Assignment 1

CSE 332: Data Abstractions, Spring 2011
University of Washington
March 30, 2011
due: Friday, April 8, 2:30 p.m.

Instructions: Create a paper representation of your answers. This can either be computer-formatted or handwritten. You might wish to prepare the file using LaTeX. You may use the source file of this assignment as a starting point if you use LaTeX. The "pdflatex" command installed on most Linux systems is a convenient way to translate the LaTeX source file into a PDF document. Be sure that your name is clearly visible at or near the top of the first page. Please staple your sheets of paper together, assuming you are using two or more sheets. Turn in your paper at the beginning of class on the due date.

1. (15 points) Prove by induction that $2n^2 = n + \sum_{i=0}^{n-1}(4i+1)$. State the basis and the induction hypothesis without simplifying the formula. (You may perform simplification as you prove the basis and the induction step.) Clearly mark the BASIS, INDUCTION HYPOTHESIS, and the INDUCTION STEP. (The induction hypothesis is what you'll assume at the beginning of the induction step, and the induction step is your proof that if the hypothesis holds at k, then it must hold at $k+1$.) Clearly state any constraints on the values of variables (such as n or k) that must hold in your formula(s).

2. (10 points) Do Weiss exercise 2.10. (Algorithms for common arithmetic). And justify each answer with one or two sentences.

3. (10 points) Do Weiss exercise 2.14 (Horner's rule).

4. (20 points) For each running-time function $T(n)$ and length of time t in the following table, determine the largest size n of a problem that can be solved within t, assuming that the algorithm to solve the problem takes $T(n)$ milliseconds. You are welcome to use a calculator or write a short program to help you.

	1 second	1 minute	1 hour	1 day	1 month	1 year
$100\log_2(\log_2 n)$						
$10\log_2 n$						
$10\log_2^2 n$						
n						
$n\log_2 n$						
n^2						
$0.1n^3$						
$0.001 \cdot 2^n$						
$0.001n!$						

5. (10 points) Prove or disprove: $8n \log_{16} 4n^2 \in \Theta(n \log n)$.

6. (15 points)

   (a) Prove that $n \log_b n \in O(n \sqrt[k]{n})$, for any constant real number $b > 1$ and constant integer $k > 1$.

   (b) Prove that $n \log_b n \notin O(n)$, for any real number $b > 1$.

7. (10 points) Let $T(n)$ be the running time of the following procedure on input n. Find a function $f(n)$ such that $T(n) \in \Theta(f(n))$, and justify your answer. Your function $f(n)$ should not contain any unnecessary constants or low-order terms.

```
Procedure threeWay(integer n):
   total ← 0;
   for i from 1 to n do
      for j from n/3 to n do
         for k from i to i+100 do
            if i+k is odd
               then total ← total +1;
               else total ← total +2;
```

8. (10 points) Let $T(n)$ be the running time of the following procedure on input n. Find a function $f(n)$ such that $T(n) \in \Theta(f(n))$, and justify your answer. Your function $f(n)$ should not contain any unnecessary constants or low-order terms.

```
Procedure twoWay(integer n):
   total ← 0;
   for i from n downTo 1 do
      for j from i downTo 0 do
         total ← total +1;
```