CSE 331
Software Design & Implementation

Winter 2025

Section 9 – HW 9 Prep

Administrivia

- HW 9 written released tonight, due 3/14 (but try to do it earlier because the code is *really really* hard)

- FINAL EXAM 3/18 (next section will be primarily exam prep)
  - 2:30 pm - 4:20 pm in CSE2 G20

HW 9 Prep & Tips

- HW 9 will be adding functionality to Campus Maps from HW 3
- Section slides and ws designed to introduce you to HW 9 concepts & data structures
- Please take a look at the starter code BEFORE starting the assignment
  - This will allow you to better understand the specifications of the assignments
  - We also give you many helper functions and definitions so this will also prevent you from reinventing the wheel (i.e. calculating the length of a list)

Prep for HW 9 / Locations

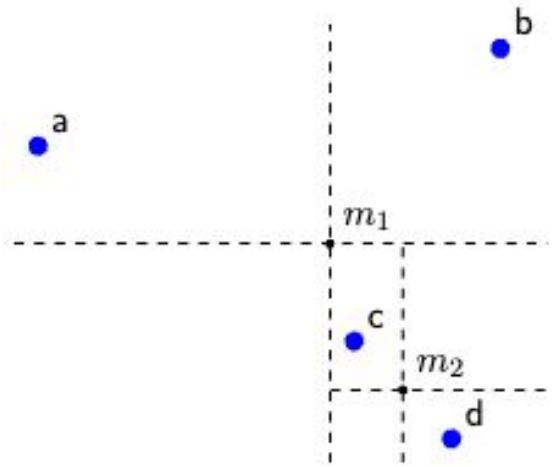
In HW 9, we will be working with Location objects again:

$$\textbf{type}\ \text{Location} := \{x : \mathbb{R},\ y : \mathbb{R}\}$$

We will be interested in finding all Location points within a given rectangle.

To keep track, we will use trees that recursively split areas on the map until each region only contains a single point

LocTree



- To represent the points in the image on the left, we would start by keeping track of one rectangle that represents the entire map
- We would then split that rectangle into 4 rectangles at m_1
- And then split the lower right rectangles into 4 rectangles at m_2.

m_1 is the average location of all 4 points, m_2 is the average location of points c and d. We call this average point the "centroid" !

This inductive data type will help us represent the tree:

```
type LocTree := empty
              | single(loc : Location)
              | split(at : Location, nw : LocTree, ne : LocTree, sw : LocTree, se : LocTree)
```

Note that nw, ne, sw, and se are lowercase! Uppercase relates to a different function.

Regions

We will keep track of the corners for regions we are interested in. The Region object will be as follows:

$$\textbf{type}\ \text{Region} := \{\text{x1}: \mathbb{R},\ \text{x2}: \mathbb{R},\ \text{y1}: \mathbb{R},\ \text{y2}: \mathbb{R}\}$$

The region *R* contains the location ℓ if and only if *R.x1* <= ℓ.x < *R.x2* and *R.y1* <= ℓ.*y* < *R.y2.* Thus, the following region includes every point in the plane:

$$\text{EVERYWHERE} := \{\text{x1}: -\infty,\ \text{x2}: \infty,\ \text{y1}: -\infty,\ \text{y2}: \infty\}$$

Location, Region, LocTree

- Go over how NW(m, R), NE(m, R), etc. work (picture shown below for reference of what the definition is)

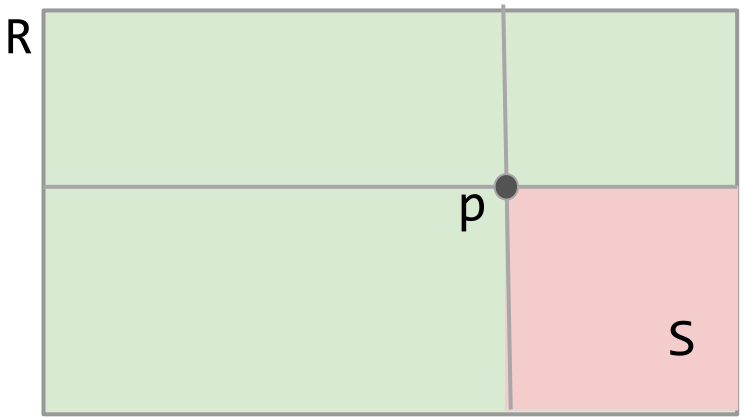
$$\mathsf{NW}(m, R) := \{\mathsf{x1}: R.\mathsf{x1},\ \mathsf{x2}: m.x,\ \mathsf{y1}: R.\mathsf{y1},\ \mathsf{y2}: m.y\}$$
$$\mathsf{NE}(m, R) := \{\mathsf{x1}: m.x,\ \mathsf{x2}: R.\mathsf{x2},\ \mathsf{y1}: R.\mathsf{y1},\ \mathsf{y2}: m.y\}$$
$$\mathsf{SW}(m, R) := \{\mathsf{x1}: R.\mathsf{x1},\ \mathsf{x2}: m.x,\ \mathsf{y1}: m.y,\ \mathsf{y2}: R.\mathsf{y2}\}$$
$$\mathsf{SE}(m, R) := \{\mathsf{x1}: m.x,\ \mathsf{x2}: R.\mathsf{x2},\ \mathsf{y1}: m.y,\ \mathsf{y2}: R.\mathsf{y2}\}$$

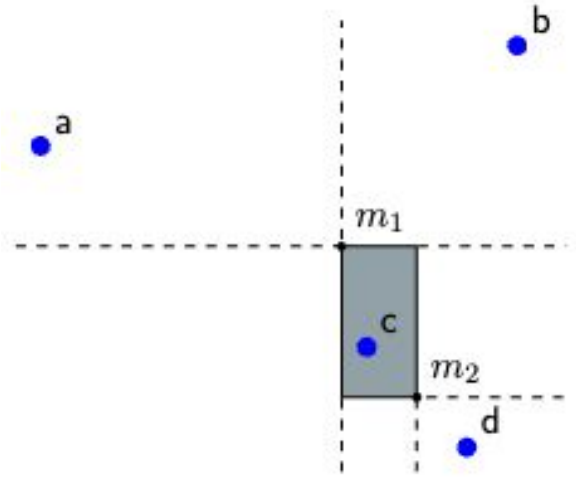
Region functions

Functions NW… SW : (Location, Region) -> Region

- take a location point and a region rectangle
- return the subregion of parameter that is split at the location point and the in indicated direction

EX: SE(p, R) = S





EX: NW(m2, SE(m1, EVERYWHERE))

Question 1

We have a function overlap : (Region, Region) $\rightarrow$ Bool that returns true if two regions overlap. Two regions overlap if they share any area in common. Write an expression that returns true if 2 regions R_1 and R_2 overlap.

Question 2a

(a) Define a function FindAll : (LocTree, Region) → List⟨Location⟩ that returns a list of all locations in the LocTree that fall within the given region. The order of the locations in the list does not matter but there should be no duplicate entries. Assume we have the following function contains : (Region, Location) → Bool that returns true if the location is within the region.

Question 2b

(b) Improve the algorithm by excluding any quadrants that do not overlap with the region passed in. This will avoid traversing any subtrees that cannot contain any locations in the region. We can do this by defining an improved function fa : $(\text{LocTree}, \text{Region}, \text{Region}) \rightarrow \text{List}\langle\text{Location}\rangle$ that takes in an additional Region parameter. The second Region parameter is the region that we are looking for locations in. The first Region parameter is a region containing all the points in the tree. It will use this region parameter to avoid recursing into quadrants of split nodes when they cannot contain a location in the second Region parameter.

Question 2c

(c) Prove that if region S contains all locations in the tree T, then $\mathsf{fa}(T, S, R) = \mathsf{FindAll}(T, R)$. Your proof should be by structural induction on T.

Feel free to use the fact that, if S contains all the locations in $\mathsf{split}(m, \mathsf{nw}, \mathsf{ne}, \mathsf{sw}, \mathsf{se})$, then $\mathsf{NW}(m, S)$ contains all the locations in nw and likewise for ne, sw, and se. (This follows from the representation invariant for split nodes and the definitions of these functions.)

$$
\begin{aligned}
\mathsf{FindAll}(\mathsf{empty}, R) &:= [] \\
\mathsf{FindAll}(\mathsf{single}(\mathsf{s}), R) &:= [\mathsf{s}] && \text{if } \mathsf{contains}(R, s) \\
\mathsf{FindAll}(\mathsf{single}(\mathsf{s}), R) &:= [] && \text{if not } \mathsf{contains}(R, s) \\
\mathsf{FindAll}(\mathsf{split}(m, \mathsf{nw}, \mathsf{ne}, \mathsf{sw}, \mathsf{se}), R) &:= \mathsf{FindAll}(\mathsf{nw}, \mathsf{R}) \\
&\quad +\!\!+ \mathsf{FindAll}(\mathsf{ne}, \mathsf{R}) \\
&\quad +\!\!+ \mathsf{FindAll}(\mathsf{sw}, \mathsf{R}) \\
&\quad +\!\!+ \mathsf{FindAll}(\mathsf{se}, \mathsf{R})
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{fa}(\mathsf{empty}, S, R) &:= [] \\
\mathsf{fa}(\mathsf{single}(\mathsf{s}), S, R) &:= [\mathsf{s}] && \text{if } \mathsf{contains}(R, s) \\
\mathsf{fa}(\mathsf{single}(\mathsf{s}), S, R) &:= [] && \text{if not } \mathsf{contains}(R, s) \\
\mathsf{fa}(\mathsf{split}(m, \mathsf{nw}, \mathsf{ne}, \mathsf{sw}, \mathsf{se}), S, R) &:= [] && \text{if not } \mathsf{overlap}(S, R) \\
\mathsf{fa}(\mathsf{split}(m, \mathsf{nw}, \mathsf{ne}, \mathsf{sw}, \mathsf{se}), S, R) &:= \mathsf{fa}(\mathsf{nw}, \mathsf{NW}(m, S), R) \\
&\quad +\!\!+ \mathsf{fa}(\mathsf{ne}, \mathsf{NE}(m, S), R) \\
&\quad +\!\!+ \mathsf{fa}(\mathsf{sw}, \mathsf{SW}(m, S), R) \\
&\quad +\!\!+ \mathsf{fa}(\mathsf{se}, \mathsf{SE}(m, S), R)
\end{aligned}
$$