CSE 332

Data Abstractions

Dictionaries & Trees

Outline

ADT's So Far

Where We've Been So Far

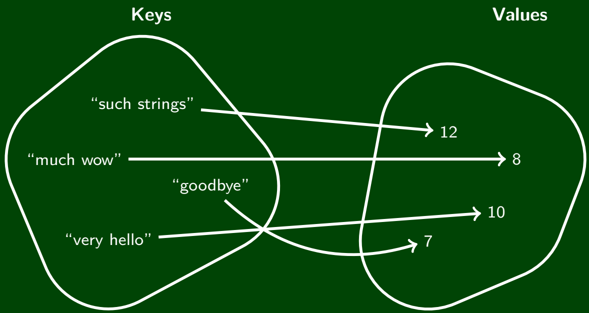
- Stack (Get LIFO)
- Queue (Get FIFO)
- Priority Queue (Get By Priority)

Today, we begin discussing **Map**s. This ADT is hugely important.

A New ADT: "Dictionaries" (Also called "Maps")

Dictionary ADT

Data	**Set** of **Comparable** (key, value) pairs
insert(**key**, **val**)	Places (**key**, **val**) in map (overwrites existing **val** entry)
find(**key**)	Returns the **val** currently associated to **key**
delete(**key**)	deletes any pair relating **key** from the map



find("such strings") → 12

Sets and Maps

Dictionaries are the **more general** structure, but, in terms of implementation, they're nearly identical.

In a `Set`, we store the `key` directly, but conceptually, there's nothing different in storing an **Entry**:

```
class Entry {
   Data key;
   Data value;
}
```

The `Set ADT` usually has our favorite operations: intersection, union, etc.

Notice that union, intersection, etc. **still make sense on maps**!

As always, depending on our usage, we might choose to add/delete things from out `ADT`.

Bottom Line: If we have a set implementation, we also have a valid dictionary implementation (and vice versa)!

Dictionaries Are The BEST!

It turns out dictionaries are super useful. They're a natural generalization of arrays. Instead of storing data at an index, we store data at **anything**.

- Networks: router tables
- Operating Systems: page tables
- Compilers: symbol tables
- Databases: dictionaries with other nice properties
- Search: inverted indexes, phone directories, . . .
- Biology: genome maps

Dictionary Implementations, Take # 1

For each of the following potential implementations, what is the worst case runtime for `insert`, `find`, `delete`?

- Unsorted Array
- Unsorted Linked List
- Sorted Linked List
- Sorted Array List

For each of the following potential implementations, what is the worst case runtime for `insert`, `find`, `delete`?

- Unsorted Array
  Insert by searching for existence and inserting which is $\mathcal{O}(n)$
  Find by linear search which is $\mathcal{O}(n)$
  Delete by linear search AND shift which is $\mathcal{O}(n)$
- Unsorted Linked List
  Insert by searching for existence and inserting which is $\mathcal{O}(n)$
  Find by linear search which is $\mathcal{O}(n)$
  Delete by linear search AND shift which is $\mathcal{O}(n)$
- Sorted Linked List
  Insert by searching for existence and inserting which is $\mathcal{O}(n)$
  Find by linear search which is $\mathcal{O}(n)$
  Delete by linear search AND shift which is $\mathcal{O}(n)$
- Sorted Array List
  Insert by binary search AND shift which is $\mathcal{O}(n)$
  Find by binary search which is $\mathcal{O}(\lg n)$
  Delete by binary search AND shift which is $\mathcal{O}(n)$

It turns out there are **many** different ways to do much better.

But they all have their own trade-offs!

So, we'll study many of them:

- "Vanilla BSTs" – today (vanilla because they're "plain")
- "Balanced BSTs" – there are many types: we'll study **AVL Trees**
- "B-Trees" – another strategy for **a lot of data**
- "Hashtables" – a completely different strategy (lack data ordering)
- We already saw another strategy: the amortized array dictionary

Binary Search is great! It's the only thing that was even sort of fast in that table. But `insert` and `delete` are really bad into a `sorted array`. Store the data in a structure where **most of the data isn't accessed**.

Interestingly, this is **very similar** to what made heaps useful!

To put it another way, by storing the data in an **array**, we're paying for the constant-time access that we're never even using!

It's **okay** that it takes more time to access certain elements.

. . . as long as it's **never** too bad.

Definition (Vanilla BST)

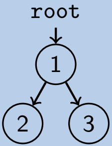
A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

To put it another way, a binary tree is a **BST** when:

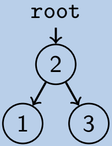
- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Am I A BST?

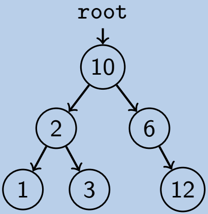
Example (Which of the following are BSTs?)



NO

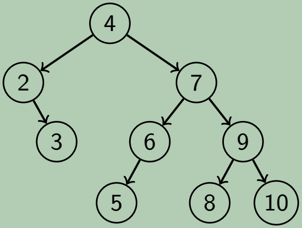


YES



NO

BST Properties



Structure Property:
0, 1, or 2 children

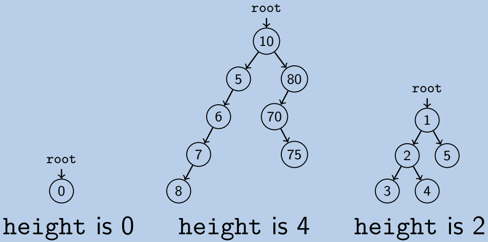
BST Property:
Keys in Left Subtree are smaller
Keys in Right Subtree are larger

Definition (Height)

The **height** of a binary tree is the length of the longest **path** from the root to a leaf.

- Height of an empty tree? **-1**
- Height of ⓧ? **0**

height




```java
private int height(Node current) {
    if (current == null) { return −1; }
    return 1 + Math.max(height(current.left), height(current.right));
}
```

Height

```
private int height(Node current) {
    if (current == null) { return -1; }
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}
```

Given that a tree has height h...

- What is the maximum number of **leaves**?
- What is the maximum number of **nodes**?
- What is the minimum number of **leaves**?
- What is the minimum number of **nodes**?

That's a big spread!

This confirms what we already know: `height` in a tree has a big impact on runtime.

Height

```java
private int height(Node current) {
   if (current == null) { return -1; }
   return 1 + Math.max(height(current.left), height(current.right));
}
```

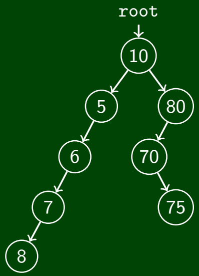
Given that a tree has height h...

- What is the maximum number of **leaves**? 2^h
- What is the maximum number of **nodes**? $2^{h+1} - 1$
- What is the minimum number of **leaves**? 1
- What is the minimum number of **nodes**? $h + 1$

That's a big spread!

This confirms what we already know: `height` in a tree has a big impact on runtime.

find Review



What about other finds?

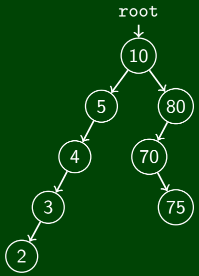
- findMin?
- findMax?
- deleteMin?

Recursive find

```
Data find(Key key, Node curr) {
   if (curr == null) { return null; }
   if (key < curr.key) {
      return find(key, curr.left);
   }
   if (key > curr.key) {
      return find(key, curr.right);
   }
   return curr.data;
}
```

Iterative find

```
Data find(Key key) {
   Node curr = root;
   while (curr != null && curr.key != key) {
      if (key < curr.key) {
         curr = curr.left;
      }
      else (key > curr.key) {
         curr = curr.right;
      }
   }
   if (curr == null) { return null; }
   return curr.data;
}
```

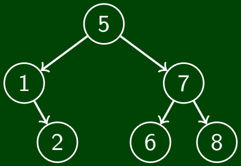


insert

- find
- create a new node

How about delete?

Consider the following tree:

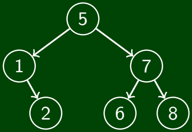


Let's try the following removals:

- `tree.delete(2)`
- `tree.delete(1)`
- `tree.delete(7)`
- `tree.delete(5)`

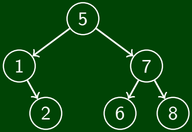
`delete` from a BST

`tree.delete(2)`



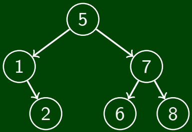
⟶

`tree.delete(1)`



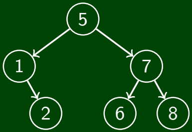
⟶

`tree.delete(7)`



⟶

`tree.delete(5)`



⟶

`delete` from a BST

`tree.delete(2)`

`tree.delete(1)`

`tree.delete(7)`

`tree.delete(5)`

`tree.delete(2)`

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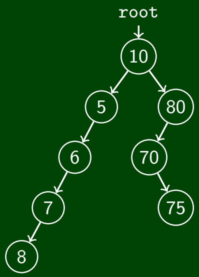


delete(x)

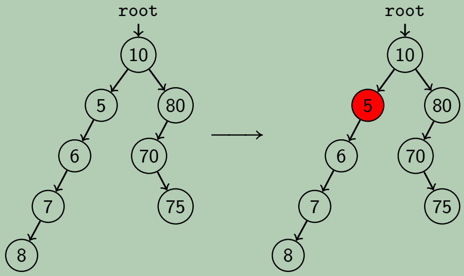
- Case 1: x is a leaf
  - Just delete x
- Case 2: x has one child
  - Replace x with its child
- Case 3: x has two children
  - Replace x with the **successor** or **predecessor** of x

The tricky case is when x has two children. If we think of the BST in sorted array form, to get the successor, we `findMin`(right subtree) (or predecessor is `findMax`(left subtree))

Instead of doing this complicated algorithm, here's an idea:

Mark the node as "deleted" instead of doing anything

`lazyDelete(5)`



Then, `insert` and `find` change slightly, but the whole thing is much simpler.

This "lazy deletion" is a very useful strategy!

Psuedocode

```
void buildTree(int[] input) {
   for (int i = 0; i < input.length; i++) {
      insert(input[i]);
   }
}
```

What's the best case? The worst case?

The worst case is a sorted input which is $\mathcal{O}(n^2)$. Ouch.

The Good News

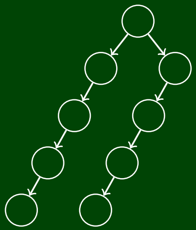
On average, we get $\mathcal{O}(\lg n)$ height (see textbook for proof). But we want it to **always** be $\mathcal{O}(\lg n)$ height...

The Solution

Add restrictions on the height of the tree. Somehow, the tree should "fix itself" so it never has too large a height.
We call this condition a **Balance Condition**.

Ideas?

- Left and right subtrees of the root have the same number of nodes
- Left and right subtrees of the root have the same **height**

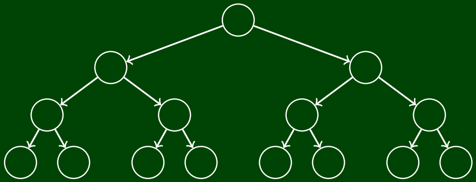


These ideas suffer from the same problem:

They're **local** conditions rather than **global** ones.

Ideas?

- Left and right subtrees ~~of the root~~ **recursively** have the same number of nodes
- Left and right subtrees ~~of the root~~ **recursively** have the same **height**



These ideas suffer from the same problem:

They're way too strong. Only **perfect** trees satisfy them.

AVL Balance Condition!

Left and right subtrees **recursively** have heights differing by at most one.

Definition (`balance`)

$$\text{balance}(\texttt{n}) = \text{abs}(\text{height}(\texttt{n.left}) - \text{height}(\texttt{n.right}))$$

Definition (AVL Balance Property)

An AVL tree is balanced when:

For every node n, balance$(n) \leq 1$

- This ensures a small depth (we'll prove this next time)
- It's relatively easy to maintain (we'll see this next time)