CSE 332: Data Structures
Binary Search Trees

Richard Anderson, Steve Seitz

Winter 2014

Announcements

- HW #2 due next Wednesday
- Project 2 out today
  - can work with partners (optional). Must sign up
  - **harder** than project 1 (16 files to implement)
  - start early!
- Read Chapter 4.1-4.3, 4.6
- No class on Monday

ADTs Seen So Far

- **Stack**
  - Push
  - Pop
- **Queue**
  - Enqueue
  - Dequeue
- **Priority Queue**
  - Insert
  - DeleteMin

None of these support "find"

The Dictionary ADT

- Data:
  - a set of
    (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)

insert(seitz, ….)

- seitz
  Steve
  Seitz
  CSE 592
- anderson
  Richard
  Anderson
  CSE 582
- kainby87
  HyeIn
  Kim
  CSE 220
- …

find(anderson)

- anderson
  Richard, Anderson,…

*The Dictionary ADT is also called the "**Map ADT**"*

Many Uses

- Networks: router tables
- Operating systems: page tables
- Compilers: symbol tables
- Search: phone directories, ...
- Biology: genome maps
- Vision: object recognition
- ...

Probably the most widely used ADT!

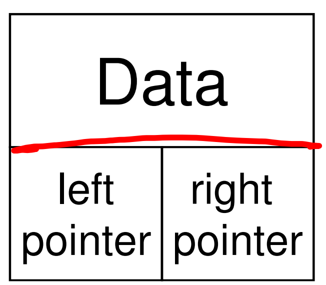
Implementations

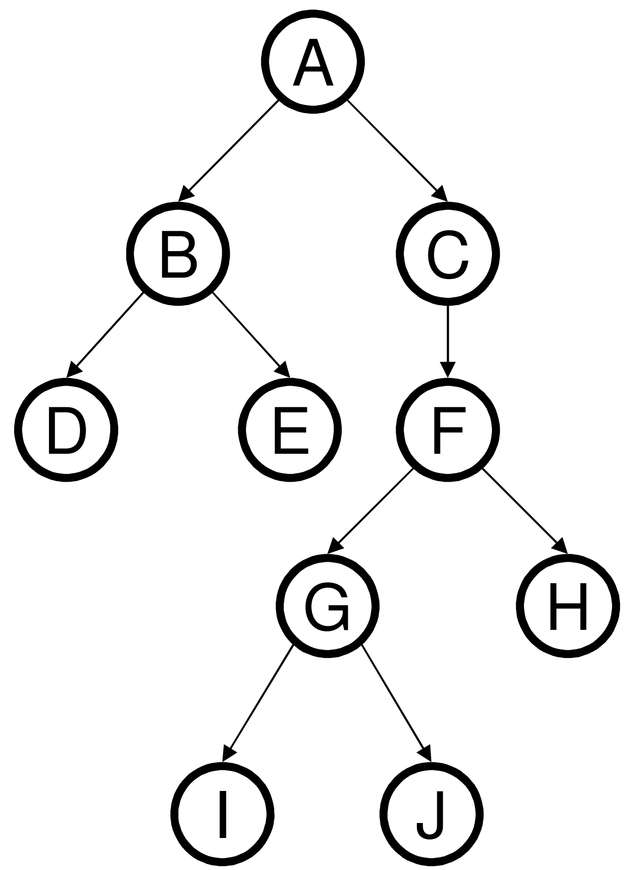
	insert	find	delete
• Unsorted Linked-list	O(1)	O(n)	O(n)
• Unsorted array	O(1) or O(n) if frst	O(n)	O(n)
• Sorted array	O(n) = O(log n) to find + O(n) to shift = O(n)	O(log n)	O(n)

Binary Trees

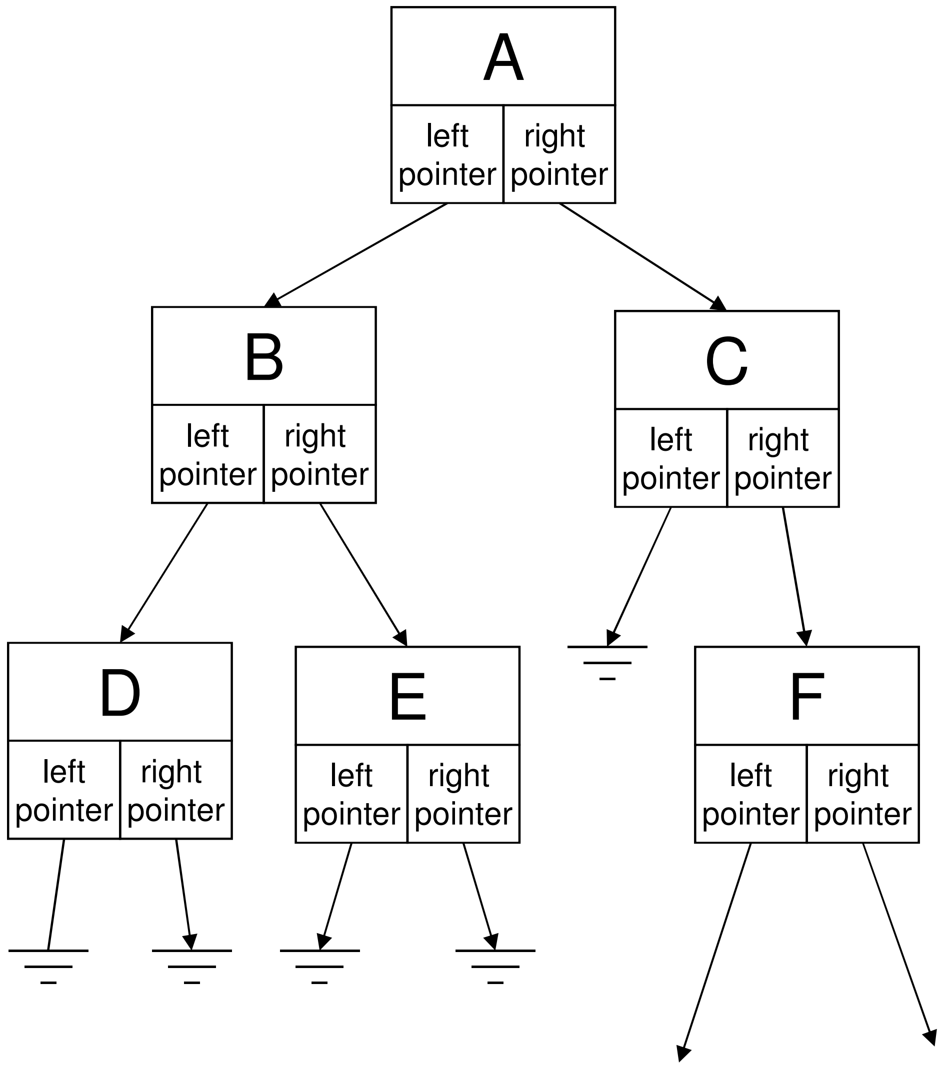
- Binary tree is
  - a root
  - left subtree *(maybe empty)*
  - right subtree *(maybe empty)*

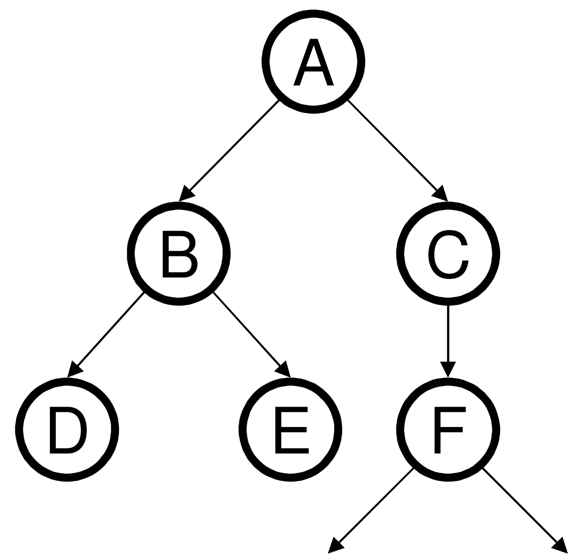
- Representation:





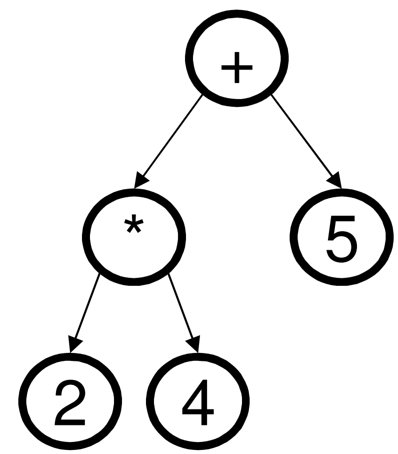
Binary Tree: Representation





Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree



(an expression tree)

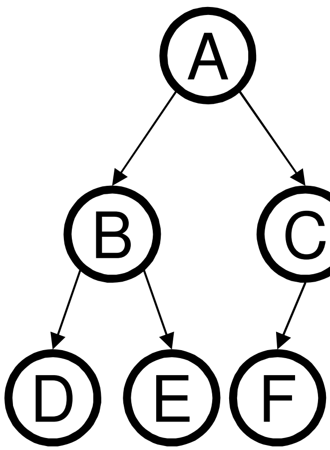
Three types:

- Pre-order: Root, left subtree, right subtree
  + * 2 4 5
- In-order: Left subtree, root, right subtree
  2 * 4 + 5
- Post-order: Left subtree, right subtree, root
  2 4 * 5 +

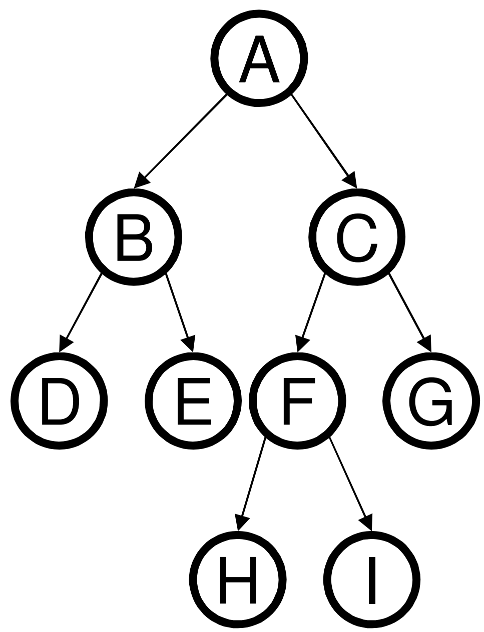
Inorder Traversal

```
void traverse(BNode t){
  if (t != NULL)
    traverse (t.left);
    process t.element;
    traverse (t.right);
  }
}
```

Binary Tree: Special Cases

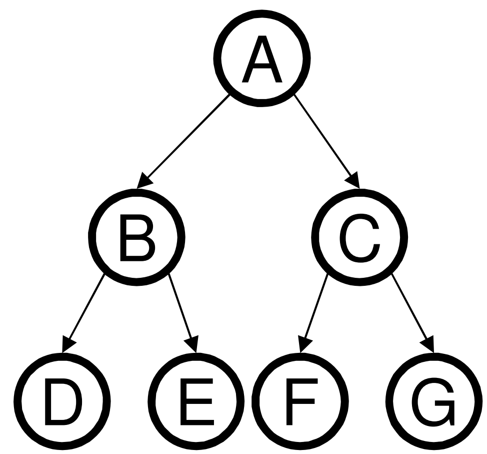


Complete Tree

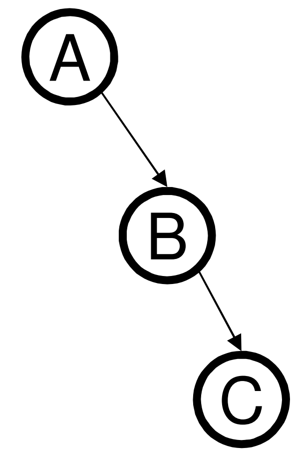


Full Tree

every node has 0 or 2
children



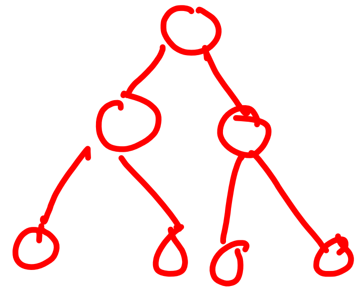
Perfect Tree



"List" Tree

Binary Tree: Some Numbers…

Recall: height of a tree = longest path from root to leaf.



$h=2$

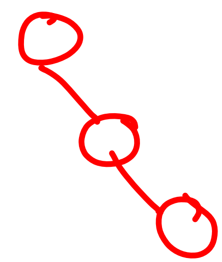
For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes: $2^{h+1}-1$

perfect trees

- min # of leaves: 1
- min # of nodes: $h+1$

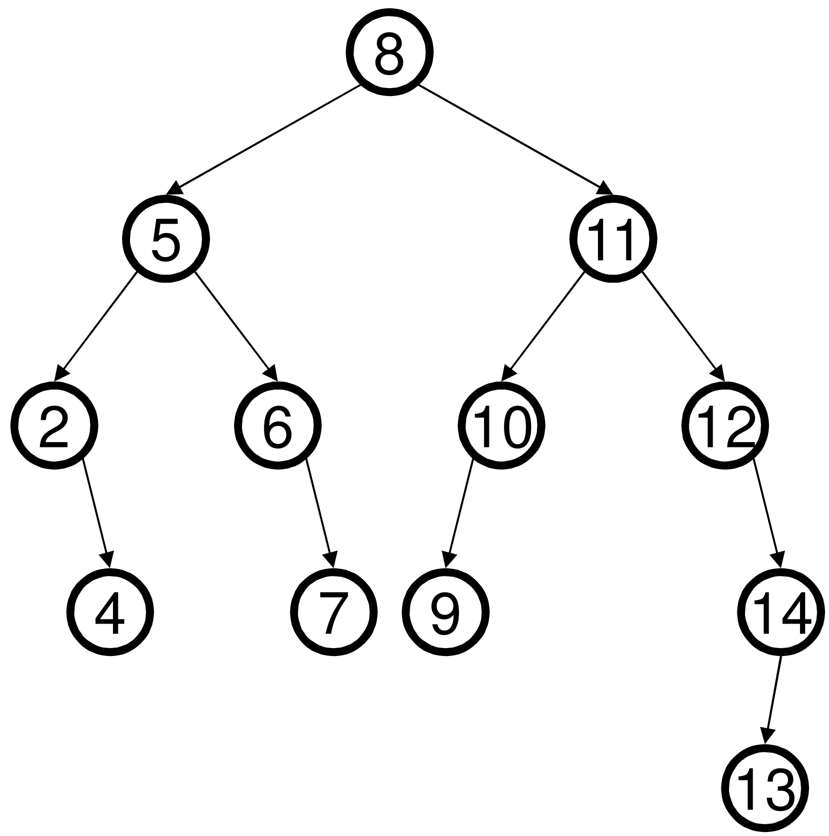
list trees



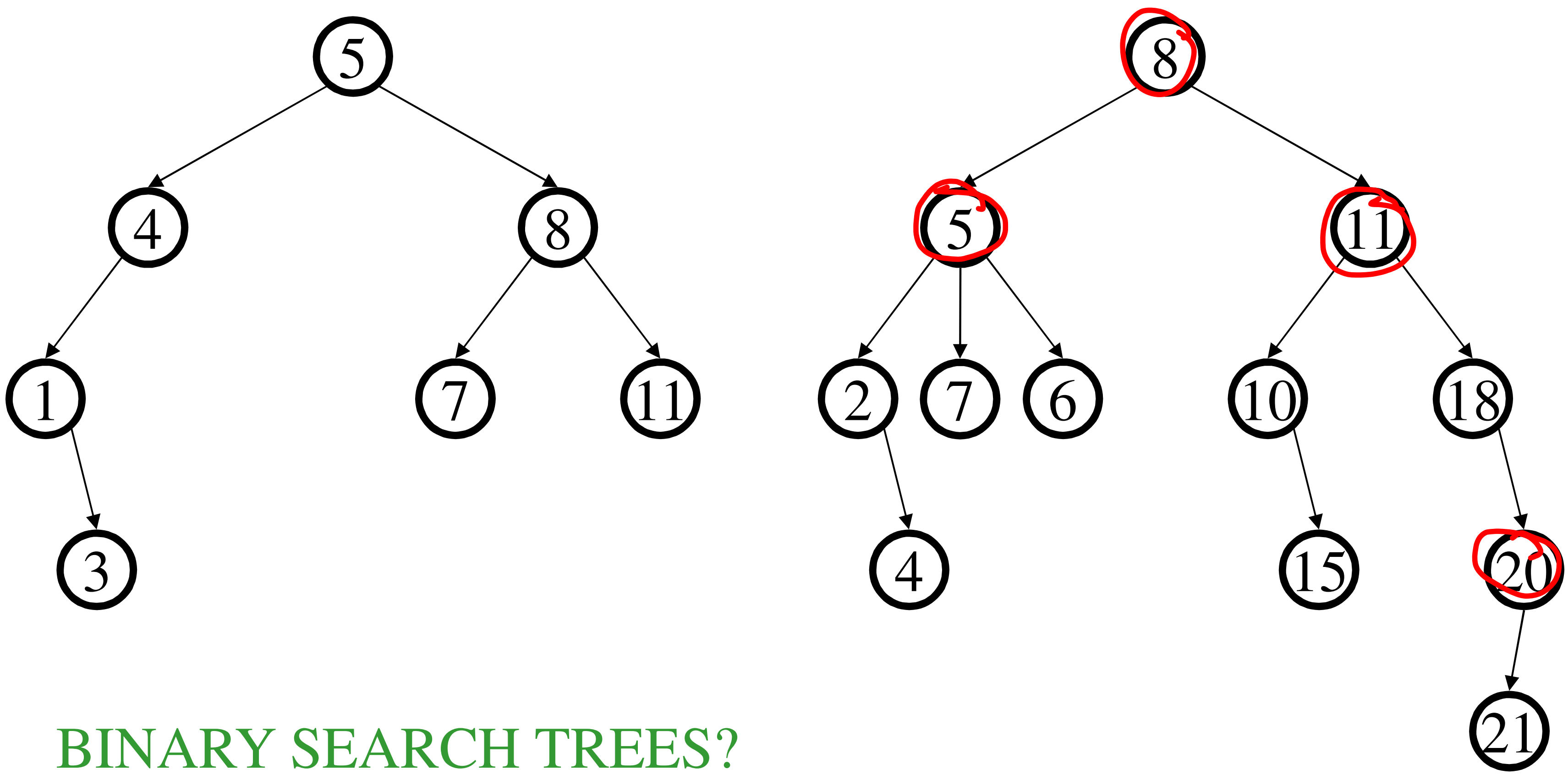
$h=2$

Binary Search Tree Data Structure

- Structural property
  - each node has ≤ 2 children
- Order property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key

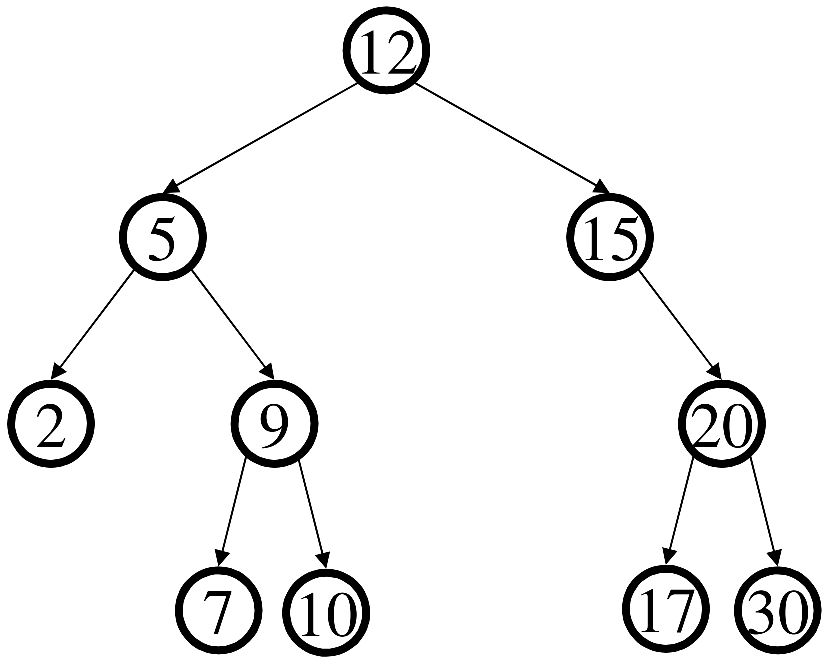


Example and Counter-Example



BINARY SEARCH TREES?

Find in BST, Recursive

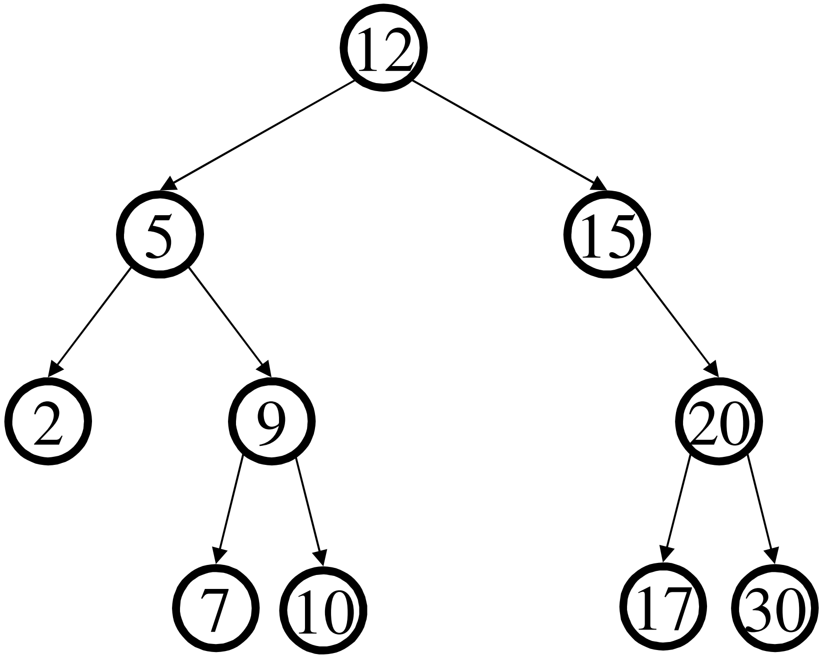



```
Node Find(Object key,
                Node root) {
  if (root == NULL)
    return NULL;

  if (key < root.key)
    return Find(key,
                root.left);
  else if (key > root.key)
    return Find(key,
                root.right);
  else
    return root;
}
```

Runtime:

Find in BST, Iterative

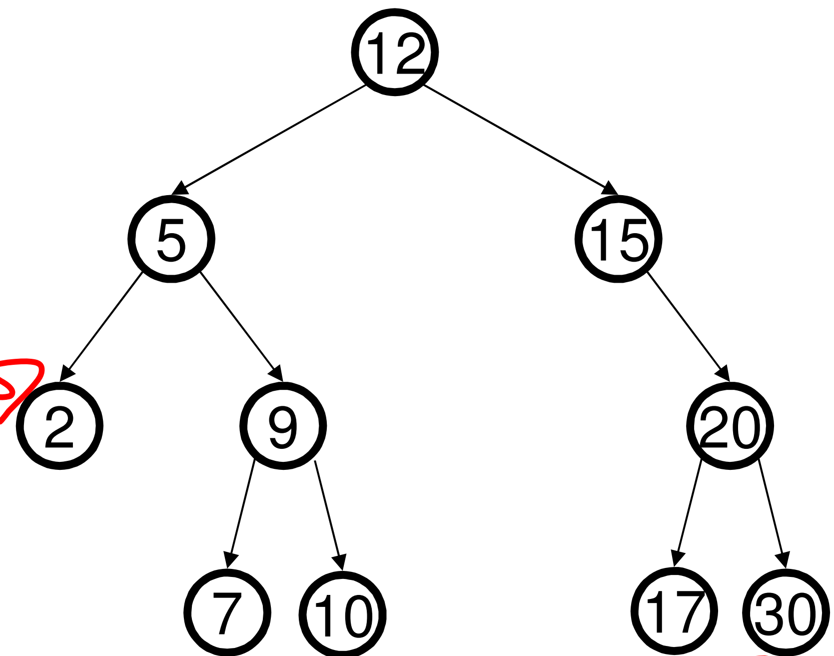
```
Node Find(Object key,
             Node root) {

  while (root != NULL &&
           root.key != key) {
    if (key < root.key)
      root = root.left;
    else
      root = root.right;
  }

  return root;
}
```



Runtime:

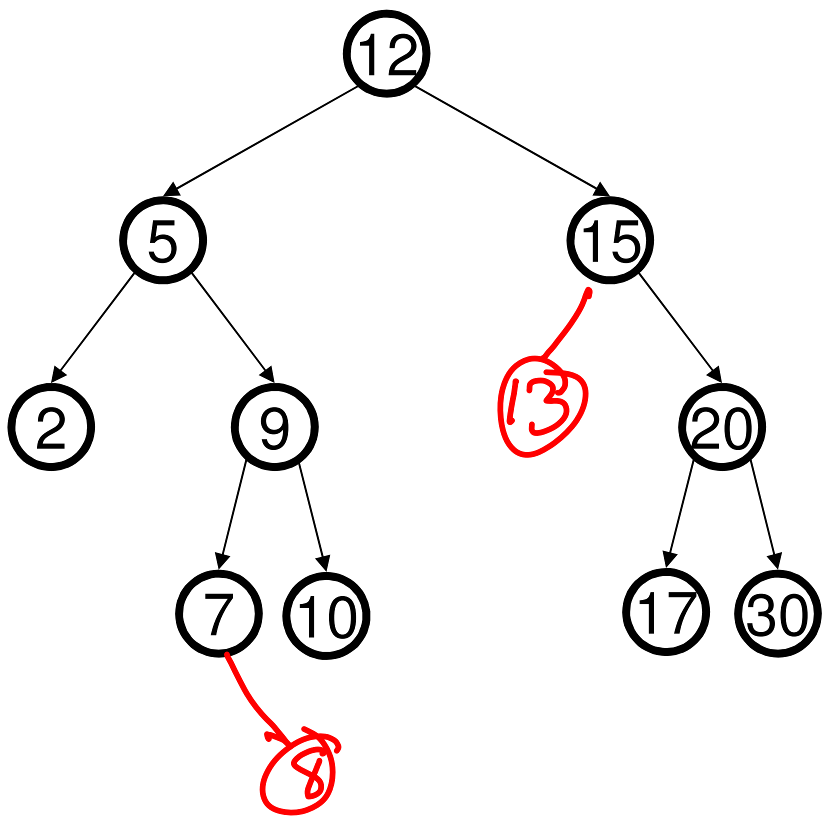
Bonus: FindMin/FindMax

- Find minimum
- Find maximum



$O(n)$

Insert in BST



Insert(13)
Insert(8)
Insert(31)

Insertions happen only at the leaves – easy!

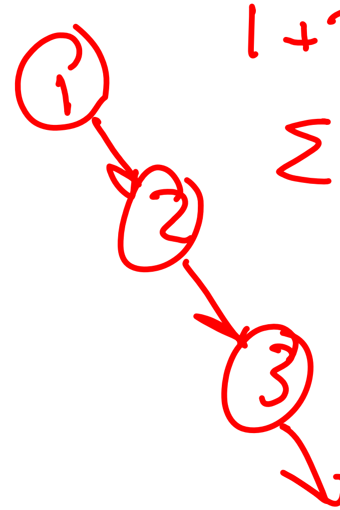
Runtime:

O(n)

BuildTree for BST

- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.

If inserted in given order, what is the tree? What big-O runtime for this kind of sorted input?



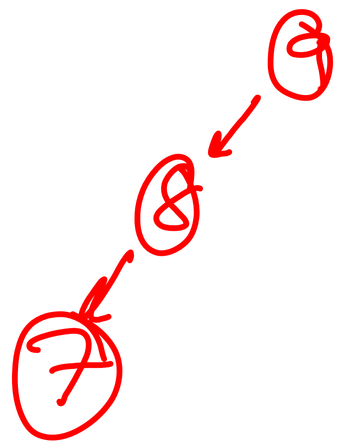
$1+2+3+\ldots$

$\sum i$

$\Rightarrow \frac{(n+1)n}{2}$

$O(n^2)$

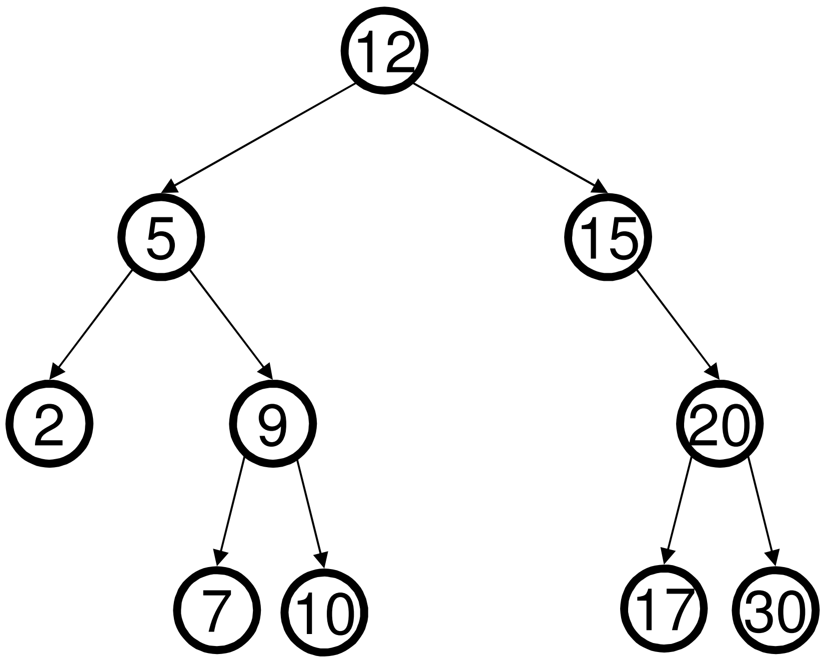
If inserted in reverse order, what is the tree? What big-O runtime for this kind of sorted input?



BuildTree for BST

- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.
  - If inserted median first, then left median, right median, etc., what is the tree? What is the big-O runtime for this kind of sorted input?

Deletion in BST



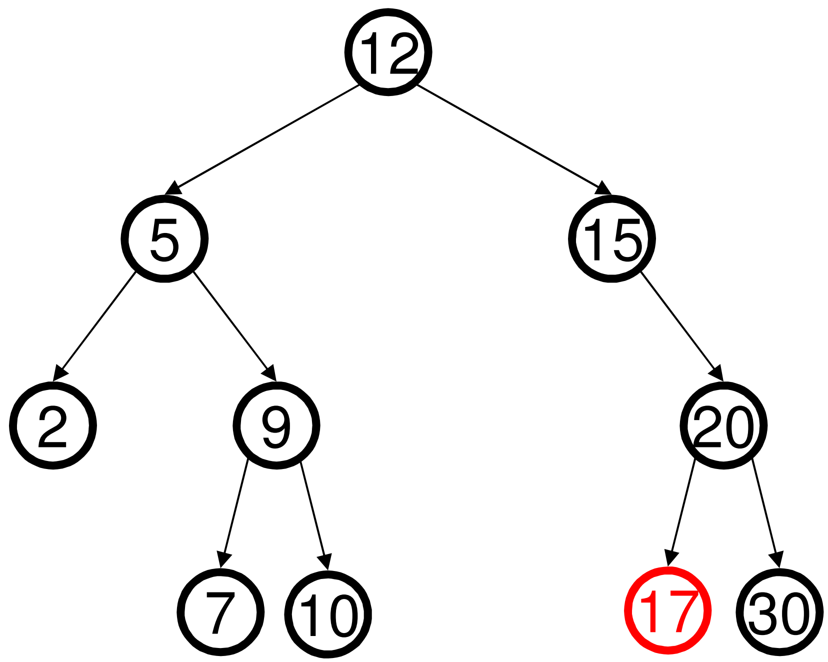
Why might deletion be harder than insertion?

Deletion

- Removing an item disrupts the tree structure.
- Basic idea: find the node that is to be removed. Then “fix” the tree so that it is still a binary search tree.
- Three cases:
  - node has no children (leaf node)
  - node has one child
  - node has two children

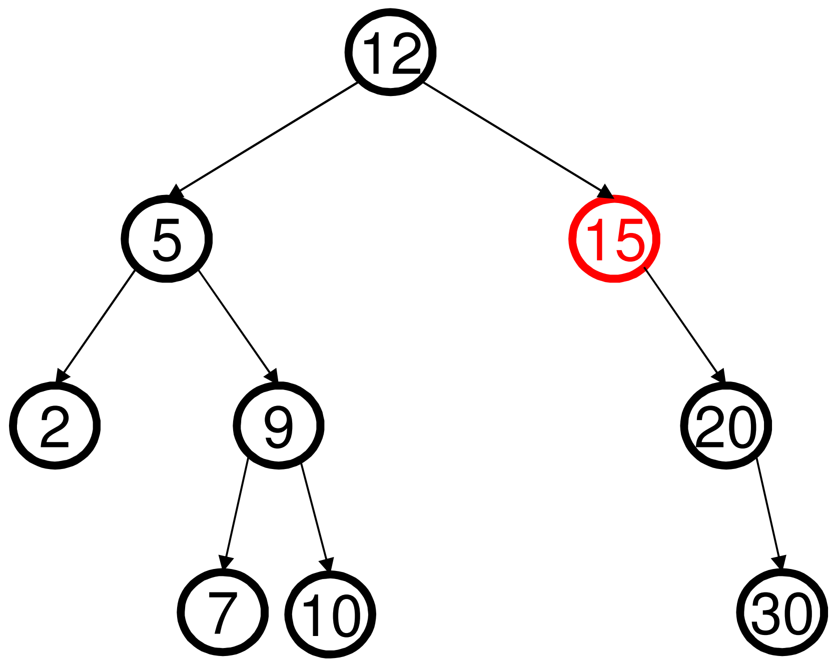
Deletion – The Leaf Case

Delete(17)



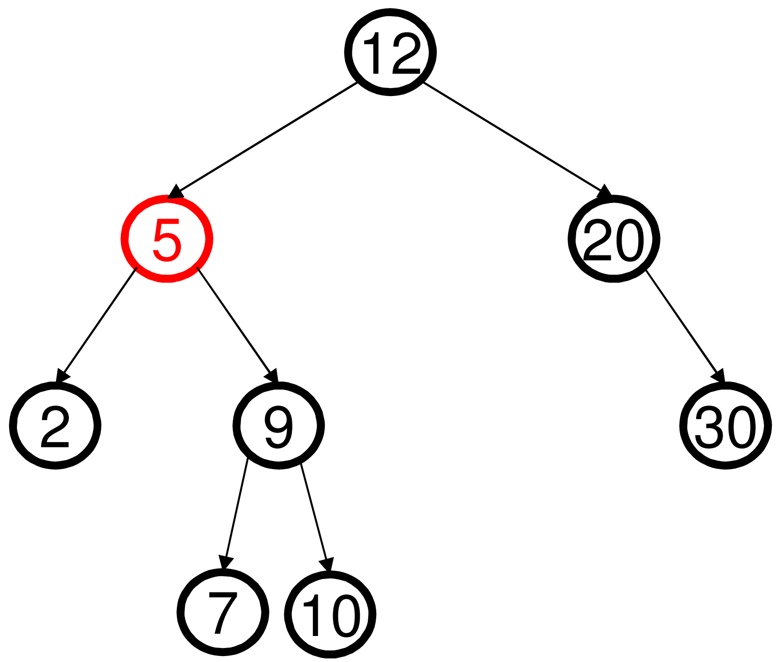
Deletion – The One Child Case

Delete(15)



Deletion – The Two Child Case

Delete(5)



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value *between* the two child subtrees

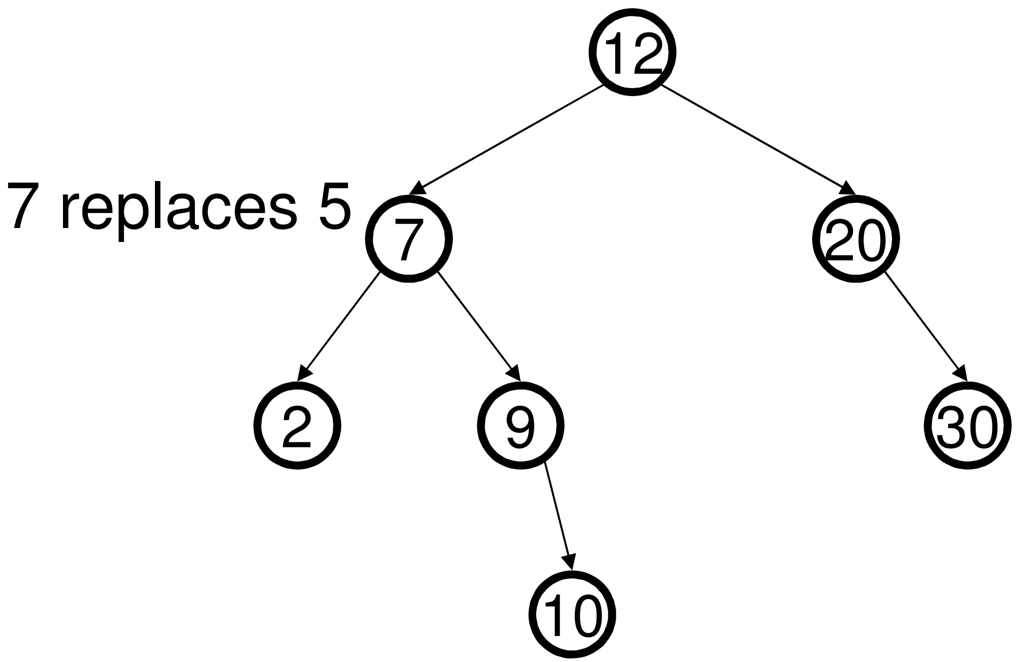
Options:

- *succ* from right subtree: findMin(t.right)
- *pred* from left subtree: findMax(t.left)

Now delete the original node containing *succ* or *pred*

- Leaf or one child case – easy!

Finally…



Original node containing
7 gets deleted

Balanced BST

Observations

- BST: the shallower the better!
- For a BST with n nodes
  - Average depth (averaged over all possible insertion orderings) is O(log n)
  - Worst case maximum depth is O(n)
- Simple cases such as insert(1, 2, 3, ..., n) lead to the worst case scenario

Solution: Require a **Balance Condition** that

1. ensures depth is O(log n) – strong enough!
2. is easy to maintain – not too strong!