CSE 332:
Analysis of Fork-Join Parallel Programs

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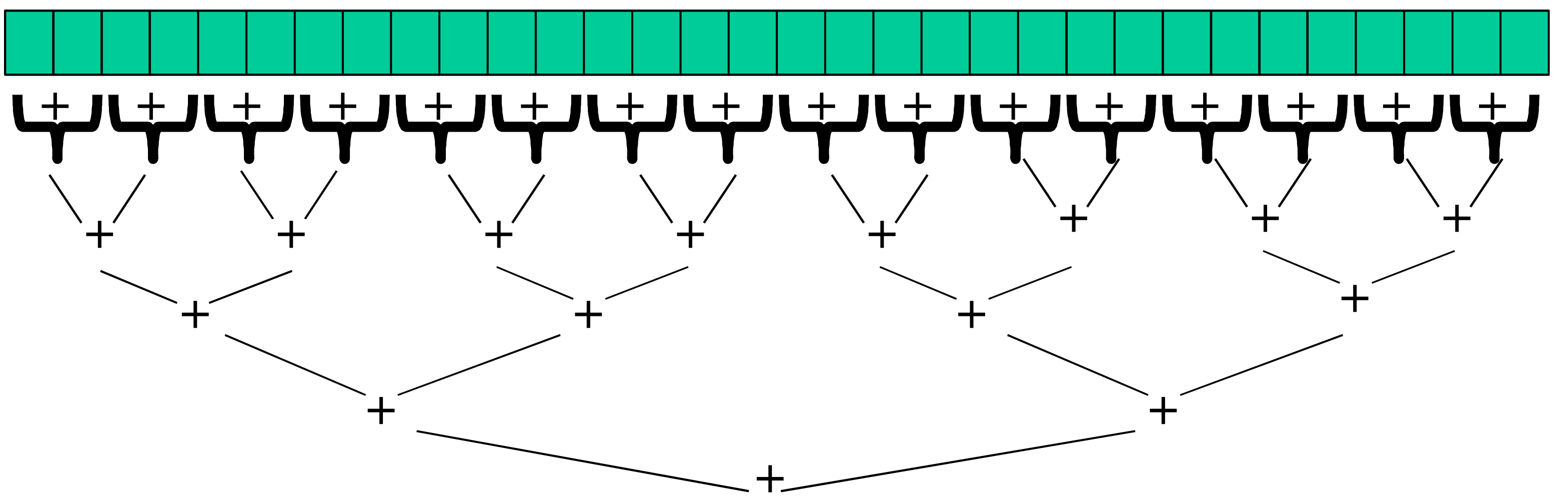
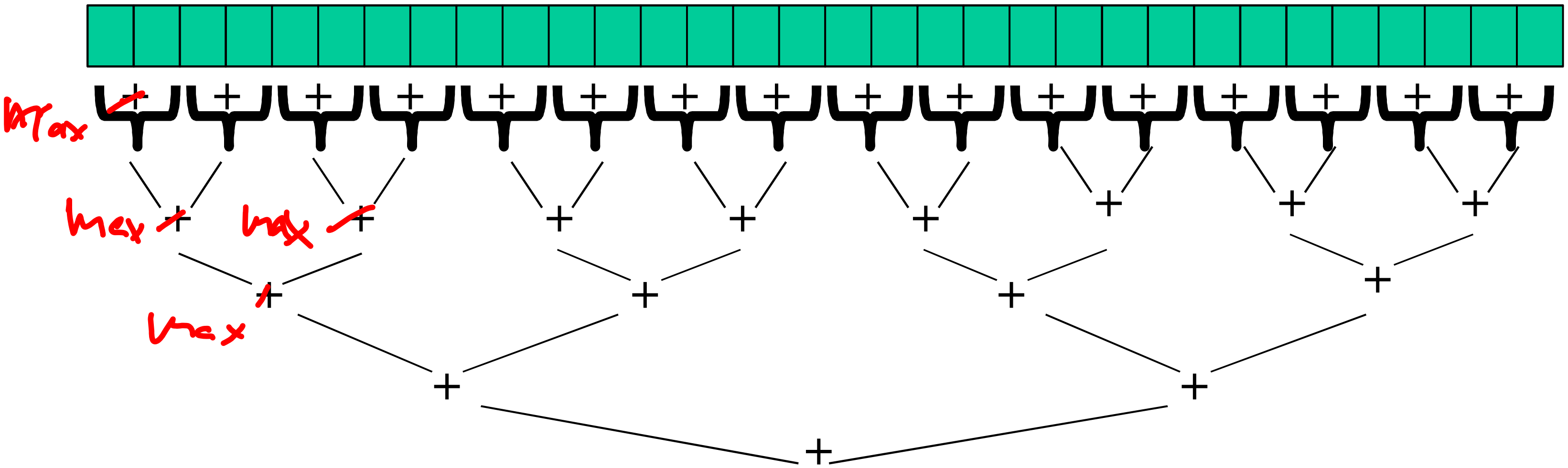
Parallel Sum

- Sum up N numbers in an array



Parallel Max?



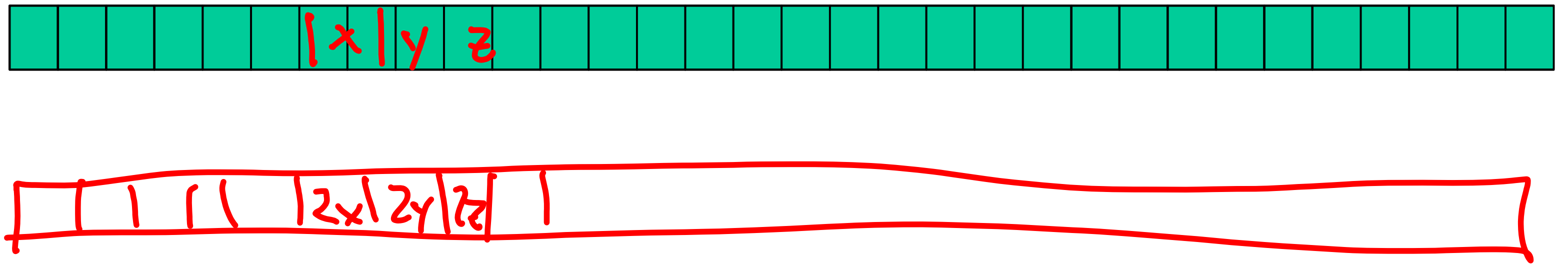
Reductions

works for associative operations $(a+b)+c$

- Same trick works for many tasks, e.g., $= a + (b+c)$
  - is there an element satisfying some property (e.g., prime)
  - left-most element satisfying some property (e.g., first prime)
  - smallest rectangle encompassing a set of points (proj3)
  - counts: number of strings that start with a vowel
  - are these elements in sorted order?
- Called a reduction, or reduce operation
  - reduce a collection of data items to a single item
    - result can be more than a single value, e.g., produce histogram from a set of test scores
- Very common parallel programming pattern

Parallel Vector Scaling

- Multiply every element in the array by 2



N proc (threads) multiply each separately
then store

Maps

- A map operates on each element of a collection of data to produce a new collection of the same size
  - each element is processed independently of the others, e.g.
    - vector scaling
    - vector addition
    - test property of each element (is it prime)
    - uppercase to lowercase
    - ...
- Another common parallel programming pattern

Maps in ForkJoin Framework

```
class VecAdd extends RecursiveAction {
  int lo; int hi; int[] res; int[] arr1; int[] arr2;
  VecAdd(int l,int h,int[] r,int[] a1,int[] a2){ … }
  protected void compute(){
    if(hi – lo < SEQUENTIAL_CUTOFF) {
      for(int i=lo; i < hi; i++)
        res[i] = arr1[i] + arr2[i];
    } else {
      int mid = (hi+lo)/2;
      VecAdd left = new VecAdd(lo,mid,res,arr1,arr2);
      VecAdd right= new VecAdd(mid,hi,res,arr1,arr2);
      left.fork();
      right.compute();
      left.join();
    }
  }
}
static final ForkJoinPool fjPool = new ForkJoinPool();
int[] add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  int[] ans = new int[arr1.length];
  fjPool.invoke(new VecAdd(0,arr.length,ans,arr1,arr2);
  return ans;
}
```

Maps and Reductions

Maps and reductions: the “workhorses” of parallel programming

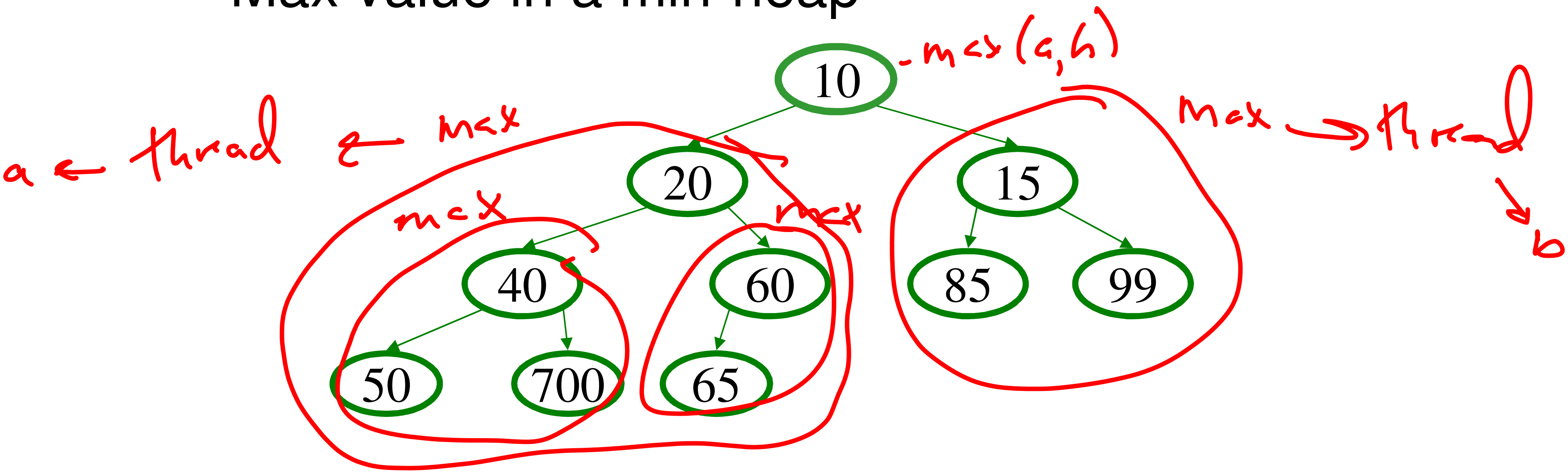
- By far the most important and common patterns
- Learn to recognize when an algorithm can be written in terms of maps and reductions
- makes parallel programming easy (plug and play)

Distributed Map Reduce

- You may have heard of Google's map/reduce
  - or open-source version called Hadoop
  - powers much of Google's infrastructure
- Idea: maps/reductions using many machines
  - same principles, applied to distributed computing
  - system takes care of distributing data, fault-tolerance
  - you just write code to handle one element, reduce a collection
- Co-developed by Jeff Dean (UW alum!)

Maps and Reductions on Trees

- Max value in a min-heap



- How to parallelize?
- Is this a map or a reduce? reduce
- Complexity? O(log N)

Analyzing Parallel Programs

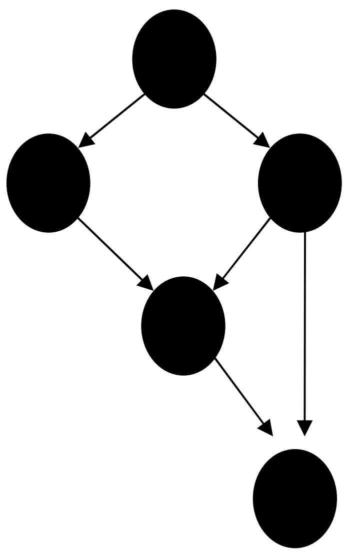
Let $\mathbf{T_P}$ be the running time on **P** processors

Two key measures of run-time:

- Work: How long it would take 1 processor = $\mathbf{T_1}$
- Span: How long it would take infinity processors = $\mathbf{T_\infty}$
  - The hypothetical ideal for parallelization
  - This is the longest "dependence chain" in the computation
  - Example: $O($`log` $n)$ for summing an array
  - Also called "critical path length" or "computational depth"

The DAG

- Fork-join programs can be modeled with a DAG
  - nodes: pieces of work
  - edges: order dependencies



A `fork` creates two children

- new thread
- continuation of current thread

A `join` makes a node with two incoming edges

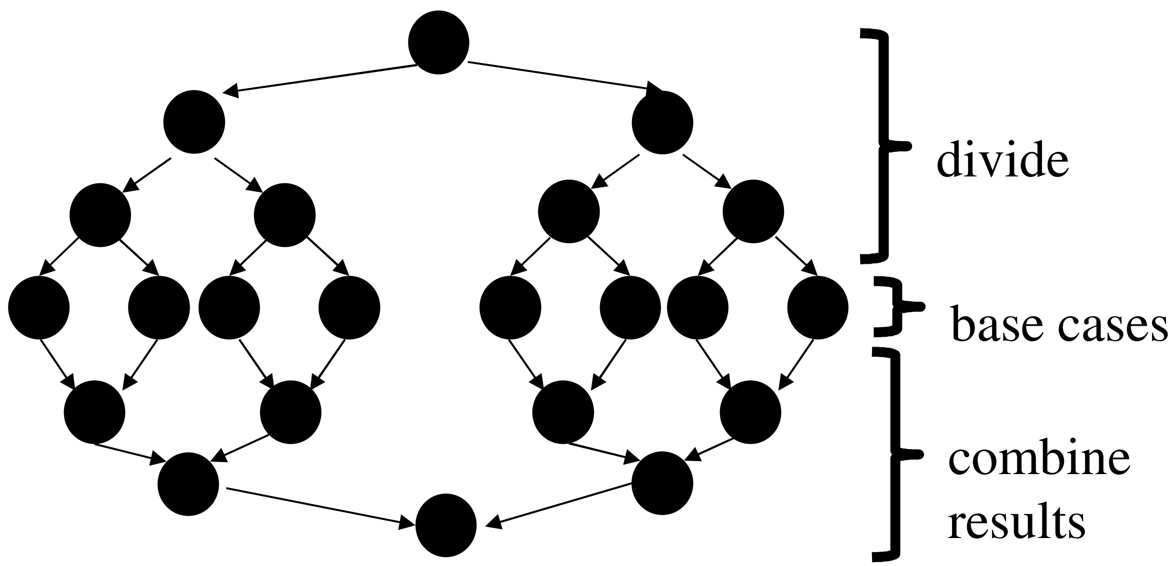
- terminated thread
- continuation of current thread

What's T_1 (work): sum of all 5 nodes

What's T_∞ (span): longest path

Divide and Conquer Algorithms

Our **`fork`** and **`join`** frequently look like this:



In this context, the span (T_∞) is:

- The longest dependence-chain; longest 'branch' in parallel 'tree'
- Example: $O(\log n)$ for summing an array; we halve the data down to our cut-off, then add back together; $O(\log n)$ steps, O(1) time for each
- Also called "critical path length" or "computational depth"

Parallel Speed-up

- Speed-up on **P** processors: $\mathbf{T_1} / \mathbf{T_P}$

  (if = P. Then)

- If speed-up is **P**, we call it perfect linear speed-up
  - e.g., doubling **P** halves running time
  - hard to achieve in practice

- Parallelism is the maximum possible speed-up: $\mathbf{T_1} / \mathbf{T}_{\infty}$
  - if you had infinite processors

Estimating T_p

- How to estimate $\mathbf{T_P}$ (e.g., P = 4)?

- Lower bounds on $\mathbf{T_P}$ (ignoring memory, caching...)
  1. $\mathbf{T_\infty}$
  2. $\mathbf{T_1 / P}$
  – which one is the tighter (higher) lower bound?

- The ForkJoin Java Framework achieves the following expected time asymptotic bound:

$$\mathbf{T_P \in O(T_\infty + T_1 / P)}$$

  – this bound is optimal

Amdahl's Law

- Most programs have
  1. parts that parallelize well
  2. parts that don't parallelize at all

- The latter become bottlenecks

Amdahl's Law

- Let T_1 = 1 unit of time
- Let S = proportion that can't be parallelized

$$1 = \mathbf{T_1 = S + (1 - S)}$$

- Suppose we get perfect linear speedup on the parallel portion:

$$\mathbf{T_P =}$$

- So the overall speed-up on P processors is (Amdahl's Law):

$$\mathbf{T_1 \,/\, T_P =}$$

$$\mathbf{T_1 \,/\, T_\infty =}$$

- If 1/3 of your program is parallelizable, max speedup is:

Pretty Bad News

- Suppose 25% of your program is sequential.
  - Then a billion processors won't give you more than a 4x speedup!
- What portion of your program must be parallelizable to get 10x speedup on a 1000 core GPU?
  - $10 <= 1 / (S + (1-S)/1000)$
- Motivates minimizing sequential portions of your programs

Take Aways

- Parallel algorithms can be a big win
- Many fit standard patterns that are easy to implement
- Can’t just rely on more processors to make things faster (Amdahl’s Law)