CSE332: Data Abstractions
Section 6

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Adapted from slides by Hye In Kim & Ruth Anderson

Today

- Announcements
- Questions?
- Graph Review
- Graph Traversals
  - Breadth First Search
  - Depth First Search

Announcements

- Midterm is Over!
- Project 2 Phase B:
  - Due Tuesday February 17th 11pm
- Written HW 4 part B
  - Tonight 11pm
- Written HW 5 is out
  - Due Friday February 20th 11pm

Questions

Questions about Written Homework 4b?

Questions about Project 2?

Other questions?

Graph Review

G = (V, E)

- Contains set of vertices and set of edges
  - | V | = number of vertices
  - | E | = number of edges
- Max | E | for undirected graph

  $|V| + (|V| - 1) + (|V| - 2) + \ldots + 1 = |V|(|V| + 1) / 2$
- Max | E | for directed graph

  $|V| + |V| + |V| + \ldots + |V| = |V| * |V| = |V|^2$

Graph Review

Path

- List of vertices $[v_0, v_1, ..., v_n]$, such that $(v_i, v_{i+1}) \in E$ for all $0 \le i < n$
  - Path length = number of edges on path
  - Path cost = sum of all edge weights on path

Cycle

- A path that begins and ends at the same node

Graph Review: Undirected

- Edges have no directions
- Connected
  - There is a path between all pairs of vertices
- Fully Connected
  - There is an edge between all pairs of vertices

Graph Review: Directed

- Edges have direction
- Weakly Connected
  - There is an undirected path between all pairs of vertices
- Strongly Connected
  - There is a directed path between all pairs of vertices
- Fully Connected
  - If there is edge (both way) between all pairs of vertices

Graph Traversals

- For an arbitrary graph and a starting node **v**, find all nodes reachable (i.e., there exists a path) from **v**
  - Possibly "do something" for each node (an iterator!)
    - E.g. Print to output, set some field, etc.
- Related:
  - Is an undirected graph connected?
  - Is a directed graph weakly / strongly connected?
    - For strongly, need a cycle back to starting node
- Basic idea:
  - Keep following nodes
  - But "mark" nodes after visiting them, so the traversal terminates and processes each reachable node exactly once

Graph Traversals: Abstract Idea

```
traverseGraph(Node start) {
   Set pending = emptySet();
   pending.add(start)
   mark start as visited
   while(pending is not empty) {
      next = pending.remove()
      for each node u adjacent to next
         if(u is not marked) {
            mark u
            pending.add(u)
         }
      }
   }
}
```

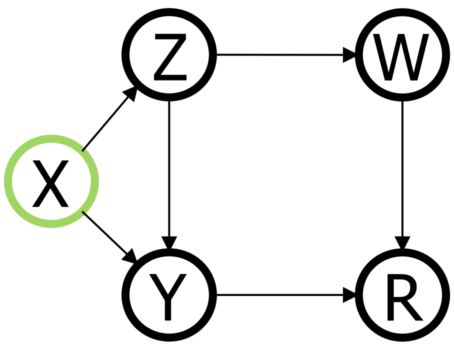
Graph Traversals: Running Time

- Assuming add and remove are O(1), entire traversal is O(|E|)
  - Use an adjacency list representation
- The order we traverse depends entirely on how add and remove work/are implemented
  - Depth-first graph search (DFS): a stack
  - Breadth-first graph search (BFS): a queue
- DFS and BFS are "big ideas" in computer science
  - Depth: recursively explore one part before going back to the other parts not yet explored
  - Breadth: Explore areas closer to the start node first

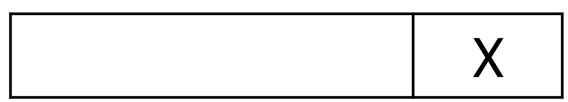
Breadth First Search

- Pick the shallowest unmarked node
  - Uses a **queue**, enqueue new nodes at the end
- BFS starting from node X

  Order Processed:



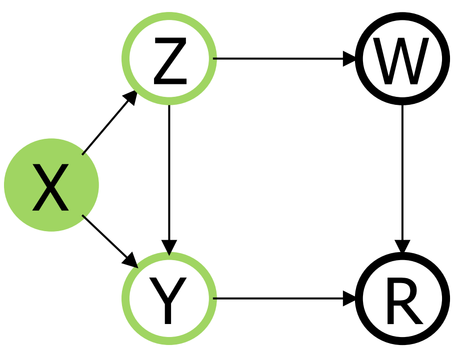
Mark X, and enqueue it



Breadth First Search

- Pick the shallowest unmarked node
  - Uses a **queue**, enqueue new nodes at the end
- BFS starting from node X

  Order Processed: X



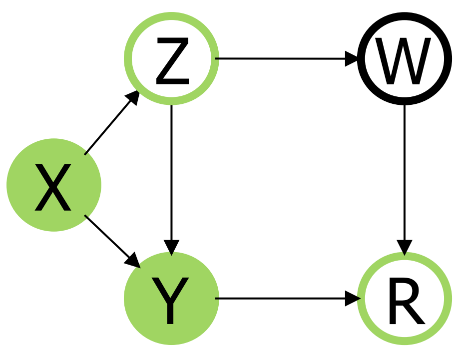
Dequeue X, process it,
mark and enqueue X's neighbors

	Z	Y

Breadth First Search

- Pick the shallowest unmarked node
  - Uses a **queue**, enqueue new nodes at the end
- BFS starting from node X

  Order Processed: X Y

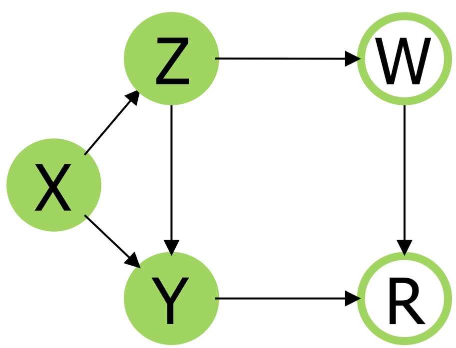


Dequeue Y, process it,
mark and enqueue Y’s neighbors

	R	Z

Breadth First Search

- Pick the shallowest unmarked node
  - Uses a **queue**, enqueue new nodes at the end
- BFS starting from node X
  Order Processed: X Y Z



Dequeue Z, process it,
mark and enqueue Z's neighbors

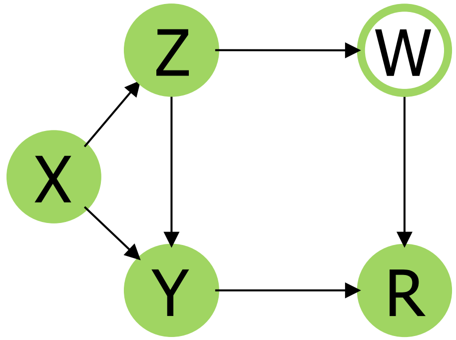
	W	R

NOTE: Do not add neighbors that have already been marked

Breadth First Search

- Pick the shallowest unmarked node
  - Uses a **queue**, enqueue new nodes at the end
- BFS starting from node X

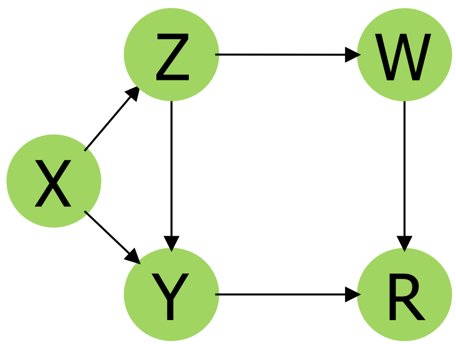
  Order Processed: X Y Z R



Dequeue R, process it,
mark and enqueue R’s neighbors

Breadth First Search

- Pick the shallowest unmarked node
  - Uses a **queue**, enqueue new nodes at the end
- BFS starting from node X
  Order Processed: X Y Z R W

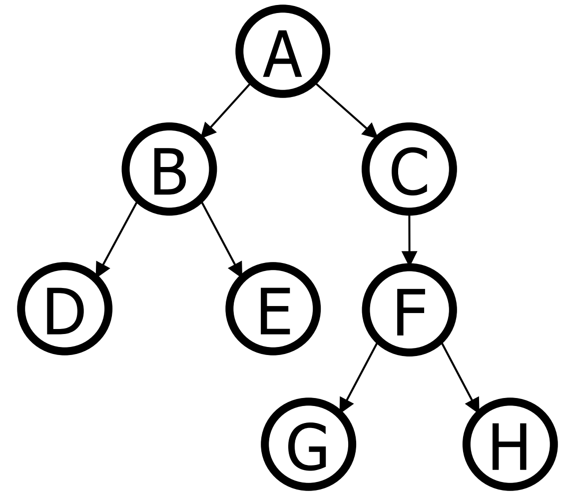


Dequeue W, process it,
mark and enqueue W's neighbors

NOTE: Do not add neighbors that have already been marked

Breadth First Search

```
BFS(Node start) {
    initialize queue q to hold start
    mark start as visited
    while(q is not empty) {
        next = q.dequeue()// and “process”
            for each node u adjacent to next
                if(u is not marked)
                    mark u and enqueue onto q
    }
}
```

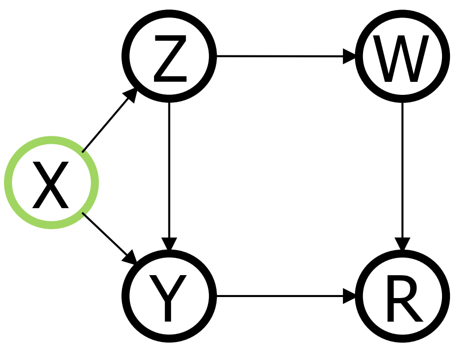


- Order Processed: A B C D E F G H
  - A “level-order” traversal

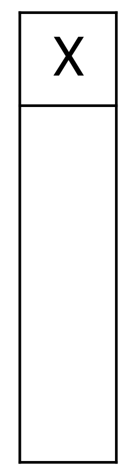
Depth First Search

- Pick the deepest unmarked node
  - Uses a **stack**, push new nodes on the top
- DFS starting from node X

  Order Processed:



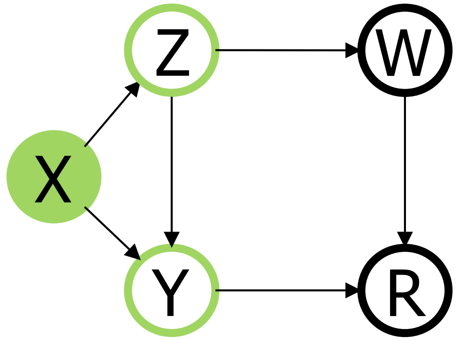
Mark X and push it



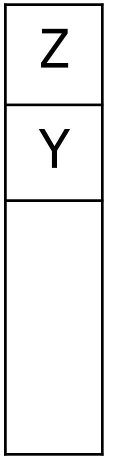
Depth First Search

- Pick the deepest unmarked node
  - Uses a **stack**, push new nodes on the top
- DFS starting from node X

  Order Processed: X



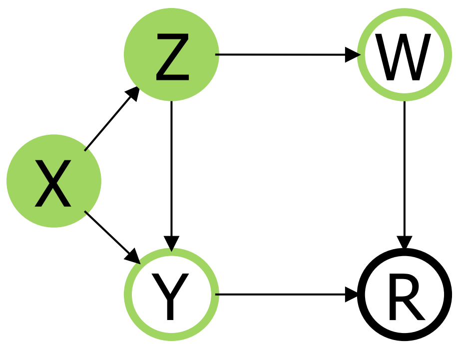
Pop X, process it, mark and push X's neighbors



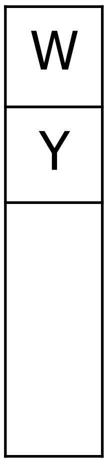
Depth First Search

- Pick the deepest unmarked node
  - Uses a **stack**, push new nodes on the top
- DFS starting from node X

  Order Processed: X Z



Pop Z, process it, mark and push Z's neighbors

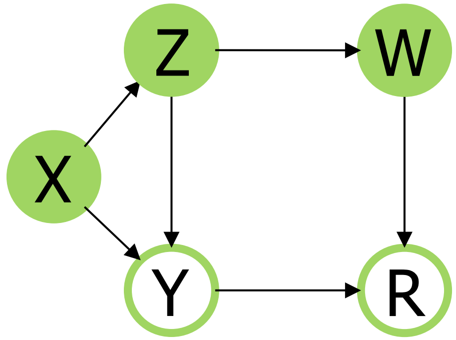


NOTE: Do not add neighbors that are already marked

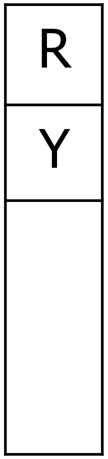
Depth First Search

- Pick the deepest unmarked node
  - Uses a **stack**, push new nodes on the top
- DFS starting from node X

  Order Processed: X Z W



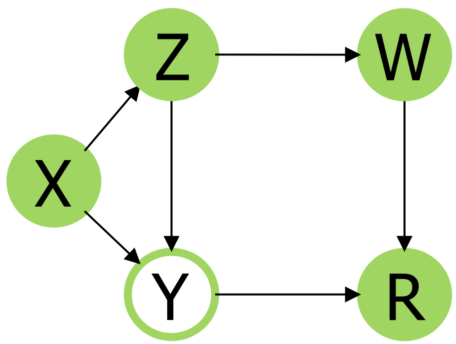
Pop W, process it, mark and push W's neighbors



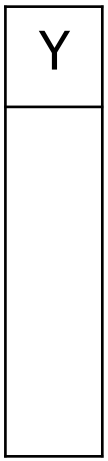
Depth First Search

- Pick the deepest unmarked node
  - Uses a **stack**, push new nodes on the top
- DFS starting from node X

  Order Processed: X Z W R

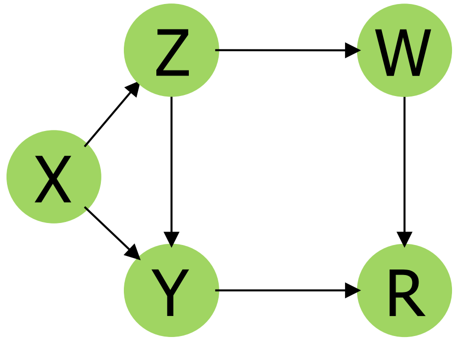


Pop R, process it, mark and push R's neighbors

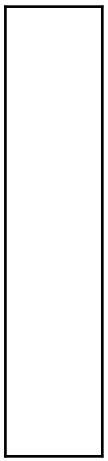


Depth First Search

- Pick the deepest unmarked node
  - Uses a **stack**, push new nodes on the top
- DFS starting from node X
  Order Processed: X Z W R Y



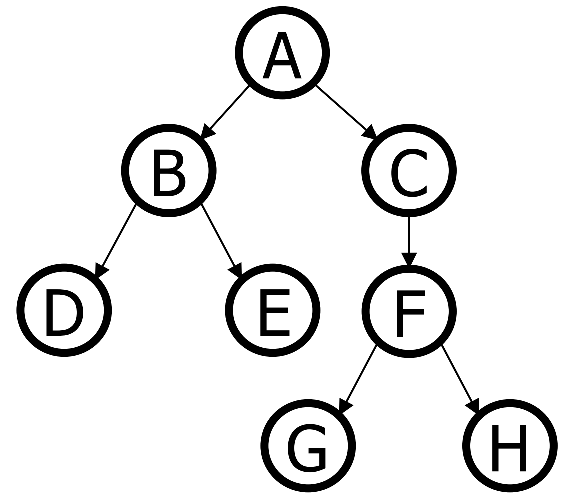
Pop Y, process it, mark and push Y's neighbors



NOTE: Do not add neighbors that are already marked

Depth First Search: Recursive

```
DFS(Node start) {
    mark and “process” (e.g. print) start
    for each node u adjacent to start
        if u is not marked
            DFS(u)
}
```

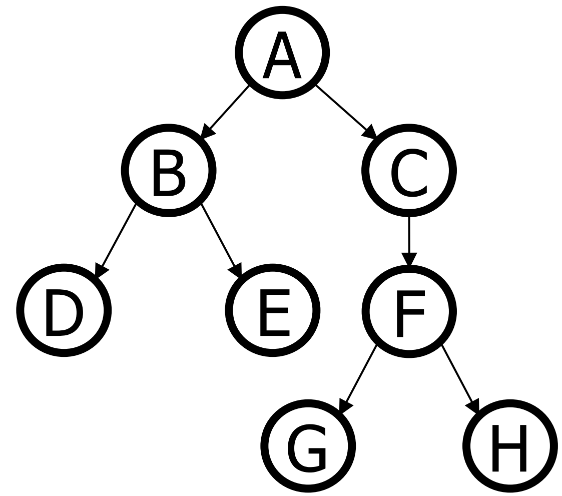


Order Processed: A B D E C F G H

- Exactly what we called a “pre-order traversal” for trees
- The marking is not needed here, but we need it to support arbitrary graphs , we need a way to process each node exactly once

Depth First Search: With a Stack

```
DFS2(Node start) {
    initialize stack s to hold start
    mark start as visited
    while(s is not empty) {
        next = s.pop() // and "process"
        for each node u adjacent to next
            if(u is not marked)
                mark u and push onto s
    }
}
```



Order Processed: A C F H G B E D

- A different but perfectly fine traversal

DFS/BFS Comparison

Breadth-first search:

- Always finds shortest paths, i.e., "optimal solutions"
  - Better for "what is the shortest path from **x** to **y**"
- Queue may hold O(|V|) nodes (e.g. at the bottom level of binary tree of height h, 2^h nodes in queue)

Depth-first search:

- Can use less space in finding a path
  - If *longest* path in the graph is **p** and highest out-degree is d then DFS stack never has more than **d*p** elements

A third approach: *Iterative deepening (IDDFS):*

- Try DFS but don't allow recursion more than **K** levels deep.
  - If that fails, increment **K** and start the entire search over
- Like BFS, finds shortest paths. Like DFS, less space.

Saving the Path

- Our graph traversals can answer the "reachability question":
  - "***<u>Is there</u>*** a path from node x to node y?"
- Q: But what if we want to ***<u>output the actual path</u>***?
  - Like getting driving directions rather than just knowing it's
- possible to get there!
- A: Like this:
  - Instead of just "marking" a node, store the **<u>previous node</u>** along the path (when processing **u** causes us to add **v** to the search, set **v.path** field to be **u**)
  - When you reach the goal, follow **path** fields backwards to where you started (and then reverse the answer)
  - If just wanted path *length*, could put the integer distance at each node instead