

Homework 5

Due Friday, July 30, 2010 at the beginning of class.

Problem 1: Graph Representations

Suppose a directed graph has a million nodes, most nodes have only a few edges, but a few nodes have hundreds of thousands of edges:

- In what way(s) would an adjacency-matrix representation of this graph lead to inefficiencies?
- In what way(s) would an adjacency-list representation of this graph lead to inefficiencies?
- Design a representation for this sort of graph that avoids all the inefficiencies in your answers to parts (a) and (b).

Problem 2: Topological Sort

Weiss, problem 9.1. For each step, show the in-degree array and the queue.

Problem 3: How to Graduate As Soon As Possible

- Given a DAG representing course pre-requisites, use precise English to describe an algorithm for computing a schedule for completing all the courses in the minimum number of academic terms. Assume that there is no limit on how many courses you can take in any given term and that every course is offered every term.
- What is the asymptotic running time of your algorithm in terms of $|V|$ and $|E|$?

Problem 4: Directed Graphs

- In a directed graph, how many edges must the graph have, at minimum, to be **weakly** connected? Explain your answer briefly.
- In a directed graph, how many edges must the graph have, at minimum, to be **strongly** connected? Explain your answer briefly.
- In lecture we stated that a directed graph can have a maximum of $|V|^2$ edges. One way to prove this would involve arguing that all edges (a,b) are legal where a & b are elements of V . Instead, for this problem, prove this inductively on the number of vertices, with a base case of $|V|=0$.