CSE 332: Data Abstractions

Leftover Asymptotic Analysis

Spring 2016

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Lecture 4a

Announcements

- Homework #1 due Wednesday, April 6

Definition of Order Notation

- h(n) ε O(f(n)) Big-O "Order"
  if there exist positive constants c and n_0
  such that h(n) ≤ c f(n) for all n ≥ n_0

O(f(n)) defines a class (set) of functions

Examples

- $3n^2 + 2$ nlog n is $O(n^2)$
- $3n^2 + 2$ nlog n is $O(n^3)$
- n^2 is $O(2n^2 + 14)$

Asymptotic Lower Bounds

- $\Omega(g(n))$ is the set of all functions asymptotically greater than or equal to $g(n)$
- $h(n) \in \Omega(g(n))$ iff
  There exist c>0 and n_0>0 such that $h(n) \geq c\, g(n)$ for all $n \geq n_0$

Asymptotic Tight Bound

- $\theta(f(n))$ is the set of all functions asymptotically equal to $f(n)$
- $h(n) \in \theta(f(n))$ iff
  $h(n) \in O(f(n))$ and $h(n) \in \Omega(f(n))$
  - This is equivalent to:

$$\lim_{n\to\infty} h(n)/f(n) = c \neq 0$$

Full Set of Asymptotic Bounds

- O($f(n)$) is the set of all functions asymptotically less than or equal to $f(n)$
  - o($f(n)$) is the set of all functions asymptotically strictly less than $f(n)$
- Ω($g(n)$) is the set of all functions asymptotically greater than or equal to $g(n)$
  - ω($g(n)$) is the set of all functions asymptotically strictly greater than $g(n)$
- θ($f(n)$) is the set of all functions asymptotically equal to $f(n)$

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Formal Definitions (for reference)

- $h(n) \in O(f(n))$ iff
  There exist c>0 and n_0>0 such that $h(n) \leq c f(n)$ for all $n \geq n_0$
- $h(n) \in o(f(n))$ iff
  There exists an n_0>0 such that $h(n) < c f(n)$ for all c>0 and $n \geq n_0$
  - This is equivalent to: $\lim_{n\to\infty} h(n)/f(n) = 0$
- $h(n) \in \Omega(g(n))$ iff
  There exist c>0 and n_0>0 such that $h(n) \geq c\, g(n)$ for all $n \geq n_0$
- $h(n) \in \omega(g(n))$ iff
  There exists an n_0>0 such that $h(n) > c\, g(n)$ for all c>0 and $n \geq n_0$
  - This is equivalent to: $\lim_{n\to\infty} h(n)/g(n) = \infty$
- $h(n) \in \theta(f(n))$ iff
  $h(n) \in O(f(n))$ and $h(n) \in \Omega(f(n))$
  - This is equivalent to: $\lim_{n\to\infty} h(n)/f(n) = c \neq 0$

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Big-Omega et al. Intuitively

Asymptotic Notation	Mathematics Relation
O	≤
Ω	≥
θ	=
o	<
ω	>

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Complexity cases (revisited)

Problem size **N**

- **Worst-case complexity**: **max** # steps algorithm takes on "most challenging" input of size **N**
- **Best-case complexity:** **min** # steps algorithm takes on "easiest" input of size **N**
- **Average-case complexity**: **avg** # steps algorithm takes on *random* inputs of size **N**
- **Amortized complexity**: **max** total # steps algorithm takes on **M** "most challenging" *consecutive* inputs of size **N**, divided by **M** (i.e., divide the max total by **M**).

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