CSE 332: Sorting II

Spring 2016

Announcements

- Midterm: Friday April 29

Sorting: *The Big Picture*

Simple algorithms: $O(n^2)$	Fancier algorithms: $O(n \log n)$	Comparison lower bound: $\Omega(n \log n)$	Specialized algorithms: $O(n)$	Handling huge data sets
Insertion sort Selection sort …	Heap sort Merge sort Quick sort (avg) …		Bucket sort Radix sort	External sorting

Quicksort

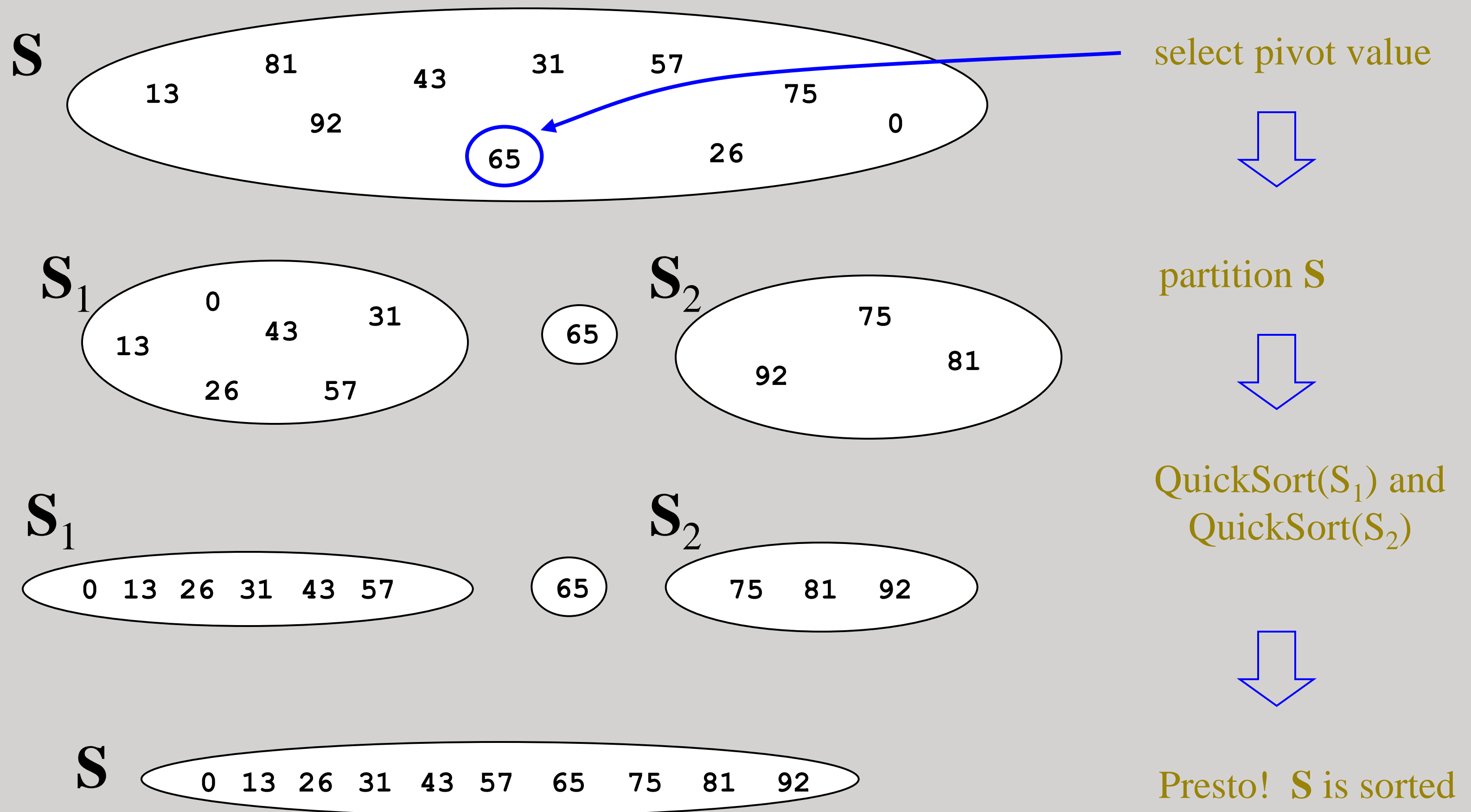
Quicksort uses a divide and conquer strategy, but does not require the O(N) extra space that MergeSort does.

Here's the idea for sorting array **S**:

1. Pick an element *v* in **S**. This is the ***pivot*** value.
2. Partition **S**-{*v*} into two disjoint subsets, $\mathbf{S}_1$ and $\mathbf{S}_2$ such that:
   - elements in $\mathbf{S}_1$ are all $\leq v$
   - elements in $\mathbf{S}_2$ are all $\geq v$
3. Return concatenation of QuickSort($\mathbf{S}_1$), *v*, QuickSort($\mathbf{S}_2$)

Recursion ends when Quicksort( ) receives an array of length 0 or 1.

The steps of Quicksort



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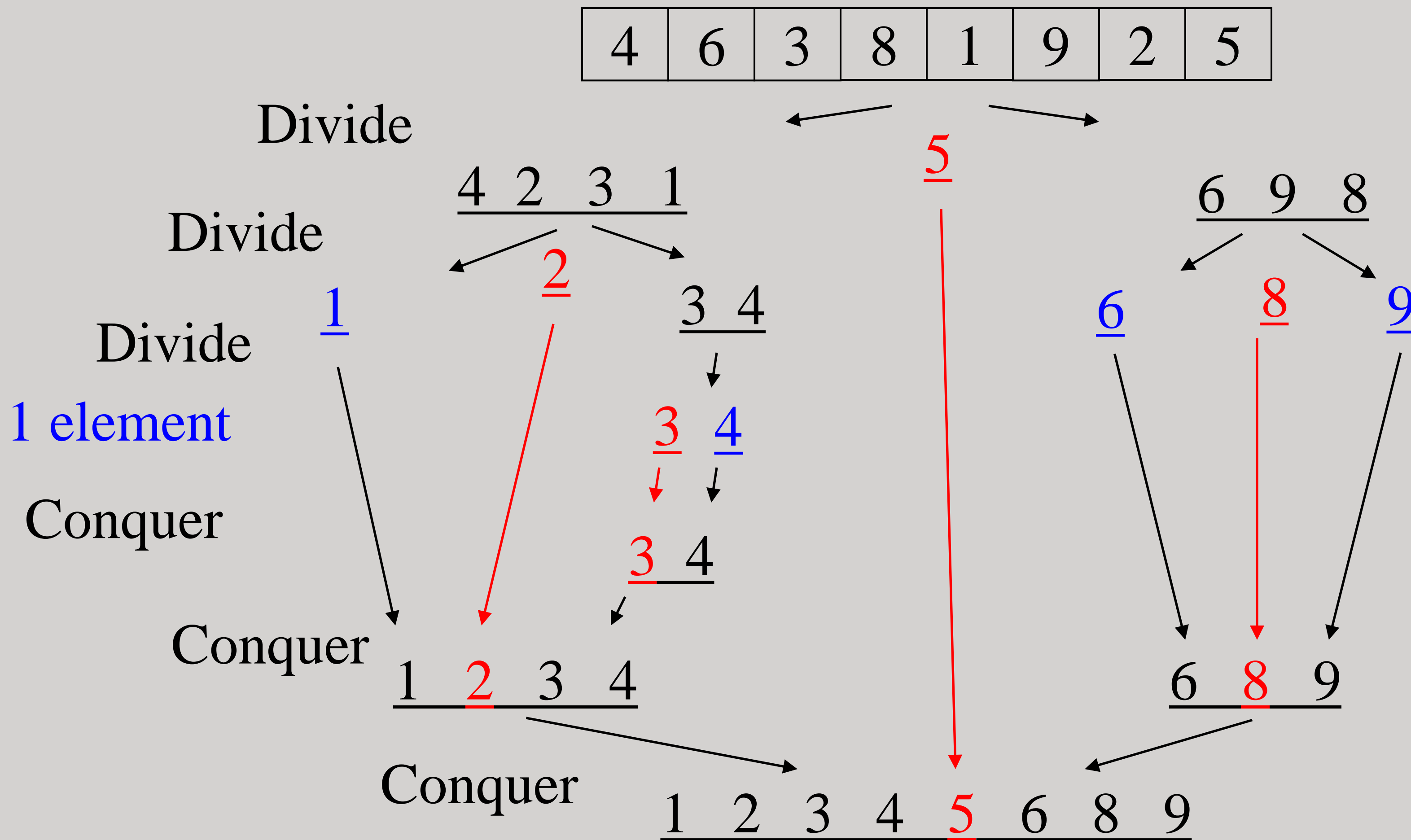
Quicksort Example



Pivot Picking and Partitioning

The tricky parts are:

- **Picking the pivot**
  - Goal: pick a pivot value so that $|S_1|$ and $|S_2|$ are roughly equal in size.
- **Partitioning**
  - Preferably in-place
  - Dealing with duplicates

Picking the Pivot

Median of Three Pivot

0	1	2	3	4	5	6	7	8	9
8	1	4	9	**6**	3	5	2	7	**0**

`medianOf3Pivot(…)`

0	1	4	9	7	3	5	2	6	8

Choose the pivot as the median of three.

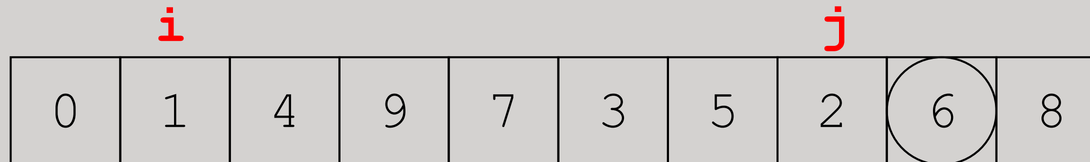
Place the pivot and the largest at the right and the smallest at the left.

Quicksort Partitioning

- Partition the array into left and right sub-arrays such that:
  - elements in left sub-array are ≤ pivot
  - elements in right sub-array are ≥ pivot
- Can be done in-place with another “two pointer method”
  - Sounds like mergesort, but here we are *partitioning*, not sorting…
  - …and we can do it in-place.

Partioning In-place

Setup: `i` = start and `j` = end of un-partioned elements:

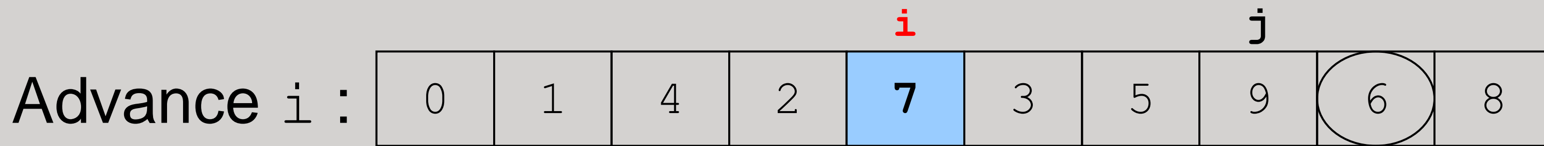


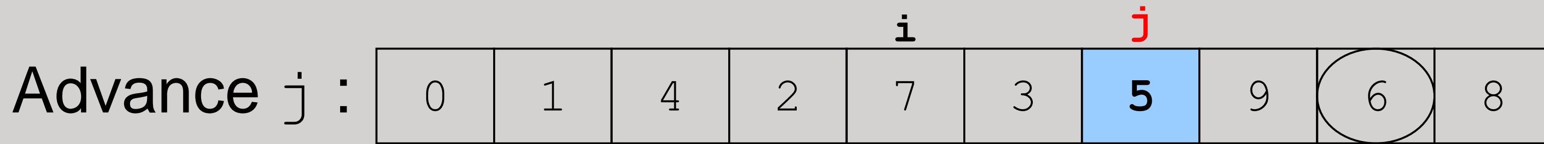
Advance `i` until element ≥ pivot:

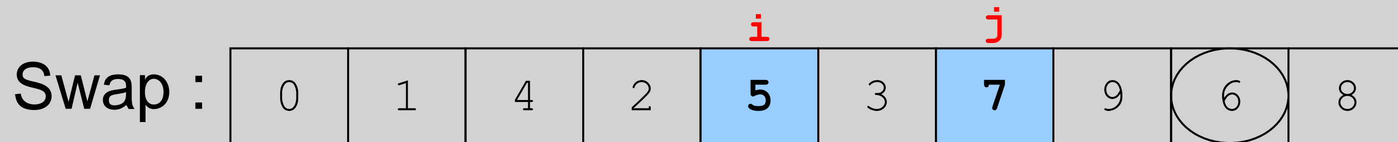
Advance `j` until element ≤ pivot:

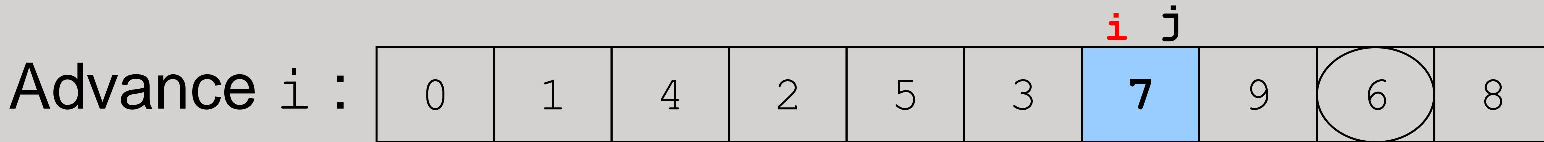
If `j` > `i`, then swap:

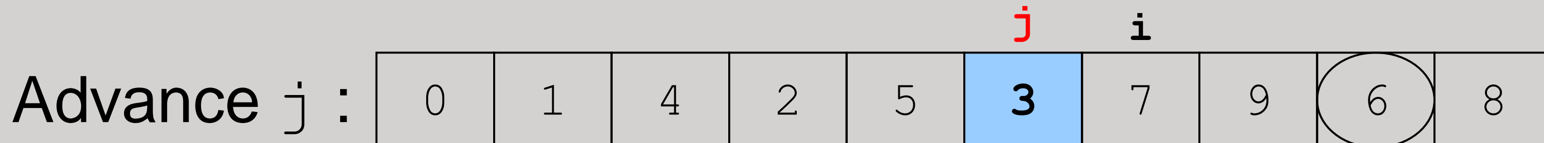
Partioning In-place



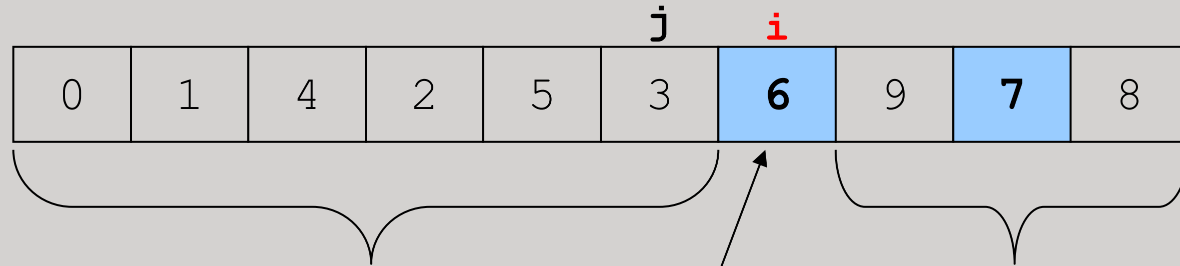








i > j, swap in pivot, partition done!



$S_1 \leq$ pivot pivot $S_2 \geq$ pivot

Partition Pseudocode

```
Partition(A[], left, right) {
  v = A[right]; // Assumes pivot value currently at right
  i = left;     // Initialize left side, right side pointers
  j = right-1;

  // Do i++, j-- until they cross, swapping values as needed
  while (1) {
    while (A[i] < v) i++;
    while (A[j] > v) j--;
    if (i < j) {
      Swap(A[i], A[j]);
      i++; j--;
    }
    else
      break;
  }
  Swap(A[i], A[right]); // Swap pivot value into position
  return i;             // Return the final pivot position
}
```

Complexity for input size *n*?

Quicksort Pseudocode

Putting the pieces together:

```
Quicksort(A[], left, right) {
  if (left < right) {
    medianOf3Pivot(A, left, right);
    pivotIndex = Partition(A, left+1, right-1);

    Quicksort(A, left, pivotIndex – 1);
    Quicksort(A, pivotIndex + 1, right);
  }
}
```

QuickSort: Best case complexity

```
Quicksort(A[], left, right) {
  if (left < right) {
    medianOf3Pivot(A, left, right);
    pivotIndex = Partition(A, left+1, right-1);

    Quicksort(A, left, pivotIndex – 1);
    Quicksort(A, pivotIndex + 1, right);
  }
}
```

QuickSort:
Worst case complexity

```
Quicksort(A[], left, right) {
  if (left < right) {
    medianOf3Pivot(A, left, right);
    pivotIndex = Partition(A, left+1, right-1);

    Quicksort(A, left, pivotIndex – 1);
    Quicksort(A, pivotIndex + 1, right);
  }
}
```

QuickSort: Average case complexity

Turns out to be **O(n log n)**.

See Section 7.7.5 for an idea of the proof.
Don't need to know proof details for this course.

Many Duplicates?

An important case to consider is when an array has many duplicates.

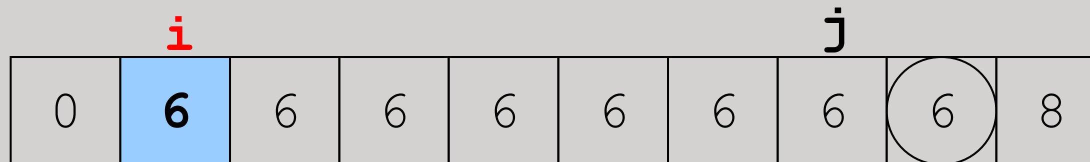
0	1	2	3	4	5	6	7	8	9
8	6	6	6	**6**	6	6	6	6	**0**

0	6	6	6	6	6	6	6	6	8

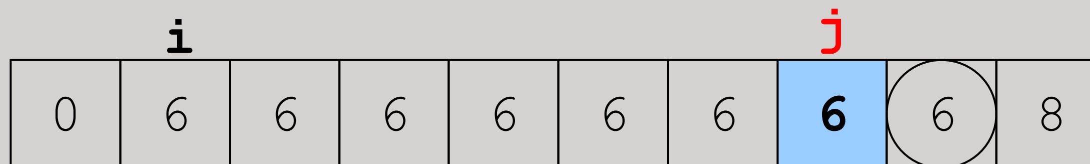
Partitioning with Duplicates

Setup: `i` = start and `j` = end of un-partioned elements:

Advance `i` until element ≥ pivot:

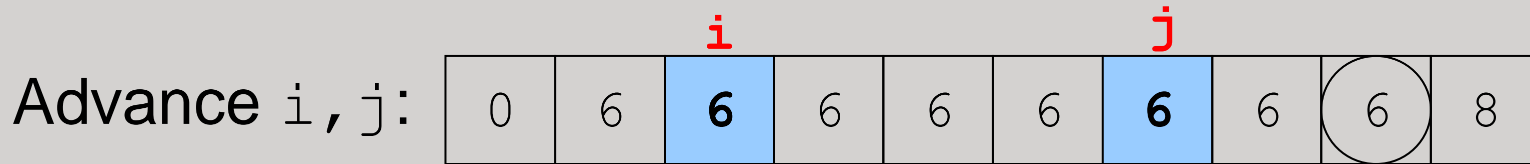


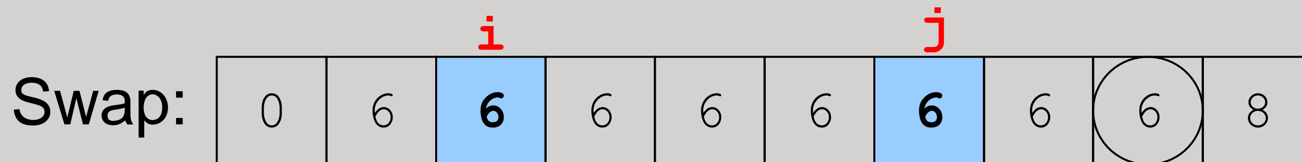
Advance `j` until element ≤ pivot:

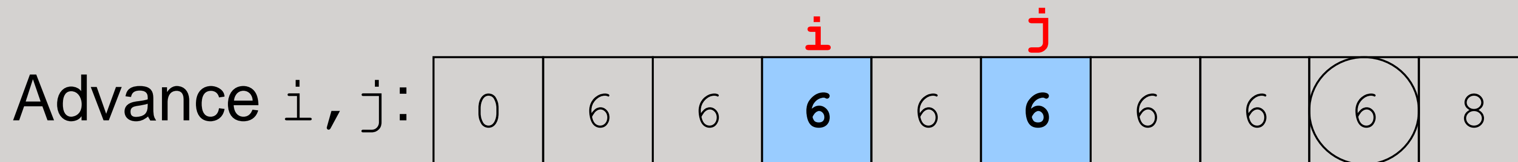


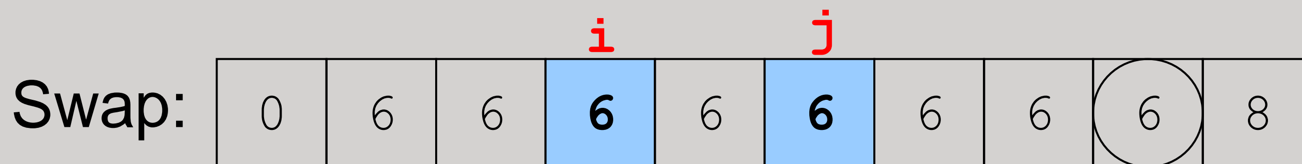
If `j` > `i`, then swap:

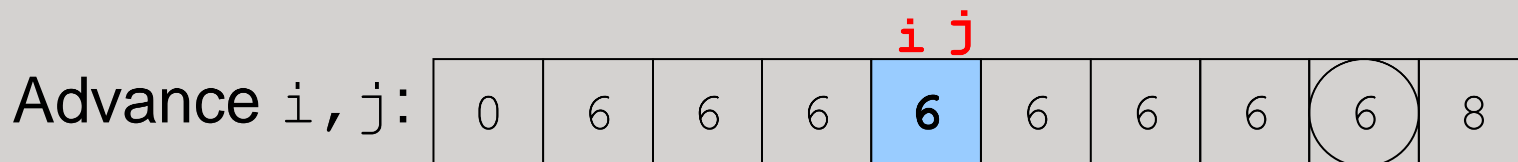
Partitioning with Duplicates

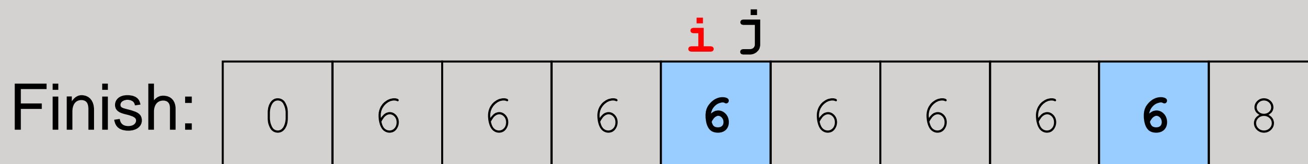






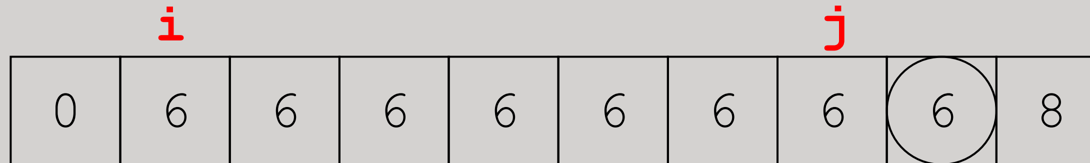




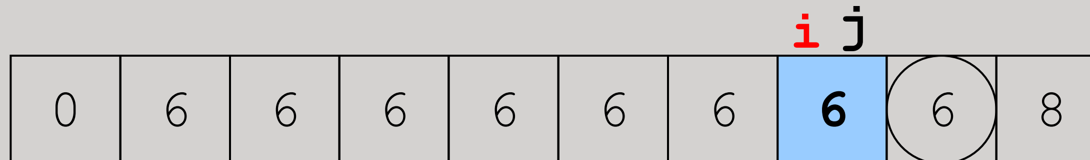


Partitioning with Duplicates:Take Two

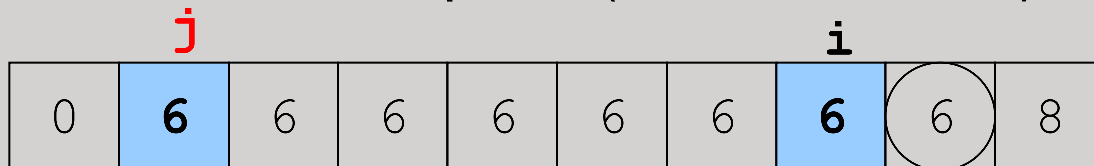
Start `i` = start and `j` = end of un-partioned elements:

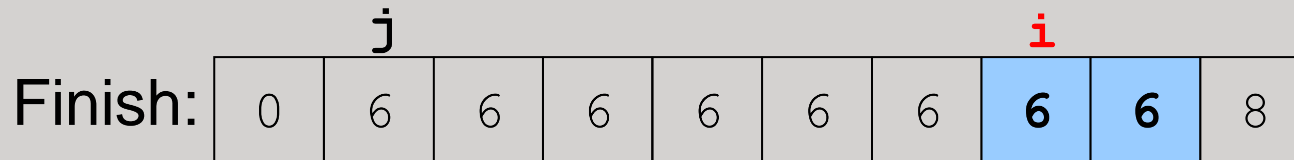


Advance `i` until element **>** pivot (and in bounds):



Advance `j` until element **<** pivot (and in bounds):





Is this better?

Partitioning with Duplicates: Upshot

It's better to stop advancing pointers when elements are equal to pivot, and then just do swaps.

Complexity of quicksort on an array of identical values?

Can we do better?

Important Tweak

Insertion sort is actually better than quicksort on small arrays. Thus, a better version of quicksort:

```
Quicksort(A[], left, right) {
  if (right - left ≥ CUTOFF) {
    medianOf3Pivot(A, left, right);
    pivotIndex = Partition(A, left+1, right-1);

    Quicksort(A, left, pivotIndex – 1);
    Quicksort(A, pivotIndex + 1, right);

  } else {
    InsertionSort(A, left, right);
  }
}
```

CUTOFF = 16 is reasonable.

Properties of Quicksort

- $O(N^2)$ worst case performance, but
  $O(N \log N)$ average case performance.
- Pure quicksort not good for small arrays.
- No iterative version (without using a stack).
- "In-place," but uses auxiliary storage because of recursive calls.
- Stable?
- Used by Java for sorting arrays of primitive types.