

## Section 1: Asymptotics

### 0. Big-Oh Proofs

For each of the following, prove that  $f \in \mathcal{O}(g)$ .

(a)  $f(n) = 7n$   $g(n) = \frac{n}{10}$

(b)  $f(n) = 1000$   $g(n) = 3n^3$

(c)  $f(n) = 7n^2 + 3n$   $g(n) = n^4$

(d)  $f(n) = n + 2n \lg n$   $g(n) = n \lg n$

## 1. Asymptotics Disproof

Prove that  $n^2 \notin \mathcal{O}(n)$ .

## 2. Is Your Program Running? Better Catch It!

For each of the following, determine the asymptotic worst-case runtime in terms of  $n$ .

(a)

```
1 int x = 0;
2 for (int i = n; i >= 0; i--) {
3     if ((i % 3) == 0) {
4         break;
5     }
6     else {
7         x += n;
8     }
9 }
```

(b)

```
1 int x = 0;
2 for (int i = 0; i < n; i++) {
3     for (int j = 0; j < (n * n / 3); j++) {
4         x += j;
5     }
6 }
```

(c)

```
1 int x = 0;
2 for (int i = 0; i <= n; i++) {
3     for (int j = 0; j < (i * i); j++) {
4         x += j;
5     }
6 }
```

## 3. Induction Shminduction

Prove  $\sum_{i=0}^n 2^i = 2^{n+1} - 1$  by induction on  $n$ .

## 4. The Implications of Asymptotics

For each of the following, determine if the statement is true or false.

(a)  $f(n) \in \Theta(g(n)) \rightarrow f(n) \in \mathcal{O}(g(n))$

(b)  $f(n) \in \Theta(g(n)) \rightarrow g(n) \in \Theta(f(n))$

(c)  $f(n) \in \Omega(g(n)) \rightarrow g(n) \in \mathcal{O}(f(n))$

## 5. Asymptotic Analysis

For each of the following, determine if  $f \in \mathcal{O}(g)$ ,  $f \in \Omega(g)$ ,  $f \in \Theta(g)$ , several of these, or none of these.

(a)  $f(n) = \log n$   $g(n) = \log \log n$

(b)  $f(n) = 2^n$   $g(n) = 3^n$

(c)  $f(n) = 2^{2n}$   $g(n) = 2^n$