Quiz Section 9: Points

In this section, we will be working with a data structure called a LocTree. For more information on the data structures and definitions used, please refer to the resource found here: `https://courses.cs.washington.edu/courses/cse331/25wi/homework/hw9def.pdf`.

Task 1 – Regions in Hot Pursuit!

We have a function overlap : (Region, Region) → Bool that returns true if two regions overlap. Two regions overlap if they share any area in common. Write an expression that returns true if 2 regions R_1 and R_2 overlap.

Task 2 – Tree-search takes root!

(a) Define a function findAll : (LocTree, Region) → List⟨Location⟩ that returns a list of all locations in the LocTree that fall within the given region. The order of the locations in the list does not matter but there should be no duplicate entries. Assume we have the following function contains : (Region, Location) → Bool that returns true if the location is within the region.

(b) Improve the algorithm by excluding any quadrants that do not overlap with the region passed in. This will avoid traversing any subtrees that cannot contain any locations in the region. We can do this by defining an improved function fa : (LocTree, Region, Region) → List⟨Location⟩ that takes in an additional Region parameter. The second Region parameter is the region that we are looking for locations in. The first Region parameter is a region containing all the points in the tree. It will use this region parameter to avoid recursing into quadrants of split nodes when they cannot contain a location in the second Region parameter.

(c) Prove that if region S contains all locations in the tree T, then $\mathsf{fa}(T, S, R) = \mathsf{findAll}(T, R)$. Your proof should be by structural induction on T.

Feel free to use the fact that, if S contains all the locations in $\mathsf{split}(m, \mathsf{nw}, \mathsf{ne}, \mathsf{sw}, \mathsf{se})$, then $\mathsf{NW}(m, S)$ contains all the locations in nw and likewise for ne, sw, and se. (This follows from the representation invariant for split nodes and the definitions of these functions.)