

Let x and L be `LinkedList` Nodes.

Analyzing append

```

1 append(x, L) {
2     Node curr = L;
3     while (curr != null && curr.next != null) {
4         curr = curr.next;
5     }
6     curr.next = x;
7 }
```

$O(n)$

LinkedList Reversal

```

1 reverse(L) {
2     if (L == null) {
3         return null;
4     } → else if (L.next == null) {
5         return L;
6     } else {
7         Node front = L;
8         Node rest = L.next;
9         L.next = null;
10
11         Node restReversed = reverse(rest);
12         append(front, restReversed);
13     }
14 }
```

$$T(n) = \begin{cases} d_0 & \text{if } n = 0 \text{ or } 1 \\ c_0 + c_1 n + T(n-1) & \text{otherwise} \end{cases}$$

$$\begin{aligned} \rightarrow T(n) &= (c_0 + c_1 n) + T(n-1) \\ &= (c_0 + c_1 n) + (c_0 + c_1(n-1)) + T(n-2) \\ &= (c_0 + c_1 n) + (c_0 + c_1(n-1)) + (c_0 + c_1(n-2)) + \dots + (c_0 + c_1(1)) + d_0 \end{aligned}$$

$$= \sum_{i=1}^n (c_0 + c_1 i) + d_0$$

$$= n \cdot c_0 + c_1 \sum_{i=1}^n i + d_0$$

$$= n \cdot c_0 + c_1 \frac{n(n+1)}{2} + d_0 = O(n^2)$$

```

int sum(int[] arr) {
    return help(arr, 0, arr.length);
}

int help(int[] arr, int lo, int hi) {
    if (lo == hi) return 0;
    if (lo == hi - 1) return arr[lo];
    int mid = (hi + lo) / 2;
    return help(arr, lo, mid) + help(arr, mid, hi);
}

```

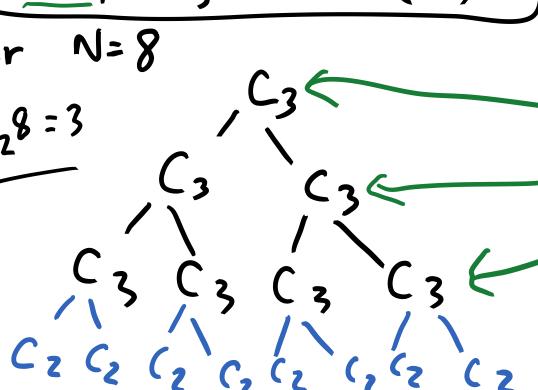
$$T(0) = C_1$$

$$T(1) = C_2$$

$$T(N) = C_3 + 2 \cdot T\left(\frac{N}{2}\right)$$

For $N=8$

$$\log_2 8 = 3$$



"level"

$$T\left(\frac{N}{2}\right) \quad T\left(\frac{N}{2}\right)$$

nodes
at this level

Total Work
at this level

$$1 \cdot C_3$$

$$2 \cdot C_3$$

$$4 \cdot C_3$$

$$2^i \cdot C_3$$

$$T(N) = \sum_{i=0}^{\lfloor \log_2 N \rfloor - 1} 2^i C_3 + C_2 \cdot N$$

$$= C_3 \sum_{i=0}^{\lfloor \log_2 N \rfloor - 1} 2^i + C_2 \cdot N$$

$$= C_3 \cdot \left(2^{\lfloor \log_2 N \rfloor} - 1 \right) + C_2 \cdot N$$

$$= C_3 \cdot (N - 1) + C_2 \cdot N = C_3 \cdot N - C_3 + C_2 \cdot N = O(N)$$

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$