CSE 332: Data Abstractions
Assignment #3
July 6, 2011
due: Wednesday, July 13, 10:50 a.m.

There are 2 pages to this assignment.

1. This exercise is about ordinary binary search trees, not balanced search trees. That is, use the algorithms of Figures 4.22 and 4.25.

   (a) Show the binary search tree that results from inserting the keys 186, 039, 991, 336, 778, 066, 564, 154, 538, and 645, in this order, into an initially empty tree.

   (b) Show the result of deleting the key 186 from the final tree of part 1a.

2. (a) Find the AVL tree that results from inserting the keys 186, 039, 991, 336, 778, 066, 564, 154, 538, and 645, in this order, into an initially empty tree. Show the AVL tree after each insertion (including possible rotations) is completed.

   (b) Show the result of deleting the key 186 from the final tree of part 2a.

   (c) Show the result of deleting the key 538 from the final tree of part 2b.

3. Find the AVL tree that results from inserting the keys 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, in this order, into an initially empty tree. Show the AVL tree after each of the 10 insertions (including possible rotations) has completed.

4. Give pseudocode for a linear-time algorithm that verifies that an AVL tree is correctly maintained. Assume every node has fields `key`, `height`, `left`, and `right` and that keys can be compared with $<$, $==$, and $>$. The algorithm should verify all of the following:

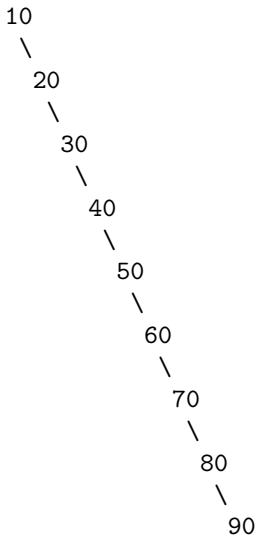
   (a) The tree is a binary search tree.

   (b) The height information of every node is correct.

   (c) Every node is balanced.

   Use `assert` statements so that your pseudocode has an assertion failure if and only if the AVL tree is incorrect.

5. (a) Consider the following splay tree T:

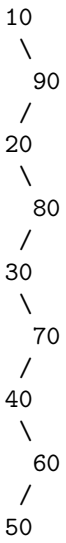


Find the result of splaying 90 in T. Show your tree after every zig-zig rotation.

(b) Suppose that splaying was done by performing single rotations, bottom up, as described in Section 4.5.1 of the textbook. For the splay tree T of part 5a, show what the result of splaying 90 would have been.

(c) Explain now why it would be a terrible idea to implement zig-zig rotations as suggested in part 5b. Describe a series of operations on T that would perform very poorly.

6. (a) Consider the following splay tree T:



Find the result of splaying 50 in T. Show your tree after every zig-zag rotation.

(b) What has happened to the trees after the splays of problems 5a and 6a and why is this a good thing?