Announcements

• **Project 1** – phase A due Wed 11pm via catalyst
• **Homework 1** – due Friday at *beginning* of class

• Links to materials from section – useful for project 1
• Info sheets?
Today

- Finish up Asymptotic Analysis
- New ADT! Priority Queues
Scenario

What is the difference between waiting for service at a pharmacy versus an ER?

Pharmacies usually follow the rule
First Come, First Served

Emergency Rooms assign priorities based on each individual's need
Scenario

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Emergency Rooms assign priorities based on each individual's need
A new ADT: Priority Queue

- Textbook Chapter 6
  - We will go back to binary search trees (ch4) and hash tables (ch5) later
  - Nice to see a new and surprising data structure first
- A priority queue holds compare-able data
  - Unlike stacks and queues need to compare items
    - Given $x$ and $y$, is $x$ less than, equal to, or greater than $y$
    - What this means can depend on your data
    - Much of course will require comparable data: e.g. sorting
  - Integers are comparable, so will use them in examples
    - But the priority queue ADT is much more general
    - Typically two fields, the priority and the data
Priority Queue ADT

- Assume each item has a “priority”
  - The lesser item is the one with the greater priority
  - So “priority 1” is more important than “priority 4”
  - Just a convention, could also do a maximum priority

- Main Operations:
  - insert
  - deleteMin

- Key property: deleteMin returns and deletes from the queue the item with greatest priority (lowest priority value)
  - Can resolve ties arbitrarily
Aside: We will use ints as data and priority

For simplicity in lecture, we’ll often suppose items are just ints and the int is also the priority

• So an operation sequence could be
  
  ```
  insert 6
  insert 5
  x = deleteMin  // Now x = 5.
  ```

  – int priorities are common, but really just need comparable

• Not having “other data” is very rare

  – Example: print job has a priority and the file to print is the data
Priority Queue Example

After execution:

insert a with priority 5
insert b with priority 3
insert c with priority 4
w = deleteMin
x = deleteMin
insert d with priority 2
insert e with priority 6
y = deleteMin
z = deleteMin

Analogy: insert is like enqueue, deleteMin is like dequeue
But the whole point is to use priorities instead of FIFO

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Priority Queue Example

To simplify our examples, we will just use the priority values from now on

insert $a$ with priority 5
insert $b$ with priority 3
insert $c$ with priority 4
$w = \text{deleteMin}$
$x = \text{deleteMin}$
insert $d$ with priority 2
insert $e$ with priority 6
$y = \text{deleteMin}$
$z = \text{deleteMin}$

after execution:

$w = b$
$x = c$
$y = d$
$z = a$

Analogy: insert is like enqueue, deleteMin is like dequeue
But the whole point is to use priorities instead of FIFO
Applications

Like all good ADTs, the priority queue arises often
  – Sometimes “directly”, sometimes less obvious

- Run multiple programs in the operating system
  – “critical” before “interactive” before “compute-intensive”
  – Maybe let users set priority level

- Treat hospital patients in order of severity (or triage)
- Select print jobs in order of decreasing length?
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (cf. CSE143)
- Sort: `insert` all, then repeatedly `deleteMin`
  – Much like Project 1 uses a stack to implement reverse
More applications

• “Greedy” algorithms
  – Select the ‘best-looking’ choice at the moment
  – Will see an example when we study graphs in a few weeks
• Discrete event simulation (system modeling, virtual worlds, …)
  – Simulate how state changes when events fire
  – Each event $e$ happens at some time $t$ and generates new events $e_1, \ldots, e_n$ at times $t+t_1, \ldots, t+t_n$
  – Naïve approach: advance “clock” by 1 unit at a time and process any events that happen then
  – Better:
    • $Pending\ events$ in a priority queue (priority = time happens)
    • Repeatedly: $\text{deleteMin}$ and then $\text{insert}$ new events
    • Effectively, “set clock ahead to next event”
# Preliminary Implementations of Priority Queue ADT

<table>
<thead>
<tr>
<th></th>
<th>insert</th>
<th>deleteMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted Array</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unsorted Linked-List</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sorted Circular Array</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sorted Linked-List</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary Search Tree</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Worst case, Assume arrays have enough space
Need a good data structure!

- Next we will show an efficient, non-obvious data structure for this ADT
  - But first let’s analyze some “obvious” ideas for $n$ data items
  - All times worst-case; assume arrays “have room”

<table>
<thead>
<tr>
<th>data</th>
<th>insert algorithm / time</th>
<th>deleteMin algorithm / time</th>
</tr>
</thead>
<tbody>
<tr>
<td>unsorted array</td>
<td>add at end $O(1)$</td>
<td>search $O(n)$</td>
</tr>
<tr>
<td>unsorted linked list</td>
<td>add at front $O(1)$</td>
<td>search $O(n)$</td>
</tr>
<tr>
<td>sorted circular array</td>
<td>search / shift $O(n)$</td>
<td>move front $O(1)$</td>
</tr>
<tr>
<td>sorted linked list</td>
<td>put in right place $O(n)$</td>
<td>remove at front $O(1)$</td>
</tr>
<tr>
<td>binary search tree</td>
<td>put in right place $O(n)$</td>
<td>leftmost $O(n)$</td>
</tr>
</tbody>
</table>
Aside: More on possibilities

- Note: If priorities are inserted in random order, binary search tree will likely do better than $O(n)$
  - $O(\log n)$ insert and $O(\log n)$ deleteMin on average
  - Could get same performance from a balanced binary search tree (e.g. AVL tree we will study later)

- One more idea: if priorities are 0, 1, ..., $k$ can use array of lists
  - insert: add to front of list at $\text{arr}[\text{priority}]$, $O(1)$
  - deleteMin: remove from lowest non-empty list $O(k)$
Our Data Structure: The Heap

The Heap:
- Worst case: $O(\log n)$ for insert
- Worst case: $O(\log n)$ for deleteMin
- If items arrive in random order, then the average-case of insert is $O(1)$
- Very good constant factors

Key idea: Only pay for functionality needed
- We need something better than scanning unsorted items
- But we do not need to maintain a full sorted list

- We will visualize our heap as a tree, so we need to review some tree terminology
Reviewing Some Tree Terminology

root(T):

leaves(T):

children(B):

parent(H):

siblings(E):

ancestors(F):

descendents(G):

subtree(G):
Some More Tree Terminology

\[ \text{depth}(B): \]

\[ \text{height}(G): \]

\[ \text{height}(T): \]

\[ \text{degree}(B): \]

\[ \text{branching factor}(T): \]
Reviewing Some Tree Terminology

**root**(T):

**leaves**(T):

**children**(B):

**parent**(H):

**siblings**(E):

**ancestors**(F):

**descendents**(G):

**subtree**(G):

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Tree T

- A
- D-F, I, J-N
- D, E, F
- G
- D, F
- B, A
- H, I, J-N
- G and its children
Some More Tree Terminology

- **depth(B):** 1
- **height(G):** 2
- **height(T):** 4
- **degree(B):** 3
- **branching factor(T):** 0-5

Tree T
Types of Trees

Binary tree: Every node has $\leq 2$ children

n-ary tree: Every node has $\leq n$ children

Perfect tree: Every row is completely full

Complete tree: All rows except possibly the bottom are completely full, and it is filled from left to right

Perfect Tree

Complete Tree
Some Basic Tree Properties

Nodes in a perfect binary tree of height $h$?
$$2^{h+1} - 1$$

Leaf nodes in a perfect binary tree of height $h$?
$$2^h$$

Height of a perfect binary tree with $n$ nodes?
$$\lfloor \log_2 n \rfloor$$

Height of a complete binary tree with $n$ nodes?
$$\lfloor \log_2 n \rfloor$$
Properties of a Binary Min-Heap

More commonly known as a binary heap or simply a heap

– Structure Property:
  A complete [binary] tree

– Heap Property:
  The priority of every non-root node is greater than the priority of its parent

How is this different from a binary search tree?
Properties of a Binary Min-Heap

More commonly known as a binary heap or simply a heap

- **Structure Property:**
  A complete [binary] tree

- **Heap Order Property:**
  The priority of every non-root node is greater than the priority of its parent

A Heap

```
          10
         /  \
       20   80
      /  \
    40   60
   /  \
  50   700
```

Not a Heap

```
         30
        /  \
       20   80
      /  \
    13   25
```

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Properties of a Binary Min-Heap

• Where is the minimum priority item?
  At the root

• What is the height of a heap with n items?
  $\lfloor \log_2 n \rfloor$
Heap Operations

- `findMin`: 
- `deleteMin`: percolate down.
- `insert(val)`: percolate up.
Operations: basic idea

- **findMin:**
  return root.data

- **deleteMin:**
  1. answer = root.data
  2. Move right-most node in last row to root to restore structure property
  3. “Percolate down” to restore heap order property

- **insert:**
  1. Put new node in next position on bottom row to restore structure property
  2. “Percolate up” to restore heap order property

Overall strategy:
- **Preserve complete tree structure property**
- **This may break heap order property**
- **Percolate to restore heap order property**
**DeleteMin Implementation**

1. Delete value at root node (and store it for later return)

2. There is now a "hole" at the root. We must "fill" the hole with another value, must have a tree with one less node, and it must still be a complete tree

3. The "last" node is the obvious choice, but now the heap order property is violated

4. We **percolate down** to fix the heap order:
   - While greater than either child
   - Swap with smaller child
Percolate Down:

• Keep comparing with both children
• Move smaller child up and go down one level
• Done if both children are \( \geq \) item or reached a leaf node
• Why does this work? What is the run time?
DeleteMin: Run Time Analysis

- Run time is $O(\text{height of heap})$

- A heap is a complete binary tree

- Height of a complete binary tree of $n$ nodes?
  - height $= \lceil \log_2(n) \rceil$

- Run time of `deleteMin` is $O(\log n)$
Insert

- Add a value to the tree
- Structure and heap order properties must still be correct afterwards
Insert: Maintain the Structure Property

- There is only one valid tree shape after we add one more node!
- So put our new data there and then focus on restoring the heap order property.
Maintain the heap order property

Percolate up:
• Put new data in new location
• If parent larger, swap with parent, and continue
• Done if parent ≤ item or reached root
• Why does this work? What is the run time?
A Clever Trick for Storing the Heap…

Clearly, insert and deleteMin are worst-case O(log n)
• But we promised average-case O(1) insert (how??)

Insert requires access to the “next to use” position in the tree
• Walking the tree from root to leaf requires O(log n) steps
• Insert and Deletemin would have to update the “next to use” reference each time: O(log n)

We should only pay for the functionality we need!!
• Why have we insisted the tree be complete? 😊

All complete trees of size n contain the same edges
• So why are we even representing the edges?

Here comes the really clever bit about implementing heaps!!!
Array Representation of a Binary Heap

From node i:
- left child:
- right child:
- parent:

- We skip index 0 to make the math simpler
- Actually, it can be a good place to store the current size of the heap
Array Representation of a Binary Heap

From node i:
- left child: 2i
- right child: 2i+1
- parent: i / 2

- We skip index 0 to make the math simpler
- Actually, it can be a good place to store the current size of the heap