

Quiz Section 2: Proofs

Task 1 – Calc You Later

[12 pts]

Let a, b, c, d, e be integers. Complete each of the following proofs **by calculation**.

Include an explanation on each step where a given fact is used. You can skip such an explanation only if the claim written in that step is itself, *literally* a known fact. It is also fine to cite a fact that is equivalent (via simple algebra) to a known fact.

a) Given that $a = 1$ and $b = 1$, it follows that $a = 2b - 1$.

b) Given that $a = 1$, $b = 2a - 1$, and $c > 0$, it follows that $(b - 1)^2 < c$.

c) Given that $d = b + 1$, $c = a - 8$, and $e = a + 8b$, it follows that $e = c + 8d$.

d) Given that $b = 2a - 1$, $d = a^2$, and $d + b + 2 < c$, it follows that $(a + 1)^2 < c$.

Task 2 – Absolutely Positive

[6 pts]

Let x be an integer and L a list. Complete the following proof **by cases**.

Your individual calculations should include explanations as in the previous problems. When citing a function definition that uses a side condition, your explanation must not only say which function's definition is being used but also what side condition is known to hold so that the reader can see what line of the definition you are referring to.

Let $\text{abs} : \mathbb{Z} \rightarrow \mathbb{Z}$ be defined as follows:

$$\text{abs}(x) = -x \quad \text{if } x < 0$$

$$\text{abs}(x) = x \quad \text{if } x \geq 0$$

Prove that $\text{abs}(\text{abs}(x)) = \text{abs}(x)$.

Task 3 – Keeping It Cool

[8 pts]

The functions $\text{keep}, \text{skip} : (\text{List}) \rightarrow \text{List}$ remove half the elements in a list. These two functions keep and skip every other element of the passed in list. The keep function includes the first element but skips the one after it, while skip drops the first element but keeps the one after that. They are defined formally as follows:

$$\begin{aligned}\text{keep}(\text{nil}) &= \text{nil} \\ \text{keep}(x :: L) &= x :: \text{skip}(L) \\ \text{skip}(\text{nil}) &= \text{nil} \\ \text{skip}(x :: L) &= \text{keep}(L)\end{aligned}$$

Also, recall the function $\text{echo} : (\text{List}) \rightarrow \text{List}$, which was defined in class as follows:

$$\begin{aligned}\text{echo}(\text{nil}) &= \text{nil} \\ \text{echo}(x :: L) &= x :: x :: \text{echo}(L)\end{aligned}$$

and the function $\text{sum} : (\text{List}) \rightarrow \mathbb{N}$, which was defined in class as follows:

$$\begin{aligned}\text{sum}(\text{nil}) &= 0 \\ \text{sum}(x :: L) &= x + \text{sum}(L)\end{aligned}$$

You will use these functions in the problem below.

Prove that $\text{sum}(\text{skip}(\text{echo}(L))) = \text{sum}(L)$ holds by structural induction on L .