Name: ______________KEY______________________

Email address: ____

Quiz Section: ___________

SECTION A

CSE 326 Winter 2006: Midterm Exam

(closed book, closed notes, calculators o.k.)

Instructions Read the directions for each question carefully before answering. We will give partial credit based on the work you **write down**, so show your work! Use only the data structures and algorithms we have discussed in class or which were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit. There is one extra page at the end of the exam that you may use for extra space on any problem. If you detach this page it must still be turned in with your exam when you leave.

Advice You have 50 minutes, **do the easy questions first**, and work quickly!

Total: 110 points. Time: 50 minutes. Exam ends at 1:20pm

Question	Max Points	Score
1	16	
2	9	
3	20	
4	16	
5	20	
6	20	
7	9	
Total	110	

1. **(16 pts) Big-Oh and Run Time Analysis:**

   a. (10 points total) Describe the running time of the following pseudocode in Big-Oh notation in terms of the variable `n`. Assume all variables used have been declared. ***Show your work for partial credit***.

```
int foo(int k) {
   int cost;
   for (int i = 0; i < k; ++i)
     cost = cost + (i * k);
   return cost;
}
```

I.

```
answ = foo(n);
```

$O(n)$

II.

```
int sum;
for (int i = 0; i < n; ++i) {
   if (n < 1000)
     sum++
   else
     sum += foo(n);
}
```

$O(n^2)$

III.

```
for (int i = 0; i < n + 100; ++i) {
   for (int j = 0; j < i * n ; ++j){
     sum = sum + j;
   }
   for (int k = 0; k < n + n + n; ++k){
     c[k] = c[k] + sum;
   }
}
```

$O(n^3)$

IV.

```
for (int j = 4; j < n; j=j+2) {
   val = 0;
   for (int i = 0; i < j; ++i) {
     val = val + i * j;
     for (int k = 0; k < n; ++k){
       val++;
     }
   }
}
```

$O(n^3)$

V.

```
for (int i = 0; i < n * 1000; ++i) {
   sum = (sum * sum)/(n * i);
   for (int j = 0; j < i; ++j) {
     sum += j * i;
   }
}
```

$O(n^2)$

1. (cont.) **Big-Oh and Run Time Analysis**

   b. (6 pts) Consider the following function:

```
int mystery(int n) {

   int answer;
   if (n > 0) {
      answer =(mystery(n-2)+3*mystery(n/2) + 5);
      return answer;
   }
   else
     return 1;
}
```

Write down the complete recurrence relation, T(n), for the running time of `mystery(n).` Be sure you include a base case T(0). **You do not have to actually solve this relation, just write it down.**

T(n) = T(n-2) + T(n/2) + C,	*for N > 0*
T(n) = 1,	*for N = 0*

2. (9 points) **Trees**

a) What is the minimum and maximum number of nodes in a *complete* binary tree of height h?

min = 2^h
max = 2^{h+1} -1

b) What is the minimum and maximum number of nodes in a *perfect* binary tree of height h?

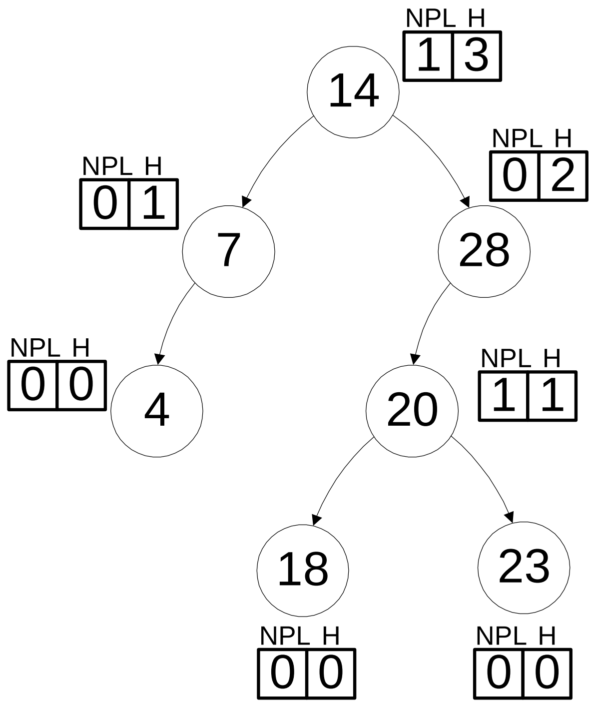
min = 2^{h+1} -1
max = 2^{h+1} -1

c) What is the AVL balance property?

At every node, the height of its left subtree differs from the height of its right subtree by no more than 1.

3. **(20 pts)**

a.) (10 pts) Mark the following properties for each node of the tree below in the space indicated for each node: **Null Path Length (NPL)** and **Height (H).**



b.) (10 pts) Also, circle **yes** or **no** to indicate whether the tree above might represent each of the following data structures. If you circle **no,** give **one specific reason** why the tree could **not** be that data structure.

- AVL tree yes **no**

Balance is broken under node 28.

- Splay tree **yes** no

Any binary search tree is a splay tree. This tree has the search tree property; so, it's a splay tree.

- Binary tree **yes** no

Every node has at most two children, so it's a binary tree.

- Skew heap yes **no**

This tree violates the heap-order property, for instance between nodes 14 and 7.

- Leftist heap yes **no**

Again, violates the heap-order property.

4. (16 pts) Running Time Analysis

Give a O-bound on the *worst case* running time for each of the following in terms of *n*. **No explanation is required**, but an explanation may help for partial credit. Assume that all keys are distinct.

(a) insert in an *AVL tree* of size *n*

worst case: O(log n)
AVL trees are balanced, and a new element goes at the fringe, so this must take take logarithmic time (best, average, worst, or any other case). Some of you suggested that we might get lucky and find a NULL child high in the tree where the new element should go. However, if such a NULL child existed, its parent would be out of balance (one child has large height, the other has height of –1).

(b) insert in a *splay tree* of size *n*

worst case: *O(n) – since the tree is unbalanced. For instance, the tree could be all left children, and then an insert of a new minimum element would have to traverse down all N children, then splay back up. Note that the amortized guarantee of a splay tree only applies to a sequence of operations – doesn't prevent individual bad cases like this.*

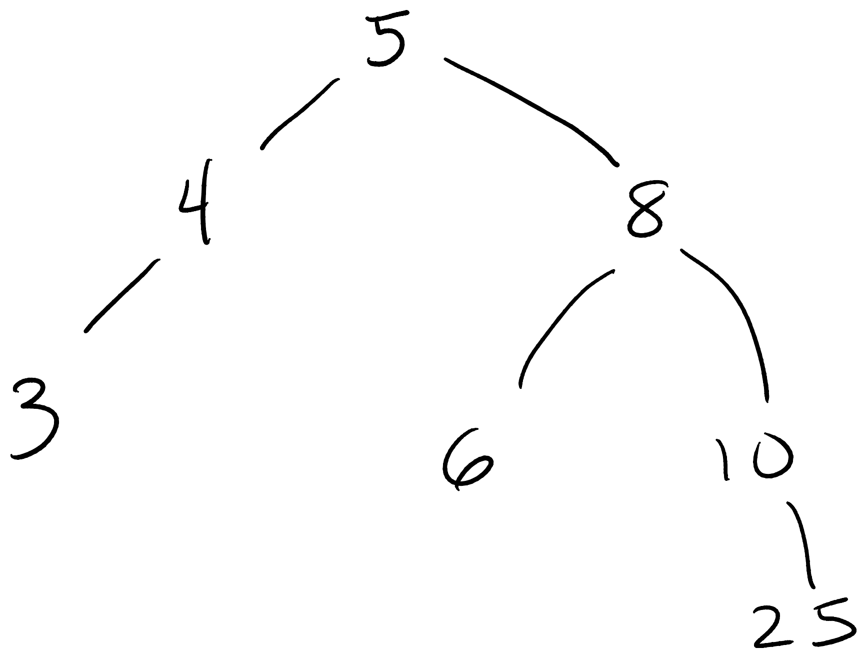
(c) deleteMin in a *d-heap* of size *n*

worst case: $O(d \log_d n)$. *At worst, the new root percolates down to the bottom, and a d heap has at most* $\log_d n$ *levels. At each level must compare with d children to decide if need to percolate down and if so which one to swap with.*

(d) Floyd's `buildHeap` to build a *binary heap* of *n* elements

worst case: *O(n)*
*This was discussed in class and is proven in the text. We check n/2 elements, and most of them can only percolate a small number of steps down. 1 pt for O(N*logN), 2 pts for explaining why you thought this bound was valid. Remember that Floyd's method is only interesting b/c it's better than the naïve O(N*logn).*

5. a. (8 pts) Draw the AVL tree that results from inserting the keys 4, 10, 3, 8, 5, 6, 25 in that order into an initially empty AVL tree. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***

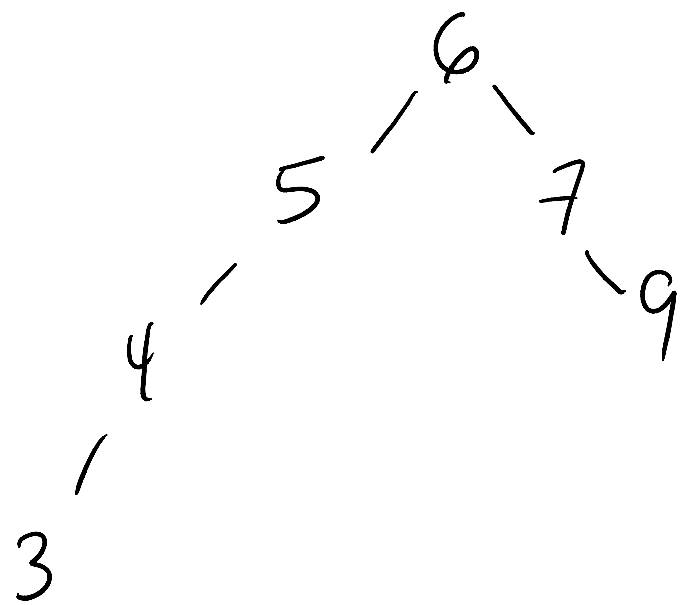


Insertion of 5 causes imbalance at node 10 – single rotation makes 8 the right child of the root (4).
Insertion of 6 causes imbalance at root (4) – double rotation needed, brings 5 to the root.
Final tree is balanced.

b. (2 pts) Give a *preorder* traversal of your final AVL tree you created in part a.

5, 4, 3, 8, 6, 10, 25

c. (8 pts) Draw the Splay tree that results from inserting the keys 4, 9, 3, 7, 5, 6 in that order into an initially empty Splay tree. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.*** You may continue your answer to this problem on the next page if needed.

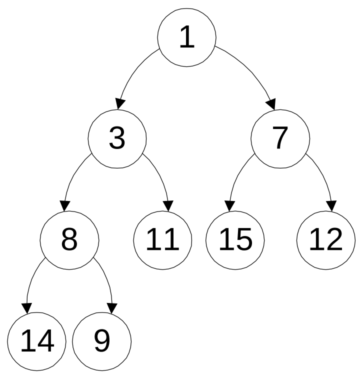


d. (2 pts) Give a *postorder* traversal of your final Splay tree you created in part c.

3, 4, 5, 9, 7, 6

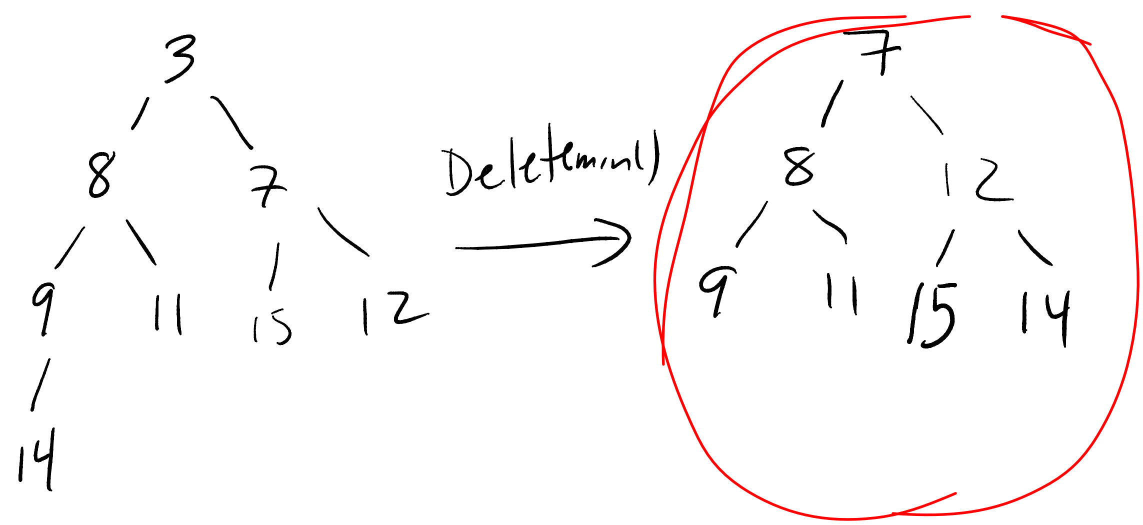
6. **(20 pts) Heaps**

a. (8 pts) Draw the binary heap that results from inserting 11, 9, 12, 14, 3, 15, 7, 8, 1 in that order into an **initially empty binary heap**. You do not need to show the array representation of the heap. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***



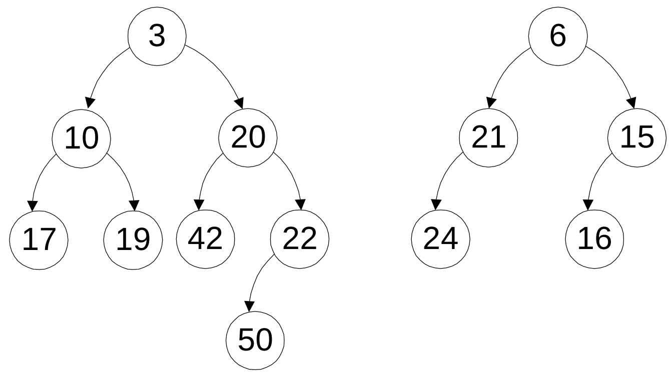
6. (cont.) **Heaps:**

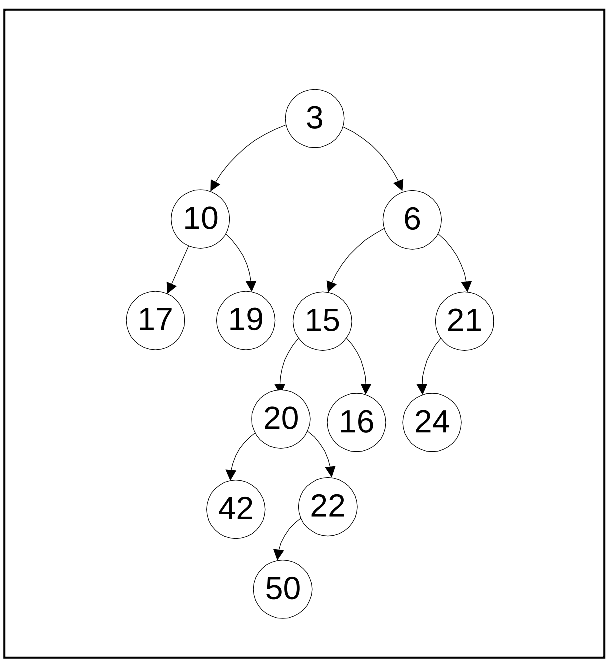
b. (4 pts) Draw the binary heap that results from doing 2 deletemins on the heap you created in part a.. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***



6. (cont.) **Heaps:**

c. (8 pts) Below are two leftist heaps. Show the result of merging them together using the algorithm we discussed in class. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.*** You may continue your answer to this question on the next page if needed.





(Extra space for Question 6. c)

7. (9 points) **Binomial Queues –**

a) What is the minimum and maximum number of nodes in a Binomial ***Tree*** of height h?

min = 2^h
max = 2^h

b) What is the minimum and maximum number of nodes in a Binomial ***Queue*** whose tallest tree is of height h?

min = 2^h
max = 2^{h+1} -1

c) Briefly describe how `Findmin()` is implemented for Binomial Queues.

The smallest value in each binomial tree will be stored at its root. So to find the minimum value in the entire queue, you just examine each root and determine the smallest root. Alternatively you can keep track of the smallest value in the queue as you add values so it can be a constant time operation to find the smallest.