CSE 332: Data Structures

Priority Queues – Binary Heaps

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Recall Queues

- FIFO: First-In, First-Out
  - Print jobs
  - File serving
  - Phone calls and operators
  - Lines at the Department of Licensing…

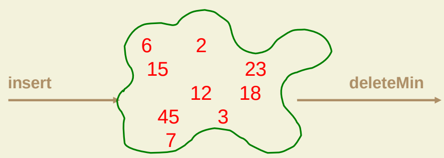
Priority Queues

Prioritize who goes first – a **priority queue**:

- treat ER patients in order of severity
- route network packets in order of urgency
- operating system can favor jobs of shorter duration or those tagged as having higher importance
- Greedy optimization: "best first" problem solving

Priority Queue ADT

- Need a new ADT
- Operations: Insert an Item,
  Remove the "Best" Item



Priority Queue ADT

1. **PQueue <u>data</u>** : collection of data with priority

2. **PQueue <u>operations</u>**
   - insert
   - deleteMin

   (also: create, destroy, is_empty)

3. **PQueue <u>property</u>**: if *x* has **<u>lower</u>** priority than *y*, *x* will be deleted before *y*

Potential implementations

	insert	deleteMin
Unsorted list (Array)		
Unsorted list (Linked-List)		
Sorted list (Array)		
Sorted list (Linked-List)		
Binary Search Tree (BST)		

Binary Heap data structure

- **binary heap** (a kind of binary tree) for priority queues:
  - O(log n) worst case for both insert and deleteMin
  - O(1) average insert
- It's optimized for priority queues. Lousy for other types of operations (e.g., searching, sorting)

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Tree Review

root(T): A

leaves(T): D-F, I-N

children(B): D-F

parent(H): G

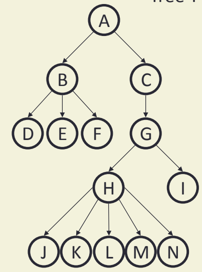
siblings(E): D,F

ancestors(F):

descendants(G):

subtree(C):

Tree T



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More Tree Terminology

depth(B):

height(G):

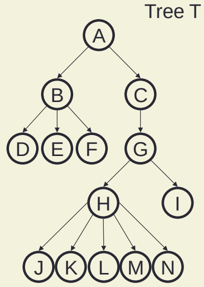
height(T):

degree(B):

branching factor(T):

n-ary tree:

Tree T



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Binary Heap Properties

A binary heap is a binary tree with two important properties that make it a good choice for priority queues:

1. **Completeness**
2. **Heap Order**

Note: we will sometimes refer to a binary heap as simply a "heap".

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Completeness Property

- A binary heap is a ***complete*** binary tree:
  - a binary tree with all levels full, except possibly the bottom level, which is filled left to right

Examples:

Height of a **complete** binary tree with n nodes?

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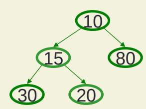
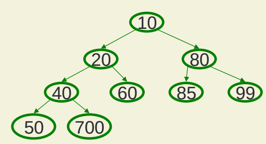
Heap **Order** Property

Heap order property: For every non-root node X, the value in the parent of X is less than (or equal to) the value in X.





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Heap Operations

- Main ops: insert, deleteMin
- Key is to maintain
  – Completeness
  – Heap Order
- Basic idea is to propagate changes up/down the tree, fixing order as we go

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Heap – insert(val)

Basic Idea:

1. Put val at last leaf position
2. Percolate up by repeatedly exchanging node with parent as long as needed

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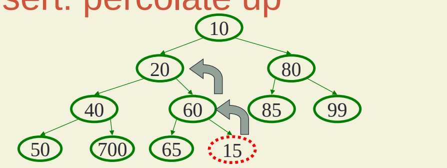
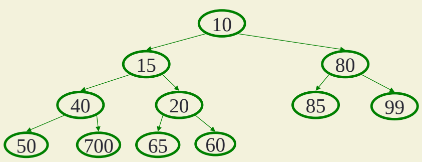
Insert: percolate up





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Heap – deleteMin

Basic Idea:

1. Remove min element
2. Put "last" leaf node value at root
3. Find smallest child of node
4. Swap node with its smallest child if needed.
5. Repeat steps 3 & 4 until no swaps needed.

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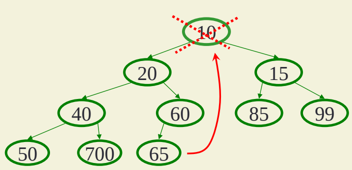
DeleteMin: percolate down



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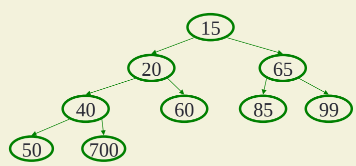
DeleteMin: percolate down



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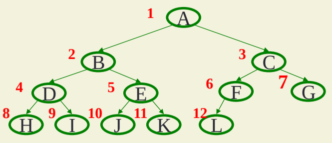
Representing Complete Binary Trees in an Array



From node **i**:

left child:
right child:
parent:

	A	B	C	D	E	F	G	H	I	J	K	L	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Why use an array?

DeleteMin Code

```
Object deleteMin() {
  assert(!isEmpty());
  returnVal = Heap[1];
  size--;
  newPos =
    percolateDown(1,
        Heap[size + 1]);
  Heap[newPos] =
    Heap[size + 1];
  return returnVal;
}
```

```
int percolateDown(int hole,
                  Object val) {
while (2*hole <= size) {
    left = 2*hole;
    right = left + 1;
    if (right ≤ size &&
        Heap[right] < Heap[left])
      target = right;
    else
      target = left;

    if (Heap[target] < val) {
      Heap[hole] = Heap[target];
      hole = target;
    }
    else
      break;
  }
  return hole;
}
```

runtime:

(Java code in book)

Insert Code

```
void insert(Object o) {
  assert(!isFull());
  size++;
  newPos =
    percolateUp(size,o);
  Heap[newPos] = o;
}
```

```
int percolateUp(int hole,
                Object val) {
  while (hole > 1 &&
         val < Heap[hole/2])
    Heap[hole] = Heap[hole/2];
    hole /= 2;
  }
  return hole;
}
```

runtime:

(Java code in book)

Insert: 16, 32, 4, 69, 105, 43, 2

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

More Priority Queue Operations

decreaseKey(nodePtr, amount):
given a pointer to a node in the queue, reduce its priority

Binary heap: change priority of node and ________________

increaseKey(nodePtr, amount):
given a pointer to a node in the queue, increase its priority

Binary heap: change priority of node and ________________

Why do we need a *pointer*? Why not simply data value?

Worst case running times?

More Priority Queue Operations

remove(objPtr):
given a pointer to an object in the queue, remove it

Binary heap: ______________________________________

findMax( ):
Find the object with the highest value in the queue

Binary heap: ______________________________________

Worst case running times?

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More Binary Heap Operations

expandHeap( ):
If heap has used up array, copy to new, larger array.
- Running time:

buildHeap(objList):
Given list of objects with priorities, fill the heap.
- Running time:

We do better with **buildHeap**...

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Building a Heap: Take 1

12	5	11	3	10	6	9	4	8	1	7	2

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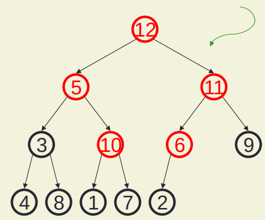
BuildHeap: Floyd's Method

12	5	11	3	10	6	9	4	8	1	7	2

Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!

Red nodes need to percolate down

Key idea: fix red nodes from **bottom-up**



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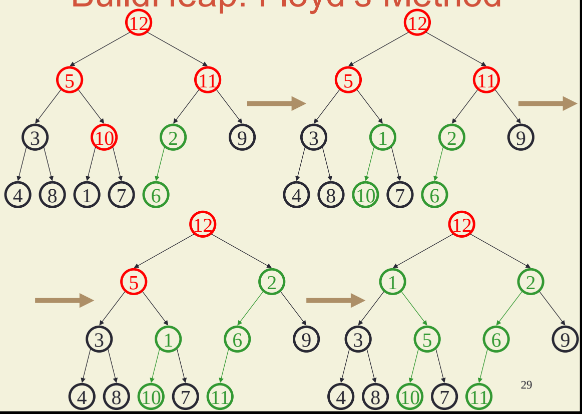
BuildHeap: Floyd's Method



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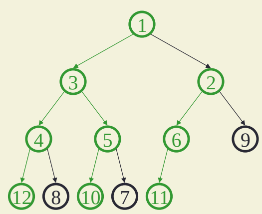
Finally…



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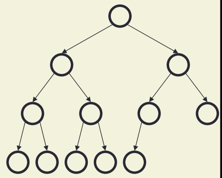
Buildheap pseudocode

```
private void buildHeap() {
  for ( int i = currentSize/2; i > 0; i-- )
     percolateDown( i );
}
```

runtime:

Buildheap Analysis

n/4 nodes percolate at most 1 level
n/8 percolate at most 2 levels
n/16 percolate at most 3 levels
...



runtime: