CSE 332: Data Abstractions

Leftover Asymptotic Analysis

Spring 2016

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Lecture 4a

Announcements

- Homework #1 due Wednesday, April 6

Definition of Order Notation

- h(n) ϵ O(f(n)) Big-O "Order"

  if there exist positive constants c and n_0

  such that $h(n) \leq c\, f(n)$ for all $n \geq n_0$

O(f(n)) defines a class (set) of functions

Examples

- $3n^2 + 2\, n\log n$ is $O(n^2)$
- $3n^2 + 2\, n\log n$ is $O(n^3)$
- n^2 is $O(2n^2 + 14)$

Asymptotic Lower Bounds

- $\Omega(g(n))$ is the set of all functions asymptotically greater than or equal to $g(n)$

- $h(n) \in \Omega(g(n))$ iff
  There exist $c>0$ and $n_0>0$ such that $h(n) \geq c\, g(n)$ for all $n \geq n_0$

Asymptotic Tight Bound

- $\theta(\ f(n)\ )$ is the set of all functions asymptotically equal to $f(n)$

- $h(n) \in \theta(\ f(n)\ )$ iff
  $h(n) \in O(\ f(n)\ )$ and $h(n) \in \Omega(f(n)\ )$
  - This is equivalent to:

$$\lim_{n \to \infty} h(n)/f(n) = c \neq 0$$

Full Set of Asymptotic Bounds

- O($f(n)$) is the set of all functions asymptotically less than or equal to $f(n)$
  - o($f(n)$) is the set of all functions asymptotically strictly less than $f(n)$

- Ω($g(n)$) is the set of all functions asymptotically greater than or equal to $g(n)$
  - ω($g(n)$) is the set of all functions asymptotically strictly greater than $g(n)$

- θ($f(n)$) is the set of all functions asymptotically equal to $f(n)$

Formal Definitions (for reference)

- $h(n) \in O(f(n))$ iff
  There exist $c>0$ and $n_0>0$ such that $h(n) \leq c f(n)$ for all $n \geq n_0$

- $h(n) \in o(f(n))$ iff
  There exists an $n_0>0$ such that $h(n) < c f(n)$ for all $c>0$ and $n \geq n_0$
  – This is equivalent to: $\lim_{n \to \infty} h(n)/f(n) = 0$

- $h(n) \in \Omega(g(n))$ iff
  There exist $c>0$ and $n_0>0$ such that $h(n) \geq c\, g(n)$ for all $n \geq n_0$

- $h(n) \in \omega(g(n))$ iff
  There exists an $n_0>0$ such that $h(n) > c\, g(n)$ for all $c>0$ and $n \geq n_0$
  – This is equivalent to: $\lim_{n \to \infty} h(n)/g(n) = \infty$

- $h(n) \in \theta(f(n))$ iff
  $h(n) \in O(f(n))$ and $h(n) \in \Omega(f(n))$
  – This is equivalent to: $\lim_{n \to \infty} h(n)/f(n) = c \neq 0$

Big-Omega et al. Intuitively

Asymptotic Notation	Mathematics Relation
O	$\leq$
Ω	$\geq$
θ	=
o	<
ω	>

Complexity cases (revisited)

Problem size **N**

- **Worst-case complexity**: **max** # steps algorithm takes on "most challenging" input of size **N**
- **Best-case complexity: min** # steps algorithm takes on "easiest" input of size **N**

- **Average-case complexity**: **avg** # steps algorithm takes on *random* inputs of size **N**
- **Amortized complexity**: **max** total # steps algorithm takes on **M** "most challenging" *consecutive* inputs of size **N**, divided by **M** (i.e., divide the max total by **M**).