CSE332: Data Abstractions

Lecture 24.5: Interlude on Intractability

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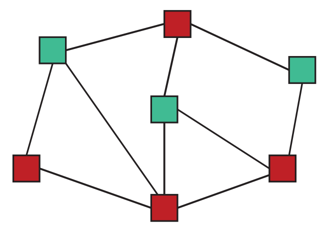
No, the material in lecture 24.5 (this one) won't be on the final
- But it's still an important high-level idea

Intractable Graph Problems

- Graph problems we studied all had efficient solutions (*roughly* $O(|V|^2)$ or better)
  - Topological sort
  - Traversals/connectedness
  - Shortest paths
  - Minimum Spanning Tree
- But there are plenty of *intractable* graph problems
  - Worst-case exponential in some aspect of the problem as far as we know
  - Topic studied in CSE312 and CSE421, but do not want to give false impression that there is always an efficient solution to everything
  - Common instances or approximate solutions can be better

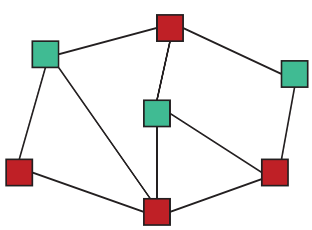
Vertex Cover: Optimal



Input: A graph (**V**,**E**)
Output: A minimum size subset **S** of **V** such that
for every edge (**u**,**v**) in **E**, at least one of **u** or **v** is in **S**

$O(2^{|V|})$ algorithm: Try every subset of vertices; pick smallest one
$O(|V|^k)$ algorithm: Unknown, probably does not exist

Vertex Cover: Decision Problem



Input: A graph (**V**,**E**) and a number **m**
Output: A subset **S** of **V** such that for every edge (**u**,**v**) in **E**, at least one of **u** or **v** is in **S** and **|S|=m** (if such an **S** exists)

$O(2^m)$ algorithm: Try every subset of vertices of size **m**
$O(m^k)$ algorithm: Unknown, probably does not exist

Good enough: Binary search on **m** can solve the original problem

Easy to *verify* a solution: See if **S** has size **m** and covers edges

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Traveling Salesman

[Like vertex cover, usually interested in the optimal solution, but we can ask a yes/no question and rely on binary search for optimal]

Input: A complete directed graph (**V**,**E**) and a number **m**
Output: A path that visits each vertex exactly once and has total cost < **m** if one exists

$O(|V|!)$ algorithm: Try every path, stop if find cheap enough one

Verifying a solution: Easy

Clique

Input: An undirected graph (**V**,**E**) and a number **m**
Output: Is there a *subgraph* of **m** nodes such that every edge in the subgraph is present?

Naïve algorithm: Try all subsets of **m** nodes

Verifying a solution: Easy

Not Just Graph Problems

- Every problem studied in CSE332 has an efficient solution
  - Correct cause and effect: *Chose* to study problems for which we know efficient solutions!
- There are plenty of intractable problems…

Subset Sum

14	**17**	5	**2**	**3**	**2**	6	**7**	6	17

31?

Input: An *array* of *n* numbers and a target-sum *sum*
Output: A subset of the numbers that add up to *sum* if one exists

$O(2^{\mathbf{n}})$ algorithm: Try every subset of array
$O(n^{\mathbf{k}})$ algorithm: Unknown, probably does not exist

Satisfiability

$$(\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee x_4) \wedge (x_2 \vee \neg x_4 \vee \neg x_5)$$

Input: a logic formula of size **m** containing **n** variables
Output: An assignment of Boolean values to the variables in the formula such that the formula is true

$O(\mathbf{m} * 2^{\mathbf{n}})$ algorithm: Try every variable assignment
$O(\mathbf{m}^{\mathbf{k}}\mathbf{n}^{\mathbf{k}})$ algorithm: Unknown, probably does not exist

So… what to do?

- Given a problem, how can you:
  - Find an efficient solution…
  - … or prove that one (probably) does not exist?
- See CSE312, CSE421, CSE431