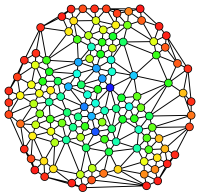
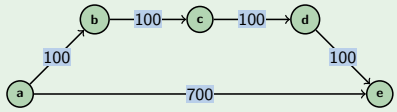
CSE 332

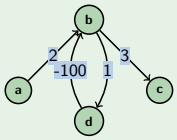
Data Abstractions

Graphs 3: Single-Source Shortest Paths

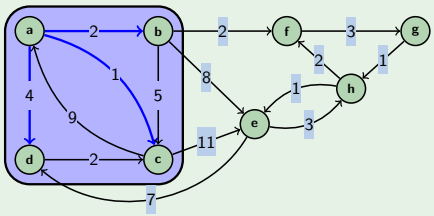


Some Initial Thoughts





The Idea



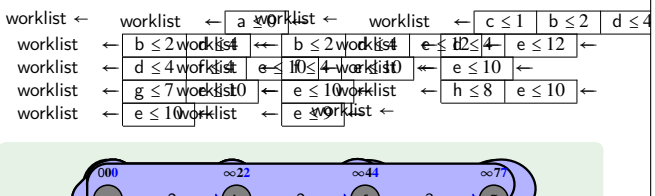
We will run a **simulation** of (infinitely many) ants exploring the graph.

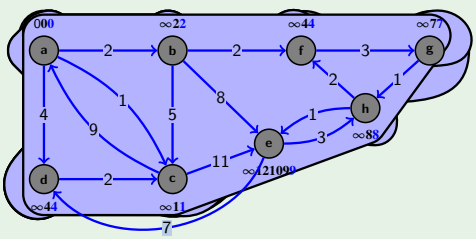
The ants all move at identical speeds.

We're interested in the **time step** that some ant first reaches each vertex.

- At each step...
  - The ants try to move along some new edge
  - We "process" a vertex at the timestep that an ant arrives there
  - When an ant arrives, they dispatch new ants to every out-edge
- We're done!

Example

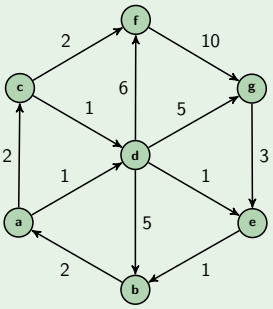




The Algorithm


```
1  dijkstra(G, source) {
2     // We will use a "sorted list" as our worklist, because the items
3     // in the work list are "events" which are processed in order
4
5     // (v, timestep) in worklist, where v is a vertex and timestep
6     // is the "time" the first ant got there
7     worklist = [];     // These ants are "currently moving"
8
9     // All the ants begin at vertex v at time step zero
10    worklist.add((source, 0));
11
12    while (worklist.hasWork()) {
13       (v, time_to_v) = next();
14
15       // Since a cluster of ants got to v, we dispatch new ants
16       for (u : v.neighbors()) {
17          // When does a cluster of ants get to u? How does it change?
18          (u, time_to_u) = worklist.get(u);
19          // w(v, u) is the edge weight from v to u
20          time_from_v_to_u = w(v, u);
21          to_u = min(time_to_u, time_to_v + time_from_v_to_u);
22          worklist.add((u, to_u));
23       }
24    }
25    return dist;
26 }
```

Example 2



Okay, and to implement this?

- Our sorted list is slow; so, replace it with a **priority queue**.
- We need a way of "changing the priority of an element"

Remember, decreaseKey? That's exactly what it does!

To make that work, we need to store a reference to the index/vertex in some dictionary.

Final Dijkstra's Algorithm


```
1  dijkstra(G, source) {
2     dist = new Dictionary();
3     worklist = [];
4     for (v : V) {
5        if (v == source) { dist[v] = 0; }
6        else             { dist[v] = ∞; }
7        worklist.add((v, dist[v]));
8     }
9
10    while (worklist.hasWork()) {
11       v = next();
12       for (u : v.neighbors()) {
13          dist[u] = min(dist[u], dist[v] + w(v, u));
14          worklist.decreaseKey(u, dist[u]);
15       }
16    }
17
18    return dist;
19 }
```

Example 3