

## Quiz Section 2: Proofs and Testing

### Task 1 – Calc You Later

[12 pts]

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Let  $a, b, c, d, e$  be integers. Complete each of the following proofs **by calculation**.

Include a justification on every step where a given fact is used. You can skip such an explanation only if the claim written in that step is itself, *literally* a known fact. It is also fine to cite a fact that is equivalent (via simple algebra) to a known fact. You do **not** need to cite algebra.

a) Given that  $a = 1$  and  $b = 1$ , it follows that  $a = 2b - 1$ .

b) Given that  $a = 1$ ,  $b = 2a - 1$ , and  $c > 0$ , it follows that  $(b - 1)^2 < c$ .

c) Given that  $d = b + 1$ ,  $c = a - 8$ , and  $e = a + 8b$ , it follows that  $e = c + 8d$ .

d) Given that  $b = 2a - 1$ ,  $d = a^2$ , and  $d + b + 2 < c$ , it follows that  $(a + 1)^2 < c$ .

## Task 2 – Absolutely Positive

[6 pts]

Let  $x$  be an integer and  $L$  a list. Complete the following proof **by cases**.

Your individual calculations should include explanations as in the previous problems. When citing a function definition that uses a side condition, your explanation must not only say which function's definition is being used but also what side condition is known to hold so that the reader can see what line of the definition you are referring to.

Let  $\text{abs} : \mathbb{Z} \rightarrow \mathbb{Z}$  be defined as follows:

$$\begin{aligned}\text{abs}(x) &= -x & \text{if } x < 0 \\ \text{abs}(x) &= x & \text{if } x \geq 0\end{aligned}$$

Prove that  $\text{abs}(\text{abs}(x)) = \text{abs}(x)$ .

### Task 3 – Keeping It Cool

[8 pts]

The functions  $\text{keep}$ ,  $\text{skip} : (\text{List}) \rightarrow \text{List}$  remove half the elements in a list. These two functions  $\text{keep}$  and  $\text{skip}$  every other element of the passed in list. The  $\text{keep}$  function includes the first element but skips the one after it, while  $\text{skip}$  drops the first element but keeps the one after that. They are defined formally as follows:

$$\begin{aligned}\text{keep}(\text{nil}) &= \text{nil} \\ \text{keep}(x :: L) &= x :: \text{skip}(L) \\ \text{skip}(\text{nil}) &= \text{nil} \\ \text{skip}(x :: L) &= \text{keep}(L)\end{aligned}$$

Also, recall the function  $\text{echo} : (\text{List}) \rightarrow \text{List}$ , which was defined in class as follows:

$$\begin{aligned}\text{echo}(\text{nil}) &= \text{nil} \\ \text{echo}(x :: L) &= x :: x :: \text{echo}(L)\end{aligned}$$

and the function  $\text{sum} : (\text{List}) \rightarrow \mathbb{N}$ , which was defined in class as follows:

$$\begin{aligned}\text{sum}(\text{nil}) &= 0 \\ \text{sum}(x :: L) &= x + \text{sum}(L)\end{aligned}$$

You will use these functions in the problem below.

Prove that  $\text{sum}(\text{skip}(\text{echo}(L))) = \text{sum}(L)$  holds by structural induction on  $L$ .

**Task 4 – The Test-Laid Plans****[12 pts]**

For each of the following functions, state the number of tests required to meet our coverage requirements and explain why that is the required number.

Then, describe a specific set of tests to use (with the same number of tests you as said before). Describe each test by giving the input (identify a specific input rather than saying, e.g., "some positive number"), stating which portion of the function it tests, and explaining why our rules require that test.

a) 

```
public static int f(int n) {
    if (n < 0) {
        return -2 * n;
    } else {
        return 3 * n;
    }
}
```

b) 

```
public static int h(int n) {
    if (n <= 0) {
        return 1;
    } else {
        return 2 + h(n / 3);
    }
}
```