

# CSE 332 Data Abstractions, Spring 2016

## Homework 8

Due: **Friday, June 3, 2016** at the BEGINNING of lecture. Your work should be readable as well as correct. (Note the shift in deadline to give an extra couple of days due to Memorial Day).

### Problem 1: Dijkstra's Algorithm

- a) Weiss, problem 9.5(a) (the problem is the same in the 2nd and 3rd editions of the textbook). Use Dijkstra's algorithm and show the results of the algorithm in the form used in lecture — a table showing for each vertex its best-known distance from the starting vertex and its predecessor vertex on the path. Also show the **order** in which the vertices are added to the “cloud” of known vertices as the algorithm progresses.

Although the final table is all that is required, for potential partial credit I would recommend showing how your table changes over time by leaving lots of space and crossing things out as they change. Also be sure that you show the value of the path variable.

- b) If there is more than one minimum cost path from  $v$  to  $w$ , will Dijkstra's algorithm always find the path with the fewest edges? If not, explain in a few sentences how to modify Dijkstra's algorithm so that if there is more than one minimum path from  $v$  to  $w$ , a path with the fewest edges is chosen. Assume no negative weight edges or negative weight cycles.
- c) Give an example where Dijkstra's algorithm gives the wrong answer in the presence of a negative-cost edge but no negative-cost cycles. Explain briefly why Dijkstra's algorithm fails on your example. The example need not be complex; it is possible to demonstrate the point using as few as 3 vertices.
- d) Suppose you are given a graph that has negative-cost edges but no negative-cost cycles. Consider the following strategy to find shortest paths in this graph: uniformly add a constant  $k$  to the cost of every edge, so that all costs become non-negative, then run Dijkstra's algorithm and return that result with the edge costs reverted back to their original values (i.e., with  $k$  subtracted).
- Give an example where this technique fails (Dijkstra's would not find what is actually the shortest path) and explain why it fails.
  - Also, give a general explanation as to **why** this technique does not work. Think about your example and why the original least cost path is no longer the least cost path after adding  $k$ .

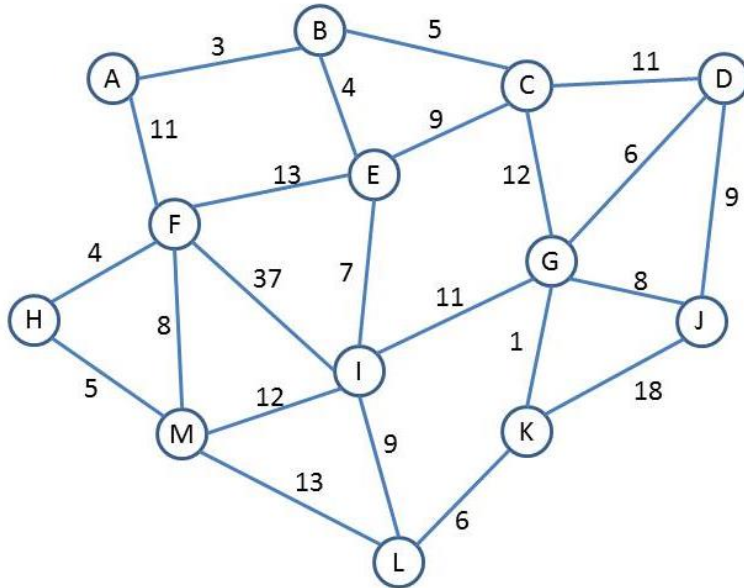
### Problem 2: Union-Find

Consider the following sequence of instructions: `union(1,2)`, `union(3,4)`, `union(3,5)`, `union(1,7)`, `union(3,6)`, `union(8,9)`, `union(1,8)`, `union(3,10)`, `union(3,11)`, `union(3,12)`, `union(3,13)`, `union(14,15)`, `union(16,0)`, `union(14,16)`, `union(1,3)`, `union(1,14)`.

- a) Show the result when the unions are arbitrary (the second points to the first.)
- b) Show the result when the unions are by size.
- c) For the trees in parts a and b, show the result of performing a single `find` operation with path compression from the deepest node.

### Problem 3: Minimum Spanning Tree

- a) Find a minimum spanning tree in the following graph using Kruskal's algorithm



- b) Is the minimum spanning tree for a graph always unique? Explain.  
c) How does the minimum spanning tree for a graph change if you add one to every edge in the graph? Explain.