



# CSE 332 Data Abstractions: B Trees and Hash Tables Make a Complete Breakfast

Kate Deibel  
Summer 2012

The national data structure of the Netherlands

## HASH TABLES

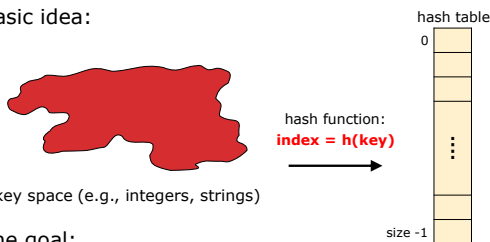
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### Hash Tables

A hash table is an array of some fixed size  
Basic idea:



The goal:

Aim for constant-time find, insert, and delete "on average" under reasonable assumptions

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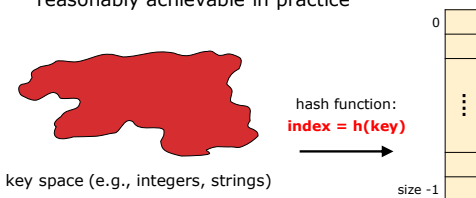
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### An Ideal Hash Functions

- Is fast to compute
- Rarely hashes two keys to the same index
  - Known as *collisions*
  - Zero collisions often impossible in theory but reasonably achievable in practice



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### What to Hash?

We will focus on two most common things to hash: **ints** and **strings**

If you have objects with several fields, it is usually best to hash most of the "identifying fields" to avoid collisions:

```
class Person {
    String firstName, middleName, lastName;
    Date birthDate;
    ...
}
```

use these four values

An inherent trade-off:

hashing-time vs. collision-avoidance

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### Hashing Integers

key space = integers

Simple hash function:  
 $h(\text{key}) = \text{key} \% \text{TableSize}$

- Client:  $f(x) = x$
- Library:  $g(x) = f(x) \% \text{TableSize}$
- Fairly fast and natural

Example:

- TableSize = 10
- Insert keys 7, 18, 41, 34, 10

0	
1	10
2	41
3	
4	
5	34
6	
7	
8	7
9	18

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## Hashing non-integer keys

If keys are not ints, the client must provide a means to convert the key to an int

Programming Trade-off:

- Calculation speed
- Avoiding distinct keys hashing to same ints

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## Hashing Strings

Key space  $K = s_0s_1s_2\dots s_{k-1}$   
 where  $s_i$  are chars:  $s_i \in [0, 256]$

Some choices: Which ones best avoid collisions?

$$h(K) = (s_0) \% \text{TableSize}$$

$$h(K) = \left( \sum_{i=0}^{k-1} s_i \right) \% \text{TableSize}$$

$$h(K) = \left( \sum_{i=0}^{k-1} s_i \cdot 37^i \right) \% \text{TableSize}$$

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## Combining Hash Functions

A few rules of thumb / tricks:

1. Use all 32 bits (be careful with negative numbers)
2. Use different overlapping bits for different parts of the hash
  - This is why a factor of  $37^i$  works better than  $256^i$
  - Example: "abcde" and "ebcda"
3. When smashing two hashes into one hash, use bitwise-xor
  - bitwise-and produces too many 0 bits
  - bitwise-or produces too many 1 bits
4. Rely on expertise of others; consult books and other resources for standard hashing functions
5. Advanced: If keys are known ahead of time, a *perfect hash* can be calculated

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Calling a State Farm agent is not an option...

**COLLISION RESOLUTION**

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## Collision Avoidance

With  $(x \% \text{TableSize})$ , number of collisions depends on

- the ints inserted
- TableSize

Larger table-size tends to help, but not always

- Example: 70, 24, 56, 43, 10  
 with TableSize = 10 and TableSize = 60

Technique: Pick table size to be prime. Why?

- Real-life data tends to have a pattern,
- "Multiples of 61" are probably less likely than "multiples of 60"
- Some collision strategies do better with prime size

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## Collision Resolution

**Collision:**

When two keys map to the same location in the hash table

We try to avoid it, but the number of keys always exceeds the table size

Ergo, hash tables generally must support some form of **collision resolution**

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## Flavors of Collision Resolution

### Separate Chaining

#### Open Addressing

- Linear Probing
- Quadratic Probing
- Double Hashing

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## Terminology Warning

We and the book use the terms

- "chaining" or "separate chaining"
- "open addressing"

Very confusingly, others use the terms

- "open hashing" for "chaining"
- "closed hashing" for "open addressing"

We also do trees upside-down

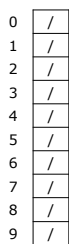


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## Separate Chaining



All keys that map to the same table location are kept in a linked list (a.k.a. a "chain" or "bucket")

As easy as it sounds

Example:

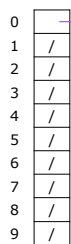
insert 10, 22, 86, 12, 42  
with  $h(x) = x \% 10$

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## Separate Chaining



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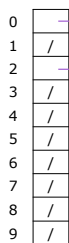
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Example:

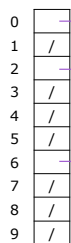
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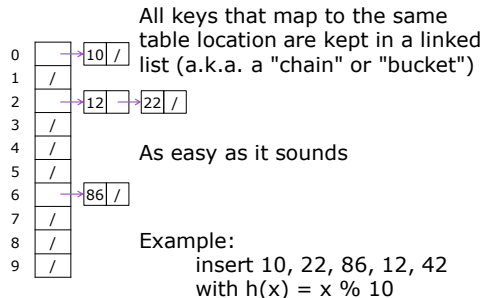
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### Separate Chaining

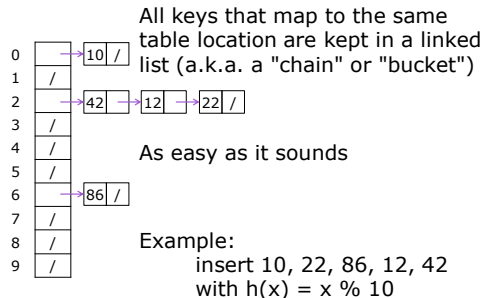


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### Separate Chaining



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### Thoughts on Separate Chaining

Worst-case time for find?

- Linear
- But only with really bad luck or bad hash function
- Not worth avoiding (e.g., with balanced trees at each bucket)
  - Keep small number of items in each bucket
  - Overhead of tree balancing not worthwhile for small n

Beyond asymptotic complexity, some "data-structure engineering" can improve constant factors

- Linked list, array, or a hybrid
- Insert at end or beginning of list
- Sorting the lists gains and loses performance
- Splay-like: Always move item to front of list

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### Rigorous Separate Chaining Analysis

The **load factor**,  $\lambda$ , of a hash table is calculated as

$$\lambda = \frac{n}{TableSize}$$

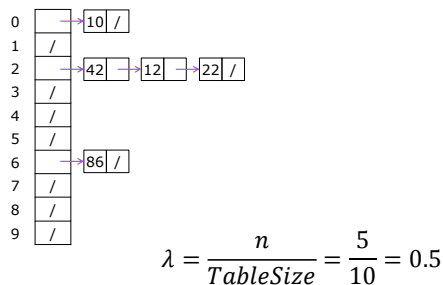
where  $n$  is the number of items currently in the table

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### Load Factor?

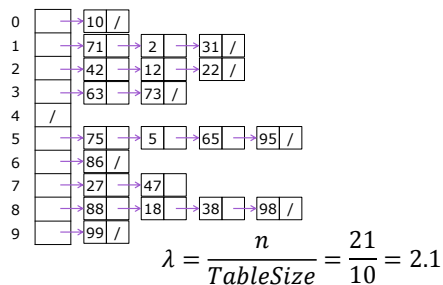


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### Load Factor?



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### Rigorous Separate Chaining Analysis

The **load factor**,  $\lambda$ , of a hash table is calculated as

$$\lambda = \frac{n}{TableSize}$$

where  $n$  is the number of items currently in the table

Under chaining, the average number of elements per bucket is \_\_\_\_

So if some inserts are followed by random finds, then on average:

- Each unsuccessful find compares against \_\_\_\_ items
- Each successful find compares against \_\_\_\_ items

How big should TableSize be??

### Rigorous Separate Chaining Analysis

The **load factor**,  $\lambda$ , of a hash table is calculated as

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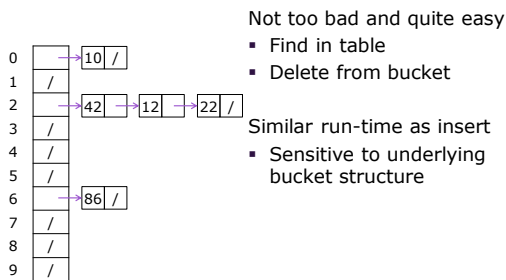
where  $n$  is the number of items currently in the table

Under chaining, the average number of elements per bucket is  $\lambda$

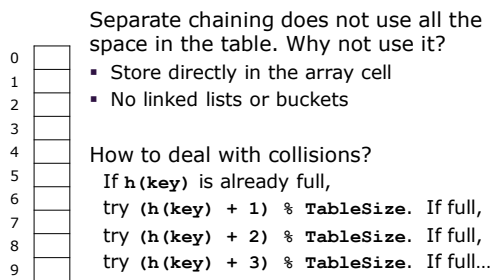
So if some inserts are followed by random finds, then on average:

- Each unsuccessful find compares against  $\lambda$  items
- Each successful find compares against  $\lambda$  items
- If  $\lambda$  is low, find and insert likely to be O(1)
- We like to keep  $\lambda$  around 1 for separate chaining

### Separate Chaining Deletion

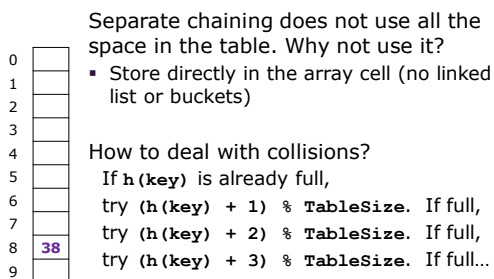


### Open Addressing: Linear Probing



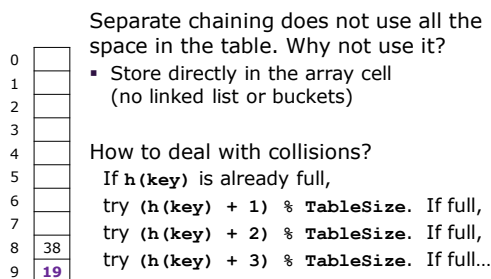
Example: insert 38, 19, 8, 79, 10

### Open Addressing: Linear Probing



Example: insert 38, 19, 8, 79, 10

### Open Addressing: Linear Probing



Example: insert 38, 19, 8, 79, 10

### Open Addressing: Linear Probing

Separate chaining does not use all the space in the table. Why not use it?

0	8
1	
2	
3	
4	
5	
6	
7	
8	38
9	19

- Store directly in the array cell (no linked list or buckets)

How to deal with collisions?

- If  $h(\text{key})$  is already full,
  - try  $(h(\text{key}) + 1) \% \text{TableSize}$ . If full,
  - try  $(h(\text{key}) + 2) \% \text{TableSize}$ . If full,
  - try  $(h(\text{key}) + 3) \% \text{TableSize}$ . If full...

Example: insert 38, 19, 8, 79, 10

### Open Addressing: Linear Probing

Separate chaining does not use all the space in the table. Why not use it?

0	8
1	79
2	
3	
4	
5	
6	
7	
8	38
9	19

- Store directly in the array cell (no linked list or buckets)

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Example: insert 38, 19, 8, 79, 10

### Load Factor?

0	8
1	79
2	10
3	
4	
5	
6	
7	
8	38
9	19

Can the load factor when using linear probing ever exceed 1.0?

Nope!!

$$\lambda = \frac{n}{\text{TableSize}} = \frac{5}{10} = 0.5$$

### Open Addressing in General

This is one example of open addressing

Open addressing means resolving collisions by trying a sequence of other positions in the table

Trying the next spot is called probing

- We just did linear probing  $h(\text{key}) + i) \% \text{TableSize}$
- In general have some probe function  $f$  and use  $h(\text{key}) + f(i) \% \text{TableSize}$

Open addressing does poorly with high load factor  $\lambda$

- So we want larger tables
- Too many probes means we lose our  $O(1)$

### Open Addressing: Other Operations

**insert** finds an open table position using a probe function

What about **find**?

- Must use same probe function to "retrace the trail" for the data
- Unsuccessful search when reach empty position

What about **delete**?

- Must use "lazy" deletion. Why?

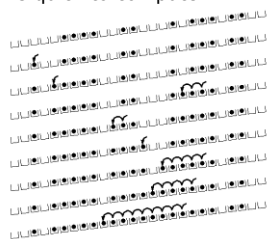
10	*	/	23	/	/	16	*	26
----	---	---	----	---	---	----	---	----

- Marker indicates "data was here, keep on probing"

### Primary Clustering

It turns out linear probing is a bad idea, even though the probe function is quick to compute (which is a good thing)

- This tends to produce clusters, which lead to long probe sequences
- This is called *primary clustering*
- We saw the start of a cluster in our linear probing example



[R. Sedgwick]

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### Analysis of Linear Probing

Trivial fact:

For any  $\lambda < 1$ , linear probing will find an empty slot

- We are safe from an infinite loop unless table is full

Non-trivial facts (we won't prove these):

Average # of probes given load factor  $\lambda$

- For an unsuccessful search as  $\text{TableSize} \rightarrow \infty$ :

$$\frac{1}{2} \left( 1 + \frac{1}{(1-\lambda)^2} \right)$$

- For an successful search as  $\text{TableSize} \rightarrow \infty$ :

$$\frac{1}{2} \left( 1 + \frac{1}{(1-\lambda)} \right)$$

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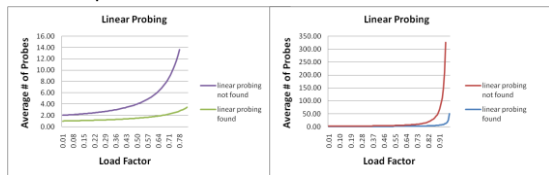
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### Analysis in Chart Form

Linear-probing performance degrades rapidly as the table gets full

- The Formula does assumes a "large table" but the point remains



Note that separate chaining performance is linear in  $\lambda$  and has no trouble with  $\lambda > 1$

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### Open Addressing: Quadratic Probing

We can avoid primary clustering by changing the probe function from just  $i$  to  $f(i)$

$$(h(\text{key}) + f(i)) \% \text{TableSize}$$

For **quadratic probing**,  $f(i) = i^2$ :

0<sup>th</sup> probe:  $(h(\text{key}) + 0) \% \text{TableSize}$

1<sup>st</sup> probe:  $(h(\text{key}) + 1) \% \text{TableSize}$

2<sup>nd</sup> probe:  $(h(\text{key}) + 4) \% \text{TableSize}$

3<sup>rd</sup> probe:  $(h(\text{key}) + 9) \% \text{TableSize}$

...

$i^{\text{th}}$  probe:  $(h(\text{key}) + i^2) \% \text{TableSize}$

Intuition: Probes quickly "leave the neighborhood"

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### Quadratic Probing Example



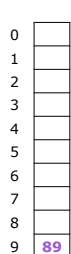
TableSize = 10  
insert(89)

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### Quadratic Probing Example



TableSize = 10  
insert(89)  
insert(18)

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### Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

TableSize = 10  
insert(89)  
insert(18)  
insert(49)

### Quadratic Probing Example

0	49
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

TableSize = 10  
insert(89)  
insert(18)  
insert(49)  
 $49 \% 10 = 9$  collision!  
 $(49 + 1) \% 10 = 0$   
insert(58)

### Quadratic Probing Example

0	49
1	
2	58
3	
4	
5	
6	
7	
8	18
9	89

TableSize = 10  
insert(89)  
insert(18)  
insert(49)  
insert(58)  
 $58 \% 10 = 8$  collision!  
 $(58 + 1) \% 10 = 9$  collision!  
 $(58 + 4) \% 10 = 2$   
insert(79)

### Quadratic Probing Example

0	49
1	
2	58
3	79
4	
5	
6	
7	
8	18
9	89

TableSize = 10  
insert(89)  
insert(18)  
insert(49)  
insert(58)  
insert(79)  
 $79 \% 10 = 9$  collision!  
 $(79 + 1) \% 10 = 0$  collision!  
 $(79 + 4) \% 10 = 3$

### Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	

TableSize = 7  
Insert:  
**76** ( $76 \% 7 = 6$ )  
40 ( $40 \% 7 = 5$ )  
48 ( $48 \% 7 = 6$ )  
5 ( $5 \% 7 = 5$ )  
55 ( $55 \% 7 = 6$ )  
47 ( $47 \% 7 = 5$ )

### Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	76

TableSize = 7  
Insert:  
76 ( $76 \% 7 = 6$ )  
**40** ( $40 \% 7 = 5$ )  
48 ( $48 \% 7 = 6$ )  
5 ( $5 \% 7 = 5$ )  
55 ( $55 \% 7 = 6$ )  
47 ( $47 \% 7 = 5$ )



### Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	40
6	76

TableSize = 7  
Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
<b>48</b>	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

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### Another Quadratic Probing Example

0	48
1	
2	
3	
4	
5	40
6	76

TableSize = 7  
Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
<b>5</b>	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

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### Another Quadratic Probing Example

0	48
1	
2	5
3	
4	
5	40
6	76

TableSize = 7  
Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
<b>55</b>	(55 % 7 = 6)
47	(47 % 7 = 5)

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### Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

TableSize = 7  
Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
<b>47</b>	(47 % 7 = 5)

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### Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

TableSize = 7  
Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
<b>47</b>	(47 % 7 = 5)

(47 + 1) % 7 = 6 collision!  
 (47 + 4) % 7 = 2 collision!  
 (47 + 9) % 7 = 0 collision!  
 (47 + 16) % 7 = 0 collision!  
 (47 + 25) % 7 = 2 collision!

**Will we ever get a 1 or 4!?!?**

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### Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

insert(47) will always fail here. Why?

For all  $n$ ,  $(5 + n^2) \% 7$  is 0, 2, 5, or 6  
 Proof uses induction and  
 $(5 + n^2) \% 7 = (5 + (n - 7)^2) \% 7$   
 In fact, for all  $c$  and  $k$ ,  
 $(c + n^2) \% k = (c + (n - k)^2) \% k$

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### From Bad News to Good News

After TableSize quadratic probes, we cycle through the same indices

The good news:

- For prime T and  $0 \leq i, j \leq T/2$  where  $i \neq j$ ,  $(h(\text{key}) + i^2) \% T \neq (h(\text{key}) + j^2) \% T$
- If TableSize is prime and  $\lambda < 1/2$ , quadratic probing will find an empty slot in at most  $\text{TableSize}/2$  probes
- If you keep  $\lambda < 1/2$ , no need to detect cycles as we just saw

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### Clustering Reconsidered

Quadratic probing does not suffer from primary clustering as the quadratic nature quickly escapes the neighborhood

But it is no help if keys initially hash the same index

- Any 2 keys that hash to the same value will have the same series of moves after that
- Called *secondary clustering*

We can avoid secondary clustering with a probe function that depends on the key: *double hashing*

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### Open Addressing: Double Hashing

Idea:

Given two good hash functions h and g, it is very unlikely that for some key,  $h(\text{key}) = g(\text{key})$   
Ergo, why not probe using g(key)?

For *double hashing*,  $f(i) = i \cdot g(\text{key})$ :

- 0<sup>th</sup> probe:  $(h(\text{key}) + 0 \cdot g(\text{key})) \% \text{TableSize}$
- 1<sup>st</sup> probe:  $(h(\text{key}) + 1 \cdot g(\text{key})) \% \text{TableSize}$
- 2<sup>nd</sup> probe:  $(h(\text{key}) + 2 \cdot g(\text{key})) \% \text{TableSize}$
- ...
- i<sup>th</sup> probe:  $(h(\text{key}) + i \cdot g(\text{key})) \% \text{TableSize}$

Crucial Detail:

We must make sure that g(key) cannot be 0

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### Double Hashing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

T = 10 (TableSize)

Hash Functions:

$h(\text{key}) = \text{key} \% T$

$g(\text{key}) = 1 + ((\text{key}/T) \% (T-1))$

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

**13**

28

33

147

43

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### Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	
8	
9	

T = 10 (TableSize)

Hash Functions:

$h(\text{key}) = \text{key} \% T$

$g(\text{key}) = 1 + ((\text{key}/T) \% (T-1))$

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

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### Double Hashing

0	
1	
2	
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T = 10 (TableSize)

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$h(\text{key}) = \text{key} \% T$

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### Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	

T = 10 (TableSize)

Hash Functions:

$h(\text{key}) = \text{key} \bmod T$

$g(\text{key}) = 1 + ((\text{key}/T) \bmod (T-1))$

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

$33 \rightarrow g(33) = 1 + 3 \bmod 9 = 4$

**147**

43

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### Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

T = 10 (TableSize)

Hash Functions:

$h(\text{key}) = \text{key} \bmod T$

$g(\text{key}) = 1 + ((\text{key}/T) \bmod (T-1))$

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

$147 \rightarrow g(147) = 1 + 14 \bmod 9 = 6$

**43**

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### Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

T = 10 (TableSize)

Hash Functions:

$h(\text{key}) = \text{key} \bmod T$

$g(\text{key}) = 1 + ((\text{key}/T) \bmod (T-1))$

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

$147 \rightarrow g(147) = 1 + 14 \bmod 9 = 6$

43  $\rightarrow g(43) = 1 + 4 \bmod 9 = 5$

**We have a problem:**

$3 + 0 = 3$     $3 + 5 = 8$     $3 + 10 = 13$

$3 + 15 = 18$     $3 + 20 = 23$

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### Double Hashing Analysis

Because each probe is "jumping" by  $g(\text{key})$  each time, we should ideally "leave the neighborhood" and "go different places from the same initial collision"

But, as in quadratic probing, we could still have a problem where we are not "safe" due to an infinite loop despite room in table

This cannot happen in at least one case:

For primes  $p$  and  $q$  such that  $2 < q < p$

$$h(\text{key}) = \text{key} \% p$$

$$g(\text{key}) = q - (\text{key} \% q)$$

### Summarizing Collision Resolution

Separate Chaining is easy

- find, delete proportional to load factor on average
- insert can be constant if just push on front of list

Open addressing uses probing, has clustering issues as it gets full but still has reasons for its use:

- Easier data representation
- Less memory allocation
- Run-time overhead for list nodes (but an array implementation could be faster)

When you make hash from hash leftovers...

**REHASHING**

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## Rehashing

As with array-based stacks/queues/lists

- If table gets too full, create a bigger table and copy everything
- Less helpful to shrink a table that is underfull

With chaining, we get to decide what "too full" means

- Keep load factor reasonable (e.g.,  $< 1$ )?
- Consider average or max size of non-empty chains

For open addressing, half-full is a good rule of thumb

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## Rehashing

What size should we choose?

- Twice-as-big?
- Except that won't be prime!

We go twice-as-big but guarantee prime

- Implement by hard coding a list of prime numbers
- You probably will not grow more than 20-30 times and can then calculate after that if necessary

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## Rehashing

Can we copy all data to the same indices in the new table?

- Will not work; we calculated the index based on TableSize

Rehash Algorithm:

Go through old table

Do standard insert for each item into new table

Resize is an  $O(n)$  operation,

- Iterate over old table:  $O(n)$
- $n$  inserts / calls to the hash function:  $n \cdot O(1) = O(n)$

Is there some way to avoid all those hash function calls?

- Space/time tradeoff: Could store  $h(\text{key})$  with each data item
- Growing the table is still  $O(n)$ ; only helps by a constant factor

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Reality is never as clean-cut as theory

## IMPLEMENTING HASHING

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## Hashing and Comparing

Our use of int key can lead to us overlooking a critical detail

- We do perform the initial hash on E
- While chaining/probing, we compare to E which requires equality testing (`compare == 0`)

A hash table needs a hash function and a comparator

- In Project 2, you will use two function objects
- The Java library uses a more object-oriented approach: each object has an equals method and a hashCode method:

```
class Object {
  boolean equals(Object o) {...}
  int hashCode() {...}
  ...
}
```

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## Equal Objects Must Hash the Same

The Java library (and your project hash table) make a very important assumption that clients must satisfy

Object-oriented way of saying it:

If `a.equals(b)`, then we must require  
`a.hashCode() == b.hashCode()`

Function object way of saying it:

If `c.compare(a,b) == 0`, then we must require  
`h.hash(a) == h.hash(b)`

If you ever override equals

- You need to override hashCode also in a consistent way
- See CoreJava book, Chapter 5 for other "gotchas" with equals

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## Comparable/Comparator Rules

We have not emphasized important "rules" about comparison for:

- all our dictionaries
- sorting (next major topic)

Comparison must impose a consistent, total ordering:

For all a, b, and c:

- If  $\text{compare}(a,b) < 0$ , then  $\text{compare}(b,a) > 0$
- If  $\text{compare}(a,b) == 0$ , then  $\text{compare}(b,a) == 0$
- If  $\text{compare}(a,b) < 0$  and  $\text{compare}(b,c) < 0$ , then  $\text{compare}(a,c) < 0$

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## A Generally Good hashCode()

```
int result = 17; // start at a prime
```

```
foreach field f
```

```
    int fieldHashCode =
```

```
        boolean: (f ? 1: 0)
```

```
        byte, char, short, int: (int) f
```

```
        long: (int) (f ^ (f >>> 32))
```

```
        float: Float.floatToIntBits(f)
```

```
        double: Double.doubleToLongBits(f), then above
        Object: object.hashCode( )
```

```
        result = 31 * result + fieldHashCode;
```

```
return result;
```



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## Final Word on Hashing

The hash table is one of the most important data structures

- Efficient find, insert, and delete
- Operations based on sorted order are not so efficient
- Useful in many, many real-world applications
- Popular topic for job interview questions

Important to use a good hash function

- Good distribution of key hashes
- Not overly expensive to calculate (bit shifts good!)

Important to keep hash table at a good size

- Keep TableSize a prime number
- Set a preferable  $\lambda$  depending on type of hashtable

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Are you ready... for an exam?

# MIDTERM EXAM

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## The Midterm

It is next Wednesday, July 18

It will take up the entire class period

It will cover everything up through today:

- Algorithmic analysis, Big-O, Recurrences
- Heaps and Priority Queues
- Stacks, Queues, Arrays, Linked Lists, etc.
- Dictionaries
- Regular BSTs, Balanced Trees, and B-Trees
- Hash Tables

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## The Midterm

The exam consists of 10 problems

- Total points possible is 110
- Your score will be out of 100
- Yes, you could score as well as 110/100

Types of Questions:

- Some calculations
- Drill problems manipulating data structures
- Writing pseudocode solutions

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### *Book, Calculator, and Notes*

The exam is closed book

You can bring a calculator if you want

You can bring a limited set of notes:

- One 3x5 index card (both sides)
- Must be handwritten (no typing!)
- You must turn in the card with your exam

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### *Preparing for the Exam*

Quiz section tomorrow is a review

→ Come with questions for David

We might do an exam review session

→ Only if you show interest

Previous exams available for review

→ Look for the link on midterm information

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### *Kate's General Exam Advice*

Get a good night's sleep

Eat some breakfast

Read through the exam before you start

Write down partial work

Remember the class is curved at the end

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## **PRACTICE PROBLEMS**

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### *Improving Linked Lists*

For reasons beyond your control, you have to work with a very large linked list. You will be doing many finds, inserts, and deletes. Although you cannot stop using a linked list, you are allowed to modify the linked structure to improve performance.

What can you do?

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### *Depth Traversal of a Tree*

One way to list the nodes of a BST is the depth traversal:

- List the root
- List the root's two children
- List the root's children's children, etc.

How would you implement this traversal?

How would you handle null children?

What is the big-O of your solution?

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### *Nth smallest element in a B Tree*

For a B Tree, you want to implement a function `FindSmallestKey(i)` which returns the  $i^{\text{th}}$  smallest key in the tree.

Describe a pseudocode solution.

What is the run-time of your code?

Is it dependent on L, M, and/or n?

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### *Hashing a Checkerboard*

One way to speed up Game AIs is to hash and store common game states. In the case of checkers, how would you store the game state of:

- The 8x8 board
- The 12 red pieces (single men or kings)
- The 12 black pieces (single men or kings)

Can your solution generalize to more complex games like chess?

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