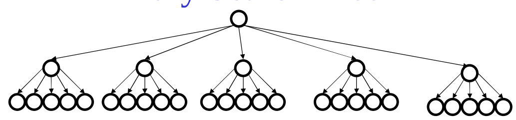
B-Trees

(4.7 in Weiss)

M-ary Search Tree



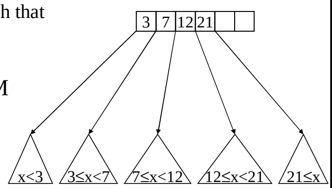
- Maximum branching factor of ***M***
- Tree with N values has height =

disk accesses for *find*:

Runtime of *find*:

Solution: B-Trees

- specialized *M*-ary search trees
- Each **node** has (up to) M-1 keys:
  - subtree between two keys *x* and *y* contains leaves with *values* *v* such that $x \le v < y$
- Pick branching factor M such that each node takes one full {*page*, *block*} of memory



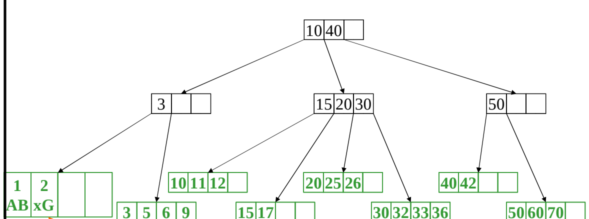
B-Trees

What makes them disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one access!
2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual *data*
   - The tree structure can be loaded into memory irrespective of data object size
   - Data actually resides in disk

B-Tree: Example

B-Tree with ***M* = 4** (# **pointers** in internal node)
and ***L* = 4** (# **data items** in **L**eaf)



Data objects, that I'll ignore in slides

Note: All leaves at the same depth!

B-Tree Properties ‡

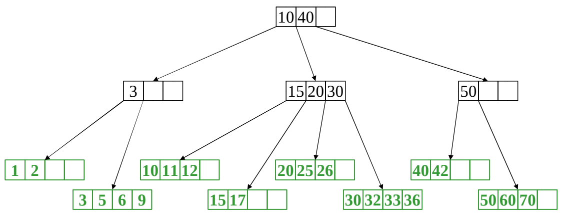
– Data is stored at the leaves
– All leaves are at the same depth and contain between $\lceil L/2 \rceil$ and *L* data items
– Internal nodes store up to *M-1* keys
– Internal nodes have between $\lceil M/2 \rceil$ and *M* children
– Root (special case) has between 2 and *M* children (or root could be a leaf)

‡These are technically B+-Trees

Example, Again

B-Tree with **M = 4**

and **L = 4**



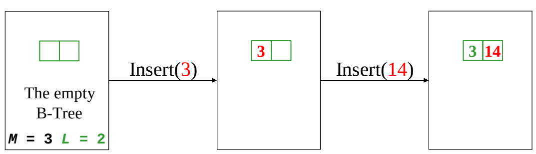
(Only showing keys, but leaves also have data!)

B-trees vs. AVL trees

Suppose we have 100 million items (100,000,000):

- Depth of AVL Tree
- Depth of B+ Tree with M = 128, L = 64

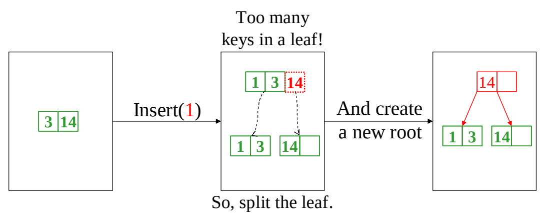
Building a B-Tree



Now, Insert(1)?

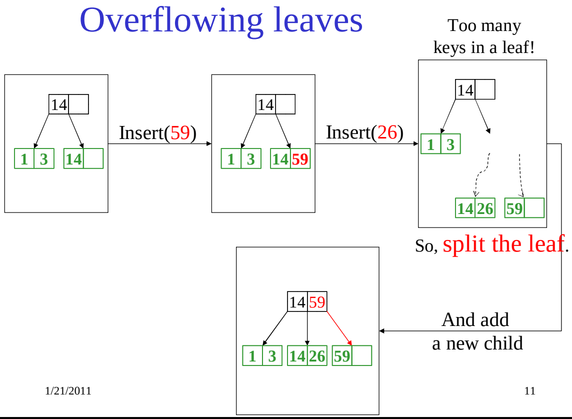
M = 3 L = 2

Splitting the Root



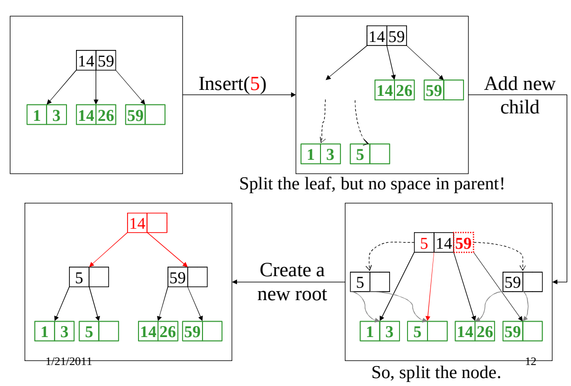
M = 3 L = 2

Overflowing leaves



M = 3 L = 2

Propagating Splits



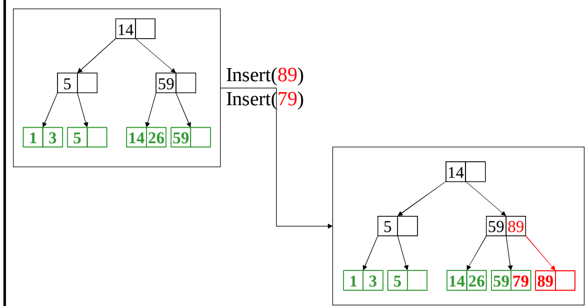
Insertion Algorithm

1. Insert the key in its leaf
2. If the leaf ends up with L+1 items, **overflow**!
   - Split the leaf into two nodes:
     - original with $\lceil (L+1)/2 \rceil$ items
     - new one with $\lfloor (L+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
3. If an internal node ends up with M+1 items, **overflow**!
   - Split the node into two nodes:
     - original with $\lceil (M+1)/2 \rceil$ items
     - new one with $\lfloor (M+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
4. Split an overflowed root in two and hang the new nodes under a new root

This makes the tree deeper!

M = 3 *L* = 2

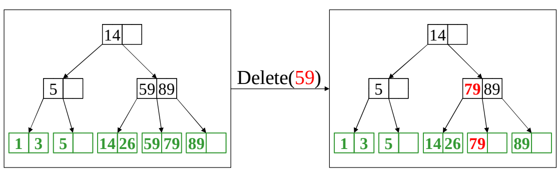
After More Routine Inserts



M = 3 *L* = 2

Deletion

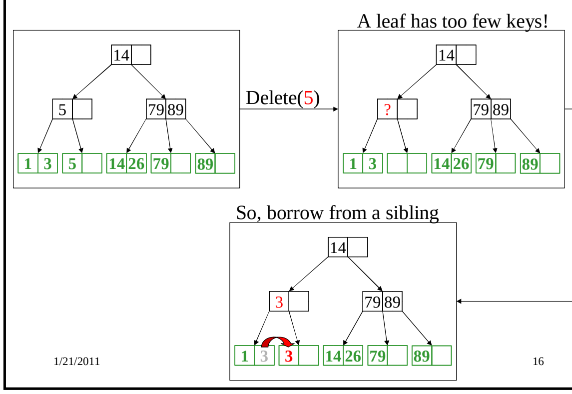
1. Delete item from leaf
2. Update keys of ancestors if necessary



What could go wrong?

M = 3 *L* = 2

Deletion and Adoption



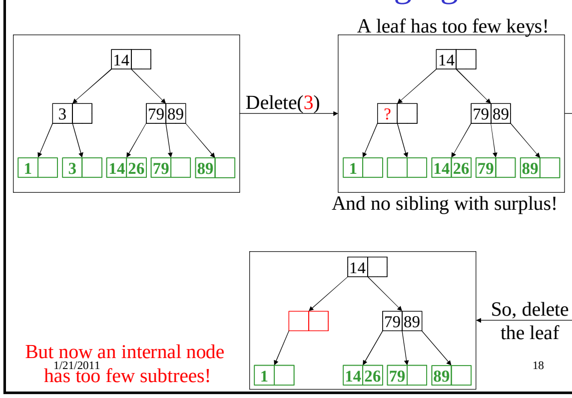
Does Adoption Always Work?

- What if the sibling doesn't have enough for you to borrow from?

  e.g. you have $\lceil L/2 \rceil$-1 and sibling has $\lceil L/2 \rceil$?

M = 3 *L* = 2

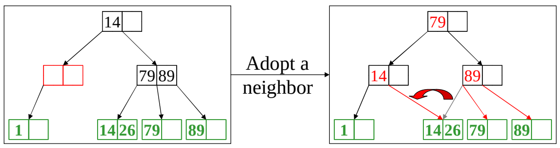
Deletion and Merging



But now an internal node has too few subtrees!

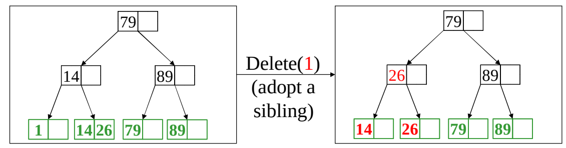
M = 3 L = 2

Deletion with Propagation (More Adoption)



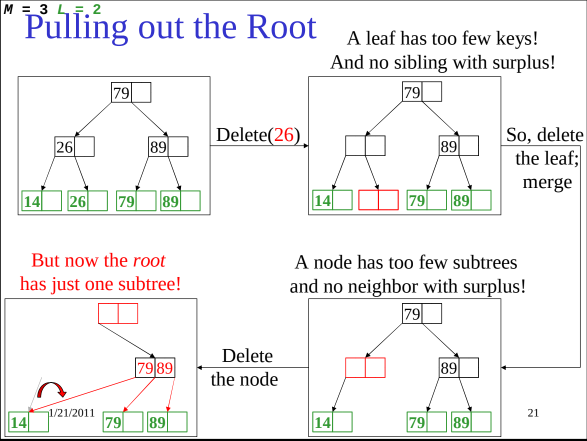
M = 3 L = 2

A Bit More Adoption



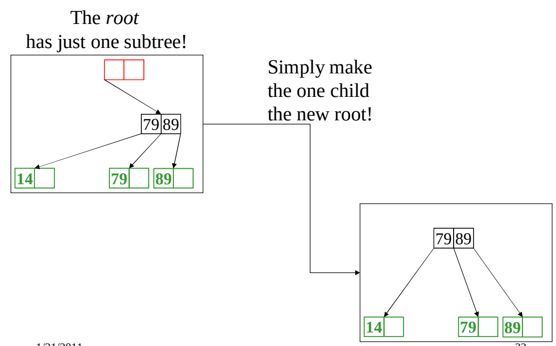
M = 3 L = 2

Pulling out the Root



M = 3 L = 2

Pulling out the Root (continued)



Deletion Algorithm

1. Remove the key from its leaf

2. If the **leaf** ends up with fewer than ⌈L/2⌉ items, **underflow**!
   - Adopt data from a sibling; update the parent
   - If adopting won't work, delete node and merge with neighbor
   - If the parent ends up with fewer than ⌈M/2⌉ items, **underflow**!

Deletion Slide Two

3. If an **internal** node ends up with fewer than ⌈M/2⌉ items, **underflow**!
   - Adopt from a neighbor; update the parent
   - If adoption won't work, merge with neighbor
   - If the parent ends up with fewer than ⌈M/2⌉ items, **underflow**!

4. If the root ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

Thinking about B-Trees

- B-Tree **insertion** can cause (expensive) splitting and propagation
- B-Tree **deletion** can cause (cheap) adoption or (expensive) deletion, merging and propagation
- Propagation is rare if ***M*** and ***L*** are large
  (Why?)
- If **`M = L = 128`**, then a B-Tree of height 4 will store at least 30,000,000 items

Tree Names You Might Encounter

FYI:
- B-Trees with **`M = 3`**, **`L = x`** are called **2-3 trees**
  - Nodes can have 2 or 3 pointers
- B-Trees with **`M = 4`**, **`L = x`** are called **2-3-4 trees**
  - Nodes can have 2, 3, or 4 pointers

Determining M and L for a B-Tree

1 Page on disk = 1 KByte

Key = 8 bytes, Pointer = 4 bytes

Data = 256 bytes per record (includes key)

M =

L =

Student Activity