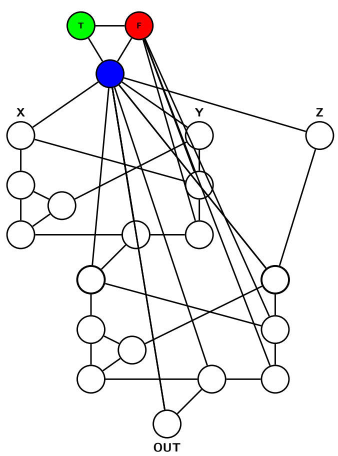
Circuit

X Y Z

OR NOT

OR

OUT



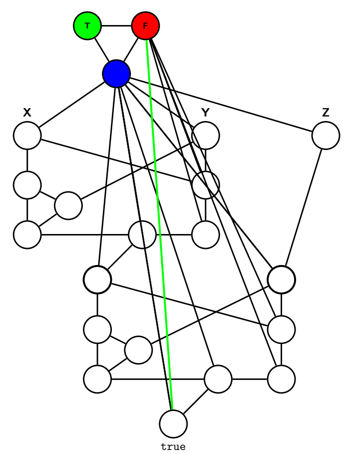
SATISFIABLE Circuit

X Y Z

OR NOT

OR

true



Lesson

We found a way to “emulate” circuit satisfiability using three coloring!

If we can find a solution to **3-COLOR**, we can solve **CIRCUITSAT** quickly.

These problems are substantially the same