Worksheet 1 Solution

Using Induction on N, prove the following:

$\sum\_{i=0}^{N}A^{i}$ = $\frac{A^{N+1}-1}{A-1}$

Base case: N=0:

A0 = 1 = (A0+1-1)/(A-1)

Inductive step:

Let’s assume that the statement is true for some k>=0; so:

A0+A1+A2+…+Ak-1+Ak = (Ak+1-1)/(A-1)

Now we just need to show that this statement holds true for k+1; that is:

A0+A1+A2+…+Ak-1+Ak+Ak+1 = (Ak+2-1)/(A-1)

Let’s start from our assumption that the statement holds for k:

A0+A1+A2+…+Ak-1+Ak = (Ak+1-1)/(A-1)

To get it to look like the k+1 version for which we’re aiming, add Ak+1 to both sides – this will make the left side look like we want it to:

A0+A1+A2+…+Ak-1+Ak + Ak+1= (Ak+1-1)/(A-1) + Ak+1

The right side isn’t quite there yet though; work the ‘Ak+1’ into the other term:

A0+A1+A2+…+Ak-1+Ak + Ak+1= (Ak+1-1 + Ak+2 – Ak+1)/(A-1)

 = (Ak+2 – 1)/(A-1)

Which finishes our inductive step.