CSE 331: Software Design & Implementation Winter 2026

Quiz Section 4: Floyd Logic – Solutions

Subscripts should *not* be used in 331 unless stated explicitly by the specification or when dealing with non-invertible operations (i.e., integer division, function calls).

Task 1 – Found Guilty of Reason [12 pts]

In this problem, you will practice proving correctness of straight-line code using **forward reasoning**.

a) Use forward reasoning to fill in the missing assertions in the following code:

$\{\{\, x \geqslant 5 \,\}\}$

`y = x - 2;`

$\{\{$ ______________________________ $\}\}$

`z = 3 * y;`

$\{\{$ ______________________________ $\}\}$

`z = z - 4;`

$\{\{\, P :$ __________________________ $\}\}$

$\{\{\, Q :\; z \geqslant 0 \,\}\}$

$\{\{\, x \geqslant 5 \,\}\}$

`y = x - 2;`

$\{\{\, x \geqslant 5$ and $y = x - 2 \,\}\}$

`z = 3 * y;`

$\{\{\, x \geqslant 5$ and $y = x - 2$ and $z = 3y \,\}\}$

`z = z - 4;`

$\{\{\, P :\; x \geqslant 5$ and $y = x - 2$ and $z + 4 = 3y \,\}\}$

$\{\{\, Q :\; z \geqslant 0 \,\}\}$

b) Show that the code is correct by proving by **calculation** that P implies Q.

We can see that Q holds since

$$\begin{aligned}
z &= 3y - 4 && \text{since } z + 4 = 3y \\
&= 3(x-2) - 4 && \text{since } y = x - 2 \\
&= 3x - 10 && \\
&\geqslant 3 \cdot 5 - 10 && \text{since } x \geqslant 5 \\
&= 5 && \\
&\geqslant 0 &&
\end{aligned}$$

c) Use forward reasoning to fill in the missing assertions in the following code:

$\{\{\, y > 5$ and $z > 2 \,\}\}$
`x = 4 * y - 3;`
$\{\{$ ______________________________________ $\}\}$
`y = y - 5;`
$\{\{$ ______________________________________ $\}\}$
`z = z * y;`
$\{\{\, P :$ ________________________________ $\}\}$
$\{\{\, Q :\ x < 2z + 20 \,\}\}$

$\{\{\, y > 5$ and $z > 2 \,\}\}$
`x = 4 * y - 3;`
$\{\{\, y > 5$ and $z > 2$ and $x = 4y - 3 \,\}\}$
`y = y - 5;`
$\{\{\, y + 5 > 5$ and $z > 2$ and $x = 4(y + 5) - 3 \,\}\}$
`z = z * y;`
$\{\{\, P :\ y + 5 > 5$ and $z/y > 2$ and $x = 4(y + 5) - 3 \,\}\}$
$\{\{\, Q :\ x < 2z + 20 \,\}\}$

d) Show that the code is correct by proving by calculation that P implies Q.

We can see that Q holds since

$$\begin{aligned} x &= 4y + 17 && \text{since } x = 4(y+5) - 3 \\ &< 2z + 17 && \text{since } z > 2y \text{ because } y > 0 \\ &< 2z + 20 \end{aligned}$$

Task 2 – Does a Duck Say "Back"? [12 pts]

In this problem, you will practice proving correctness of straight-line code using **backward reasoning**.

a) Use backward reasoning to fill in the missing assertions in the following code:

Fill in each blank by applying the rules *exactly* as taught in lecture. Then, if you want, you can simplify the resulting assertion, but do not weaken it. Separate any simplified statement from the original by "↔".

$\{\{\, P : \ c \geqslant 0 \,\}\}$

$\{\{\, Q : $ ________________________________ $\}\}$

```
b = 2*c;
```

$\{\{$ ____________________________________ $\}\}$

```
c = c - 1;
```

$\{\{$ ____________________________________ $\}\}$

```
a = b + 1;
```

$\{\{\, a \geqslant c \,\}\}$

$\{\{\, P : \ c \geqslant 0 \,\}\}$

$\{\{Q : \ 2c + 1 \geqslant c - 1 \,\}\} \quad \leftrightarrow \quad \{\{\, c \geqslant -2 \,\}\}$

```
b = 2*c;
```

$\{\{\, b + 1 \geqslant c - 1 \,\}\}$

```
c = c - 1;
```

$\{\{\, b + 1 \geqslant c \,\}\}$

```
a = b + 1;
```

$\{\{\, a \geqslant c \,\}\}$

b) Show that the code is correct by proving by calculation that P implies Q.

We can see that Q holds since $c \geqslant 0 \geqslant -2$.

c) Use backward reasoning to fill in the missing assertions in the following code:

$\{\{\, P: \;\; x < w + 1 \text{ and } w > 0 \,\}\}$

$\{\{\, Q: \;\underline{\hspace{10cm}}\,\}\}$

```
y = 4 * w;
```

$\{\{\,\underline{\hspace{11cm}}\,\}\}$

```
x = x * 2;
```

$\{\{\,\underline{\hspace{11cm}}\,\}\}$

```
z = x - 8;
```

$\{\{\, z < y \,\}\}$

$\{\{\, P: \;\; x < w + 1 \text{ and } w > 0 \,\}\}$

$\{\{Q: \;\; 2x - 8 < 4w \,\}\} \quad \leftrightarrow \quad \{\{\, 2x < 4w + 8 \,\}\} \quad \leftrightarrow \quad \{\{\, x < 2w + 4 \,\}\}$

```
y = 4 * w;
```

$\{\{\, 2x - 8 < y \,\}\}$

```
x = x * 2;
```

$\{\{\, x - 8 < y \,\}\}$

```
z = x - 8;
```

$\{\{\, z < y \,\}\}$

d) Show that the code is correct by proving by calculation that P implies Q.

We can see that Q holds since

$$\begin{aligned} x &< w + 1 \\ &< 2w + 1 \quad \text{since } w > 0 \\ &< 2w + 4 \end{aligned}$$

Task 3 – Nothing To Be If-ed At [6 pts]

Use forward reasoning to fill in the assertions. Then, prove, by cases, that what we know at the end of the conditional implies the post condition. The final assertion of the if and else branches are labeled as P1 and P2 respectively, please this abbreviation in future assertions and your proofs to refer to the same set of facts.

```
{{ $s > 0$ and $k = s^2$ }}
if (s < 5) {
  {{ ______________________________ }}
   j = k + s;
  {{ ______________________________ }}
   j = j / 2;
  {{ $P1$ : ______________________________ }}
} else {
  {{ ______________________________ }}
   j = k - s;
  {{ ______________________________ }}
   j = j + 2;
  {{ $P2$ : ______________________________ }}
}
{{ ________________ or ________________ }}
{{ $j \leqslant k + s$ }}
```

```
{{ $s > 0$ and $k = s^2$ }}
if (s < 5) {
  {{ $0 < s < 5$ and $k = s^2$ }}
   j = k + s;
  {{ $0 < s < 5$ and $k = s^2$ and $j = k + s$ }}
   j = j / 2;
  {{ $P1 : 0 < s < 5$ and $k = s^2$ and $j_0 = k + s$ and $j = \lfloor j_0/2 \rfloor$ }}
} else {
  {{ $s \geqslant 5$ and $k = s^2$ }}
   j = k - s;
  {{ $s \geqslant 5$ and $k = s^2$ and $j = k - s$ }}
   j = j + 2;
  {{ $P2 : s \geqslant 5$ and $k = s^2$ and $j - 2 = k - s$ }}
}
{{ $P1$ or $P2$ }}
{{ $j \leqslant k + s$ }}
```

We'll prove by cases that the assertion just below the conditional implies the post condition $\{\{\ j \leqslant k + s\ \}\}$:

First, assuming $P1$:

$$\begin{aligned} j &= \lfloor j_0\ /\ 2 \rfloor && \text{Since } j = \lfloor j_0\ /\ 2 \rfloor \\ &\leqslant j_0 / 2 && \text{Def of floor} \\ &= (k+s)/2 && \text{Since } j_0 = k + s \\ &\leqslant k + s && \text{Since } s > 0 \text{ and } k = s^2 > 0 \end{aligned}$$

Now, assuming $P2$:

$$\begin{aligned} j &= k - s + 2 && \text{Since } j - 2 = k - s \\ &\leqslant k + s && \text{Since } s \geqslant 5\text{, we know } -s + 2 \leqslant s \end{aligned}$$

Task 4 – Everbody Loops [12 pts]

In this problem, we will prove the correctness of a loop that finds the quotient of x divided by 10, i.e., the *largest* value y such that $10y \leqslant x$. To say that y is the largest such value means that any larger value would not satisfy the inequality, i.e., that $10(y+1) \nleqslant x$.

We denote the initial value of x at the top by x_0. This is explicitly stated in the precondition as the fact "$x = x_0$". The first two facts of the postcondition say that y is the quotient of x_0 divided by 10. The third fact says that x is the remainder, i.e., the remaining amount not divisible by 10.

This loop calculates the quotient without division. Instead, it just uses subtraction. It operates by increasing y and decreasing x each time around. The first part of the invariant says that the distance from x_0 down to $10y$ (i.e., $x_0 - 10y$) is the same as the distance from x down to 0 (i.e., $x - 0 = x$). The second part of the invariant says that x has not moved below 0 (i.e., $x \geqslant 0$).

```
{{ $x = x_0$ and $x_0 \geqslant 0$ }}
 int y = 0;
{{ Inv: $x_0 - 10y = x$ and $x \geqslant 0$ }}
 while (x >= 10) {
    y = y + 1;
    x = x - 10;
 }
{{ $10y \leqslant x_0$ and $x_0 < 10(y+1)$ and $x = x_0 - 10y$ }}
```

a) Prove that the invariant is true when we get to the top of the loop the first time.

Forward reasoning tells us that $x = x_0$, $x_0 \geqslant 0$, and $y = 0$. The first part of the invariant holds since

$$\begin{aligned} x_0 - 10y &= x - 10y && \text{since } x = x_0 \\ &= x && \text{since } y = 0 \end{aligned}$$

The second fact holds since $x = x_0 \geqslant 0$

b) Prove that, when we exit the loop, the postcondition holds.

When we exit the loop, we know that $x_0 - 10y = x$, $x \geqslant 0$, and $x < 10$. The first part of the postcondition holds since

$$\begin{aligned} 10y &= x_0 - x && \text{since } x_0 - 10y = x \\ &\leqslant x_0 && \text{since } x \geqslant 0 \end{aligned}$$

the second part of the postcondition holds since

$$\begin{aligned} x_0 &= x + 10y && \text{since } x_0 - 10y = x \\ &< 10 + 10y && \text{since } x < 10 \\ &= 10(y+1) \end{aligned}$$

and the third part is a restatement of the first fact from the invariant.

c) Prove that the invariant is preserved by the body of the loop. Use either forward or backward reasoning (your choice) to reduce the body to an implication and then check that it holds.

We can apply backward reasoning in the loop to get condition Q shown here:

$\{\{\, P:\ x_0 - 10y = x$ and $x \geqslant 0$ and $x \geqslant 10 \,\}\}$
$\{\{\, Q:\ x_0 - 10y = x$ and $x \geqslant 10 \,\}\}$
$\{\{\, x_0 - 10(y+1) = x - 10$ and $x - 10 \geqslant 0 \,\}\}$
```
y = y + 1n;
```
$\{\{\, x_0 - 10y = x - 10$ and $x - 10 \geqslant 0 \,\}\}$
```
x = x - 10n;
```
$\{\{\, x_0 - 10y = x$ and $x \geqslant 0 \,\}\}$

Both parts of Q are explicitly provided in P, so no calculations are needed.