

Exercises (Graphs)

Directions: *Submit your solutions on Canvas. Submit a pdf file.*

EX17. Dijkstra? (20 points)

- (a) [7 Points] If there is more than one minimum cost path from v to w , will Dijkstra's Algorithm always find the path with the fewest edges? If not, explain in a few sentences how to modify Dijkstra's algorithm so that if there is more than one minimum path from v to w , a path with the fewest edges is chosen. Assume no negative-cost edges.
- (b) [6 Points] Give an example where Dijkstra's Algorithm gives the wrong answer in the presence of a negative-cost edge but no negative-cost cycles. Explain briefly why Dijkstra's Algorithm fails on your example. The example need not be complex; it is possible to demonstrate the point using as few as 3 vertices.
- (c) [7 Points] Suppose you are given a graph that has negative-cost edges but no negative-cost cycles. Consider the following strategy to find shortest paths in this graph: Uniformly add a constant k to the cost of every edge, so that all costs become non-negative, then run Dijkstra's Algorithm and return that result with the edge costs reverted back to their original values (i.e., with k subtracted).
- Give an example where this technique fails (Dijkstra's would not find what is actually the shortest path) and explain why it fails.
 - Also, give a general explanation as to why this technique does not work. Think about your example and why the original least cost path is no longer the least cost path after adding k .