CSE 331

Abstract Data Types (ADTs)

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Recall: Properties of High-Quality Code

- Professionals are expected to write high-quality code
- Correctness is the most important part of quality
  - users hate products that do not work properly
- Also includes the following
  - easy to change
  - easy to understand
  - modular

abstraction provides all three properties

Recall: Procedural Abstraction

- Hide the details of the function from the caller
  - caller only needs to read the **specification**
  - ("procedure" means function)
- Caller promises to pass valid inputs
  - no promises on invalid inputs
- Implementer then promises to return correct outputs
  - does not matter how

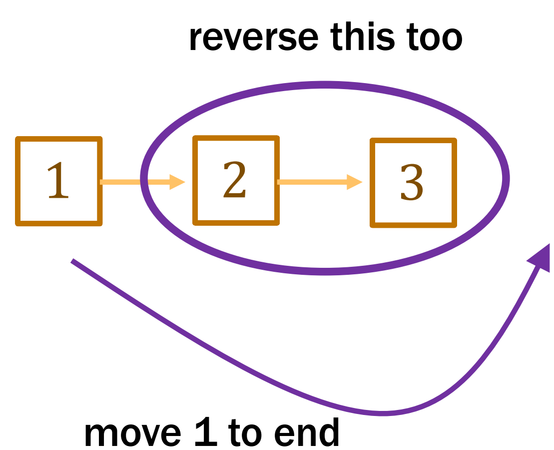
Definition of List Reversal

- **Mathematical definition of** $\mathrm{rev}(S)$

$$\begin{aligned}\mathrm{rev}(\mathrm{nil}) &:= \mathrm{nil} \\ \mathrm{rev}(x :: L) &:= \mathrm{rev}(L) \mathbin{+\!\!+} [x]\end{aligned}$$

included in the reference sheet posted on the Topics page

  - **note that** rev **uses** concat $(\mathbin{+\!\!+})$ **as a helper function**



Definition of List Reversal

- **Mathematical definition of** $\text{rev}(S)$

$$\text{rev(nil)} := \text{nil}$$
$$\text{rev}(x :: L) := \text{rev}(L) \mathbin{+\!\!+} [x]$$

  - **note that** rev **uses** concat ($\mathbin{+\!\!+}$) **as a helper function**
  - **ex:** $\text{rev}(1 :: 2 :: 3 :: \text{nil}) = 3 :: 2 :: 1 :: \text{nil}$

what other tests should we do?

$\text{rev}(1 :: 2 :: 3 :: \text{nil})$	
$= \text{rev}(2 :: 3 :: \text{nil}) \mathbin{+\!\!+} [1]$	def of rev
$= \text{rev}(3 :: \text{nil}) \mathbin{+\!\!+} [2] \mathbin{+\!\!+} [1]$	def of rev
$= \text{rev}(\text{nil}) \mathbin{+\!\!+} [3] \mathbin{+\!\!+} [2] \mathbin{+\!\!+} [1]$	def of rev
$= [] \mathbin{+\!\!+} [3] \mathbin{+\!\!+} [2] \mathbin{+\!\!+} [1]$	def of rev
$= \ldots = [3, 2, 1]$	def of concat (many times)

Procedural Abstraction Example

- **Specification of rev is imperative:**

```
// @return same numbers but in reverse order, i.e.
//   rev(nil)      := nil
//   rev(x :: L)  := rev(L) ++ [x]
public static List rev(List L) {
  return rev_acc(L, nil);  // faster way
}
```

  - code implements a different function
  - second version is $O(n)$ instead of $O(n^2)$
  - need to use reasoning to check that these two match

    can prove that rev_acc(L, nil) = rev(L) for all L by structural induction

Performance Improvements

- Faster algorithm, `rev-acc`, for reversing a list
  - rare to see this
- Most perf improvements change ***data structures***
  - different kind of abstraction barrier for data
- Let’s see an example…

Last Element of a List

$$\text{last}(x :: \text{nil}) := x$$
$$\text{last}(x :: y :: L) := \text{last}(y :: L)$$

last is undefined on nil

- Runs in $\Theta(n)$ time
  - walks down to the end of the list
  - no faster way to do this on a list
- How could we change data to make this faster?
  - we could cache the last element!

    analogous idea: store references to front and **back** nodes
  - can declare this in math as:

    $$\textbf{type}\ \text{FastLastList} := \{\text{last}: \mathbb{Z}, \text{list}: \text{List}\}$$
  - can do this in Java as well…

Fast-Last List

```
class FastLastList {
  private final int last;
  private final List list;

  FastLastList(List list) {
    this.list = list;
    this.last = last(list);
  }

  public int getLast() {
    return this.last;
  }

  public List getList() {
    return this.list;
  }
}
```

lots of real-world performance improvements look like this

in what way is this worse than just using `List`?

— less memory efficient

Fast-Last List

```
class FastLastList {
  private final int last;
  private final List list;

  public int getLast() { … }

  public List getList() { … }
}
```

- How do we switch to this type?
  - change every `List` into `FastLastList`
  - not truly hiding data structure changes yet…

Fast-Last List

```
class FastLastList {
  private final int last;
  private final List list;

  public int getLast() { … }

  public List getList() { … }
}
```

- **What if we also want the second-to-last element?**
  - this is $\Theta(n)$ to retrieve it
  - could cache the second to last element also
- **What if we just want the items at the end of the list?**
  - store it in **reverse** order!

Reversed List

```
private List L;   // regular order
private List R;   // reversed

/** @param L a list. Must be in *reverse* order */
public static void f(List L) { … }
```

- Why is this a terrible idea?
  - the type checker will not catch mistakes

    humans make mistakes. count on it
  - will unit tests catch this?

    might only show up in integration tests
  - what will the debugging be like?

Fast-Last List

```
class FastBackList {
  private final List revList;

  public int getLast() {
    return this.revList.hd;
  }

  public List getList() {
    return rev(this.revList);
  }
}
```

How could we avoid having a linear-time `getList`?

— keep `list` and `revList`

— time / space tradeoff

- How do we switch to this type?
  - change every `FastLastList` into `FastBackList`
  - not good that every data structure change is a code change

Fast-Last List

```
class FastLastList {
  private final int last;
  private final List list;
  …
}

class FastBackList {
  private final Last revList;
  …
}
```

- How can we make data structure changes without having to change all the clients every time?
  - use a Java `interface`

Fast List

```
interface FastList {
  int getLast();
  List getList();
}

class FastLastList implements FastList {
  private final int last;
  private final List list;
  …
}

class FastBackList implements FastList {
  private final List revList;
  …
}
```

Fast List

```
interface FastList {
  int getLast();
  List getList();
}
```

Interfaces contain only operations (methods) not data structures

- **Clients should use only `FastList` in declarations**
  - **only usen `FastLastList` after "`new`"**

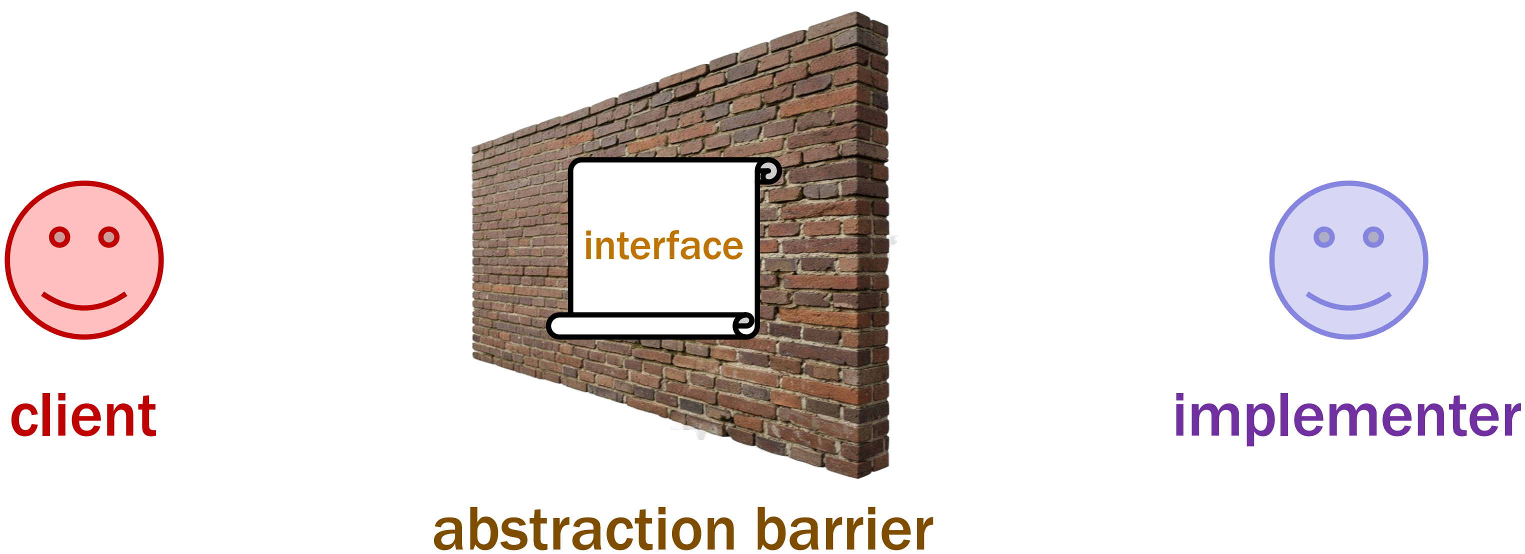
- **Standard style in Java**
  - **e.g., create an `ArrayList` but store it as `List`**

Example 1: Fast List

```
interface FastList {
  int getLast();
  List getList();
}
```

Interfaces contain only operations (methods) not data structures

- Interface provides an abstraction barrier
  - hides the details of data structures from the client



Data Abstraction

- Give clients only operations, not data
  - operations are the spec, data is hidden
- We call this an Abstract Data Type (ADT)
  - invented by Barbara Liskov in the 1970s

    won the Turing award in 2008
  - fundamental concept in computer science

    built into Java's `public` and `private`
  - data abstraction via procedural abstraction

- See recent interview with Computing History Museum

Example 2: Point in 2D Space

```
// Represents an (x, y) point in 2D space.
class Point {
  private double x;
  private double y;

  public double getX() { return this.x; }
  public double getY() { return this.y; }

  public double getR() {
    return Math.sqrt(x*x + y*y);
  }

  public double getTheta() {
    return Math.atan2(y, x);
  }
```

What if these are too slow?

— store polar coordinates

— time / space tradeoff

How do we write this as an ADT?

Example 2: Point in 2D Space

```
// Represents an (x, y) point in 2D space.
interface Point {
  double getX();
  double getY();
  double getR();
  double getTheta();
}

class SimplePoint implements Point {
  private double x, y;
}

class PolarPoint implements Point {
  private double x, y;
  private double r, theta;
}
```

What if we want to use `PolarPoint` for users with lots of memory and `SimplePoint` for users with little?

— need code to decide which to make

Example 2: Point in 2D Space

```
/** Creates a point at the given coordinates. */
public static Point makePoint(double x, double y) {
  if (Runtime.getRuntime().totalMemory() > MIN_MEM) {
    return new PolarPoint(x, y);
  } else {
    return new SimplePoint(x, y);
  }
}
```

- **This is a "factory function"**
  - an example of a design pattern

    more on these later…
  - Java SDK includes many, e.g., `Arrays.asList(..)`

Types of Operations

```java
interface Point {
  public double getX();
  public double getY();
  public double getR();
  public double getTheta();
  …
}
```

- Operations usually fall into a few classes:
  - **observers/getters**: return properties of an object
  - **mutators**: change properties of the object
  - **producers**: create new objects from an existing one

Observers/Getters on Point

```
public double getX();
public double getY();
public double getR();
public double getTheta();

/** Return distance from this point to given one. */
public double distTo(Point p);
```

- **Observers/Getters return information about the object, but do not change it in any way**

Mutators on Point

```
/** Move the point by dx in x coord and dy in y */
public void shift(double dx, double dy);

/** Move by rotating about the origin by theta. */
public void rotate(double theta);
```

- Mutators change the properties of the object, but usually do not return anything important
  - also sometimes called “setters”

Producers on Point

```
/** Move the point by dx in x coord and dy in y */
public Point shift(double dx, double dy);

/** Move by rotating about the origin by theta. */
public Point rotate(double theta);
```

- Producers create new objects
  - ***new*** object is returned
  - they ***do not*** change the existing object

Mutable vs Immutable ADTs

	Immutable	Mutable
observers	✅	✅
mutators	❌	✅
producers	✅	❌ (usually not)

- Sensible to pick one or the other
  - would be dangerous to provide both

    will see why later on

- We will stick to immutable ADTs for now
  - mutation makes reasoning & debugging harder
  - **EJ 17**: minimize mutability in classes

Recall: Fast List

```
interface FastList {
  int getLast();
  List getList();
}
```

- What kind of operations are these?
  - both are observers

- What would be examples of useful producers?

Producers on FastList

```
/** Returns the list with x added at the front. */
public FastList cons(int x);
```

- What factory functions should we provide?
  - what is the minimum we could get away with?

Creator of FastLists

```
/** @return nil. */
public static FastList emptyList() {
  return new FastBackList(null);
}
```

- How could we make this more memory efficient?
  - no need to create a new object every time

Creator of FastLists

```
public static FastList EMPTY_LIST =
    new FastBackList(null);

/** @return nil. */
public static FastList emptyList() {
  return EMPTY_LIST;
}
```

- This is the "singleton" design pattern
  - **note**: this is only possible since `FastList` is immutable!

    we will see why later on…

Producers on FastList

```
/** Returns the list with x added at the front. */
public FastList cons(int x);

/**
 * Returns the list containing the elements of
 * this list followed by those of R.
 */
public FastList concat(FastList R);
```

- How do we formalize this?
  - everything above is English

    there is possibility for confusion

Specifications for ADTs

Specifications for ADTs

- Run into problems when we try to write specs
  - for example, what goes after `@return`?

    don't want to say returns the `.list` field

    we want to hide those details from clients

```
interface FastList {
   /**
    * Returns the last element of the list.
    * @return ??
    */
   int getLast();
 };
```

- Need some terminology to clear up confusion

ADT Terminology

New terminology for specifying ADTs

Concrete State

actual fields of the record and the data stored in them

Last example: `int last, List list`

Abstract State

how clients should *think* about the object

Last example: List (i.e., nil or cons)

- **We've had different abstract and concrete types all along...**
  - **in our math, List is an inductive type (abstract)**
  - **in our code, `List` is a record (concrete)**

ADT Terminology

New terminology for specifying ADTs

Concrete State

actual fields of the record and the data stored in them

Last example: `int last, List list`

Abstract State

how clients should *think* about the object

Last example: List (i.e., nil or cons)

- Term "object" (or "obj") will refer to abstract state
  - "object" means mathematical object
  - "obj" is the mathematical value that the record represents

Specifying FastList

```
/**
 * A list of integers that can retrieve the last
 * element in O(1) time.
 */
interface FastList {
  /**
   * Returns the last element of the list (O(1) time)
   * @requires obj != nil
   * @return last(obj)
   */
  int getLast();
```

- “obj” refers to the abstract state (the list, in this case)
  - actual state will be a record with fields `last` and `list`

Specifying FastList

```
/**
 * A list of integers that can retrieve the last
 * element in O(1) time.
 */
interface FastList {
  …
  /**
   * Returns a new list with x in front of this list.
   * @return x :: obj
   */
  FastList cons(int x);
```

- Producer method: makes a new list for you
  - "obj" above is a list, so x :: obj makes sense in math

Specifying FastList

```
/**
 * A list of integers that can retrieve the last
 * element in O(1) time.
 */
interface FastList {
  …
  /**
   * Returns a new list with x in front of this list.
   * @return x :: obj
   */
  FastList cons(int x);
```

- Specification does not talk about fields, just "obj"
  - fields are ***hidden*** from clients

Specifying FastList

```
/**
 * A list of integers that can retrieve the last
 * element in O(1) time.
 */
interface FastList {
  …
  /**
   * Returns the object as a regular list of items.
   * @return ??
   */
  List getList();
```

- How do we specify this?

Specifying FastList

```
/**
 * A list of integers that can retrieve the last
 * element in O(1) time.
 */
interface FastList {
  …
  /**
   * Returns the object as a regular list of items.
   * @return obj
   */
  List getList();
```

- In math, this function does nothing (“`@return obj`”)
  - two *different* concrete representations of the same idea
  - details of the representations are ***hidden*** from clients

Specifying Point

```
/** Represents an (x, y) point in 2D space. */
interface Point {

  /** @return x */
  double getX();

  /** @return y */
  double getY();
```

- Abstract state *is* a pair (x, y)
  - i.e., we have $(x, y) := \mathrm{obj}$
  - so, we can refer to "x" and "y"

Specifying Point

```
/** Represents an (x, y) point in 2D space. */
interface Point {

  /** @return (x^2 + y^2)^(1/2) */
  double getR();

  /** @return arctan(y/x) */
  double getTheta();
```

- Imperative specifications
  - code may or may not actually do these calculations
  - `PolarPoint` just returns the value in a field

Specifying Point

```
/** Represents an (x, y) point in 2D space. */
interface Point {

  /** @return (x + dx, y + dy) */
  Point shiftBy(double dx, double dy);
```

- Describe the *abstract state* of what is returned
  - actual value returned is a `Point` of some variety

Recall: ADTs

- Abstraction over data
  - hide the details of the data representation
  - only give users a set of operations (the interface)

    data abstraction via procedural abstraction

- Interface can make clever data structures possible

- Some commonly used ADTs:
  - stack: add & remove from one end
  - queue: add to one end, remove from other
  - set: add, remove, & check if contained in list
  - map: add, remove, & get value for (key, value) pair

Immutable Queue

- A queue is a list that can *only* be changed two ways:
  - add elements to the front
  - remove elements from the back

```
// List that only supports adding to the front and
// removing from the end
interface NumberQueue {

  // @return len(obj)
  int size();

  // @return x :: obj
  NumberQueue enqueue(int x);

  // @requires len(obj) > 0
  // @return (x, Q) with obj = Q ++ [x]
  DequeueParts dequeue();
}
```

observer (size)

producer (enqueue)

producer (dequeue)

```
class DequeueParts {
  public final NumberQueue Q;
  public final int x;
}
```

Documenting an ADT Implementation

Documenting an ADT Implementation

- Last lecture, we saw how to write an ADT spec
- Key idea is the "abstract state"
  - simple definition of the object (easier to think about)
  - clients use that to reason about calls to this code
- Write specifications in terms of the abstract state
  - describe the return value in terms of "obj"
- We also need to reason about ADT implementation
  - for this, we do want to talk about fields
  - fields are hidden from clients, but visible to implementers

Documenting an ADT Implementation

- **We also need to document the ADT implementation**
  - **for this, we need two new tools**

Abstraction Function

defines what abstract state the field values currently represent

- **Maps the field values to the object they represent**
  - **object is math, so this is a *mathematical* function**

    there is no such function in the code — just a tool for reasoning
  - **will usually write this as an *equation***

    $\text{obj} = \ldots$ **right-hand side uses the fields**

Documenting the FastList ADT

```
class FastLastList implements FastList {
  // AF: obj = this.list
  private final int last;
  private final List list;
  …
}
```

- Abstraction Function (AF) gives the abstract state
  - obj = abstract state
  - this = concrete state (record with fields .last and .list)
  - AF relates abstract state to the current concrete state

    okay that "last" is not involved here
  - specifications only talk about "obj", not "this"

    "this" will appear in our reasoning

Documenting an ADT Implementation

- We also need to document the ADT implementation
  - for this, we need two new tools

Abstraction Function

defines what abstract state the field values currently represent

only needs to be defined when RI is true

Representation Invariants (RI)

facts about the field values that should always be true

defines what field values are allowed

AF only needs to apply when RI is true

Documenting the FastLastList ADT

```
class FastLastList implements FastList {
  // RI: this.last = last(this.list)
  // AF: obj = this.list
  private final int last;
  private final List list;
  …
}
```

- Representation Invariant (RI) holds info about this.last
  - fields cannot have *just any* number and list of numbers
  - they must fit together by satisfying RI

    last must be the last number in the list stored

Documenting the FastBackList ADT

```
class FastBackList implements FastList {


  private final List revList;
  …
}
```

- How can we specify this?
  - what is the AF and RI?

Documenting the FastBackList ADT

```
class FastBackList implements FastList {

  // AF: obj = rev(this.revList)
  private final List revList;
  …
}
```

- No need for an RI
  - any value for `revList` is fine

Documenting the FastBackList ADT

```
class FastBackList implements FastList {

  private final List list;
  private final List revList;
  …
}
```

- How can we specify this version?
  - what is the AF and RI?

Documenting the FastBackList ADT

```
class FastBackList implements FastList {
  // RI: this.revList = rev(this.list)
  // AF: obj = this.list
  private final List list;
  private final List revList;
  …
}
```

- Complexity moves from the AF to the RI

Documenting the PolarPoint ADT

```
/** Represents an (x, y) point in 2D space. */
class PolarPoint implements Point {


  private final double r, theta;
  …
}
```

- How can we specify this version of `Point`?
  - what is the AF and RI?

Documenting the PolarPoint ADT

```
/** Represents an (x, y) point in 2D space. */
class PolarPoint implements Point {
  // RI: r >= 0
  // AF: (r cos(theta), r sin(theta))
  private final double r, theta;
  …
}
```

- No constraints on theta
  - could restrict it by $-\pi < \theta \leq \pi$ (for example)

Documenting the PolarPoint ADT

```
/** Represents an (x, y) point in 2D space. */
class PolarPoint implements Point {


  private final double x, y;
  private final double r, theta;
  …
}
```

- **How can we specify this version of `Point`?**
  - **what is the AF and RI?**

Documenting the PolarPoint ADT

```
/** Represents an (x, y) point in 2D space. */
class PolarPoint implements Point {
  // RI: r = sqrt(x^2 + y^2) and theta = atan2(y, x)
  // AF: obj = (x, y)
  private final double x, y;
  private final double r, theta;
  …
}
```

- Complexity moves from AF to RI

Recall: Immutable Queue

- A queue is a list that can *only* be changed two ways:
  - add elements to the front
  - remove elements from the back

```
// List that only supports adding to the front and
// removing from the end
interface NumberQueue {

  // @return len(obj)
  int size();

  // @return [x] :: obj
  NumberQueue enqueue(int x);

  // @requires len(obj) > 0
  // @return (x, Q) with obj = Q ++ [x]
  DequeueParts dequeue();
}
```

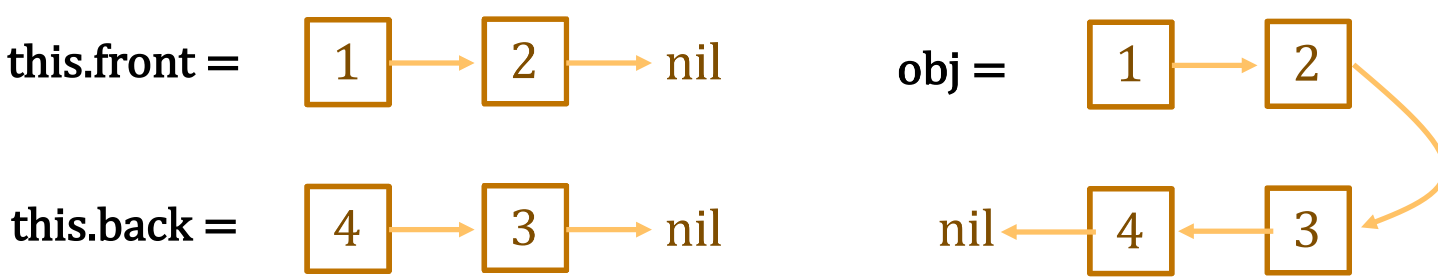
```
class DequeueParts {
  public final NumberQueue Q;
  public final int x;
}
```

Implementing a Queue with Two Lists

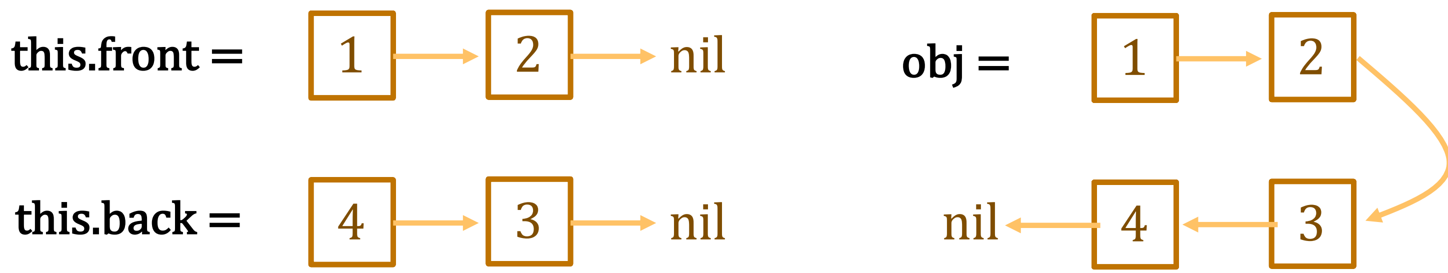
```
// Implements a queue using two lists.
class ListPairQueue implements NumberQueue {

  // AF: obj = this.front ++ rev(this.back)
  private final List front;
  private final List back;    // in reverse order
```

- Back part stored in reverse order
  - head of front is the first element
  - head of back is the *last* element



Implementing a Queue with Two Lists



- How do we enqueue (add at the front)?
  - remember that this is a producer
- How do we dequeue (remove from end)?
  - when is this not easy?
  - can we make this problem go away?

Implementing a Queue with Two Lists

```
// Implements a queue using two lists.
class ListPairQueue implements NumberQueue {

  // AF: obj = this.front ++ rev(this.back)
  // RI: if this.back = nil, then this.front = nil
  private final List front;
  private final List back;   // in reverse order
```

- **If back is nil, then the queue is *empty***
  - **if** $\text{back} = \text{nil}$, **then** $\text{front} = \text{nil}$ (**by RI**) **and thus**

$$\text{obj} =$$

Implementing a Queue with Two Lists

```
// Implements a queue using two lists.
class ListPairQueue implements NumberQueue {

  // AF: obj = this.front ++ rev(this.back)
  // RI: if this.back = nil, then this.front = nil
  private final List front;
  private final List back;   // in reverse order
```

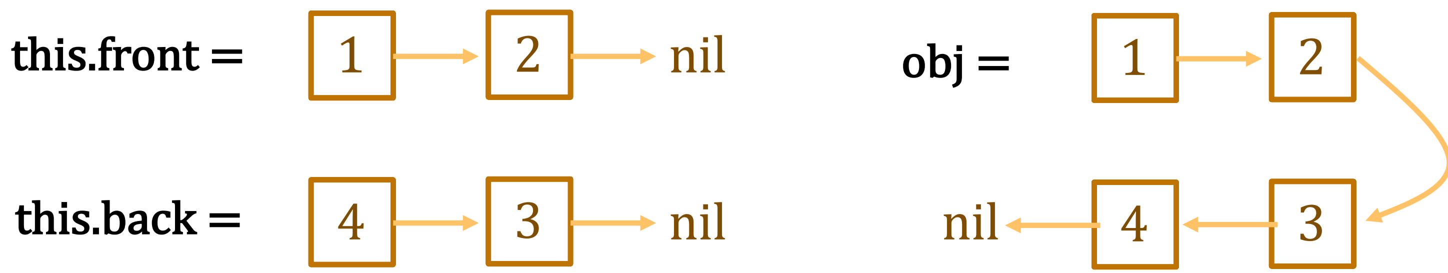
- **If back is nil, then the queue is *empty***
  - **if** back = nil, **then** front = nil **(by RI) and thus**

$$\begin{aligned} \text{obj} &= \text{nil} \mathbin{+\!\!+} \text{rev}(\text{nil}) && \textbf{by AF} \\ &= \text{rev}(\text{nil}) && \textbf{def of } \text{concat} \\ &= \text{nil} && \textbf{def of } \text{rev} \end{aligned}$$

  - **if the queue is not empty, then back is not nil**

    (**311 alert**: this is the contrapositive)

Implementing a Queue with Two Lists



- How do we dequeue (remove from end)?
  - What's different with our new RI?

- Easier to observe (find the last element), but harder to produce (the new queue)
  - i.e., easier to read but harder to write