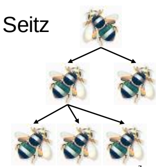
CSE 326: Data Structures
B-Trees and B+ Trees

Richard Anderson, Steve Seitz
Winter 2014



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Announcements

- This week:
  - HASHING
- Next week:
  - SORTING

- Upcoming dates
  - Wednesday: HW 3 Due
  - Thursday (11:59 pm): Project 2, Part A Due
  - Monday, February 10. Midterm

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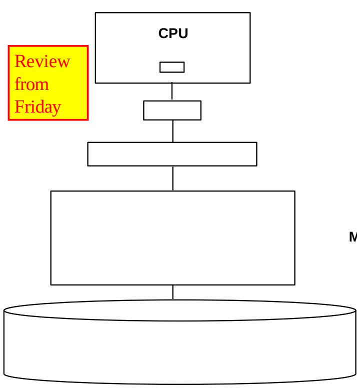
Review from Friday



Cycles to access:

Registers	1
L1 Cache	2
L2 Cache	30
Main memory	250
Disk	Random: 30,000,000
	Streamed: 5000

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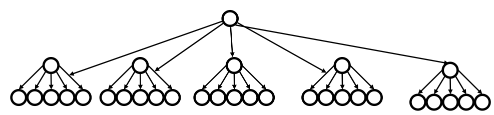
M-ary Search Tree

Consider a search tree with branching factor *M*:



Complete tree has height:

\# hops for *find*:

Runtime of *find*:

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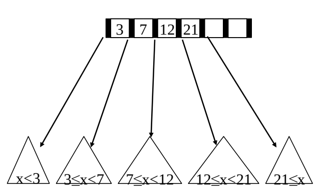
B+ Trees
(book calls these B-trees)

- Each internal node has (up to) M-1 keys:
- Order property:
  - subtree between two keys x and y contain leaves with *values* v such that $x \leq v < y$
  - Note the "≤"
- Leaf nodes have up to L **sorted** keys.



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B+ Tree Structure Properties

Internal nodes
- store up to M-1 keys
- have between $\lceil M/2 \rceil$ and M children

Leaf nodes
- where data is stored
- all at the same depth
- contain between $\lceil L/2 \rceil$ and L data items

Root (special case)
- has between 2 and ***M*** children (or root could be a leaf)

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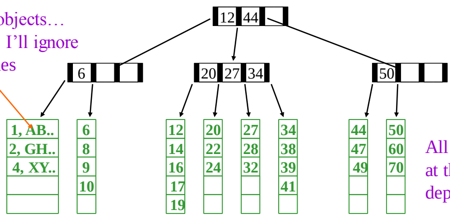
B+ Tree: Example

B+ Tree with **M = 4** (# pointers in internal node)
and **L = 5** (# data items in leaf)

Data objects… which I'll ignore in slides



All leaves at the same depth

Definition for later: "neighbor" is the next sibling to the left or right.

Disk Friendliness

What makes B+ trees disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one disk access.

2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual *data*
   - Much of tree structure can be loaded into memory irrespective of data object size
   - Data actually resides in disk

B+ trees vs. AVL trees

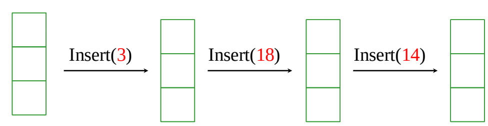
Suppose again we have $n = 2^{30} \approx 10^9$ items:

- Depth of AVL Tree

- Depth of B+ Tree with M = 256, L = 256

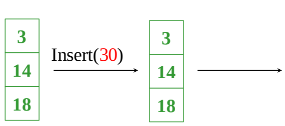
Great, but how to we actually make a B+ tree and keep it balanced…?

Building a B+ Tree with Insertions

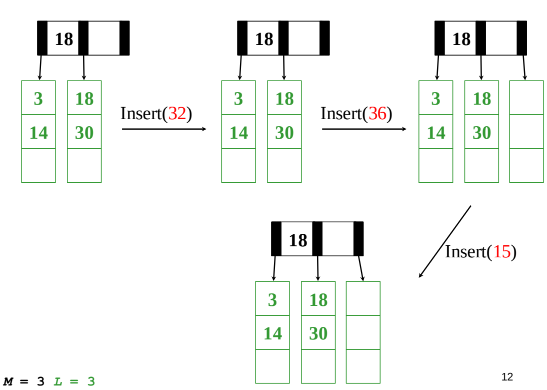


The empty B-Tree

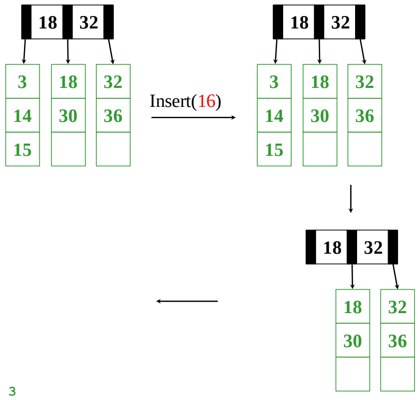
M = 3 L = 3



M = 3 L = 3



M = 3 L = 3



$M = 3$ $L = 3$

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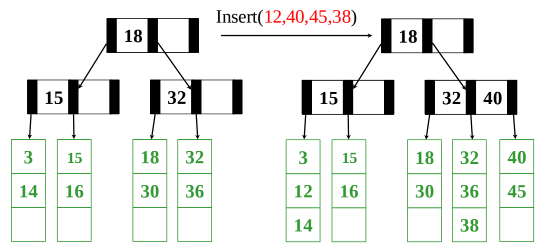

$M = 3$ $L = 3$

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Insertion Algorithm

1. Insert the key in its leaf in sorted order
2. If the leaf ends up with L+1 items, **overflow**!
   - Split the leaf into two nodes:
     - original with $\lceil (L+1)/2 \rceil$ smaller keys
     - new one with $\lfloor (L+1)/2 \rfloor$ larger keys
   - Add the new child to the parent
   - If the parent ends up with $M+1$ children, **overflow**!

This makes the tree deeper!

3. If an internal node ends up with M+1 children, **overflow**!
   - Split the node into two nodes:
     - original with $\lceil (M+1)/2 \rceil$ children with smaller keys
     - new one with $\lfloor (M+1)/2 \rfloor$ children with larger keys
   - Add the new child to the parent
   - If the parent ends up with $M+1$ items, **overflow**!
4. Split an overflowed root in two and hang the new nodes under a new root
5. Propagate keys up tree.

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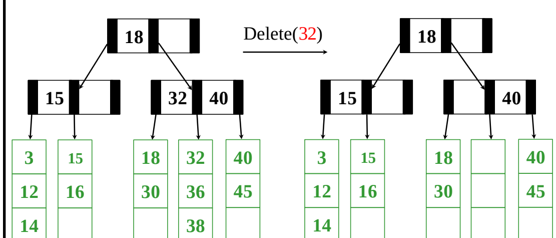
And Now for Deletion…



$M = 3$ $L = 3$

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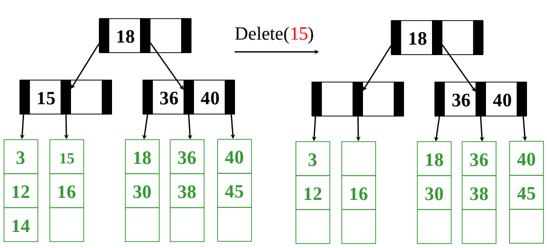

$M = 3$ $L = 3$

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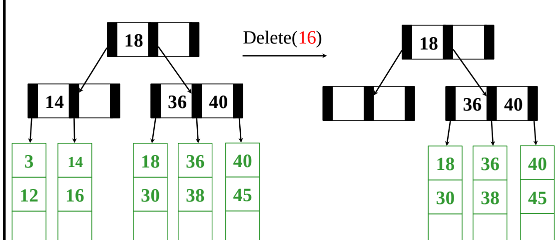

$M = 3$ $L = 3$

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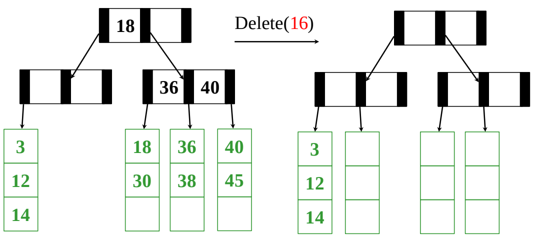

M = 3 L = 3

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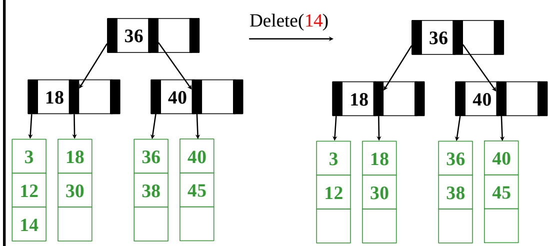

M = 3 L = 3

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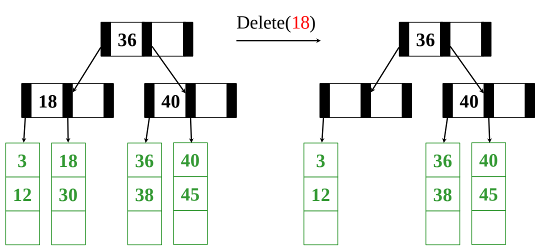

M = 3 L = 3

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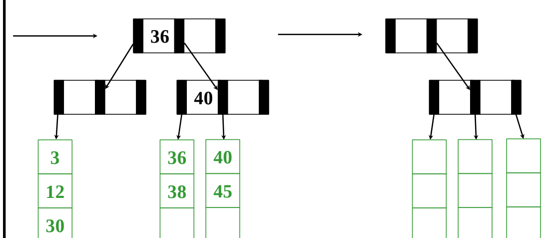

M = 3 L = 3

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Deletion Algorithm

1. Remove the key from its leaf

2. If the leaf ends up with fewer than $\lceil L/2 \rceil$ items, **underflow**!
   - Adopt data from a neighbor; update the parent
   - If adopting won't work, delete node and merge with neighbor
   - If the parent ends up with fewer than $\lceil M/2 \rceil$ children, **underflow**!

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Deletion Slide Two

3. If an internal node ends up with fewer than $\lceil M/2 \rceil$ children, **underflow**!
   - Adopt from a neighbor; update the parent
   - If adoption won't work, merge with neighbor
   - If the parent ends up with fewer than $\lceil M/2 \rceil$ children, **underflow**!

4. $\boxed{\text{If the root ends up with only one child, make the child the new root of the tree}}$

5. Propagate keys up through tree.

This reduces the height of the tree!

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Thinking about B+ Trees

- B+ Tree insertion can cause (expensive) splitting and propagation up the tree
- B+ Tree deletion can cause (cheap) adoption or (expensive) merging and propagation up the tree
- Split/merge/propagation is rare if M and L are large *(Why?)*
- Pick branching factor M and data items/leaf L such that each node takes one full page/block of memory/disk.

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Complexity

- Find:
- Insert:
  - find:
  - Insert in leaf:
  - split/propagate up:
- Claim: O(M) costs are negligible

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Tree Names You Might Encounter

- "B-Trees"
  - More general form of B+ trees, allows data at internal nodes too
  - Range of children is (key1,key2) rather than [key1, key2)
- B-Trees with $M = 3$, $L = x$ are called **2-3 trees**
  - Internal nodes can have 2 or 3 children
- B-Trees with $M = 4$, $L = x$ are called **2-3-4 trees**
  - Internal nodes can have 2, 3, or 4 children

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