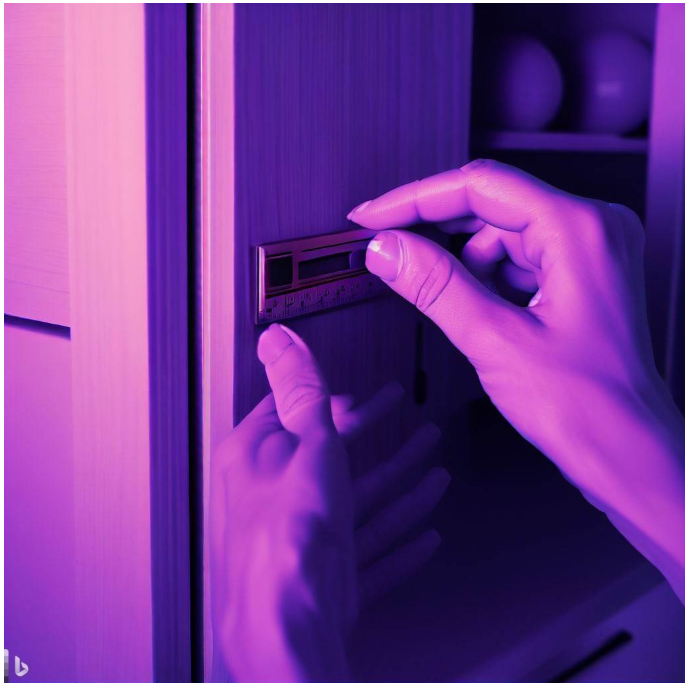


CSE 331

Correctness

Katherine Murphy

Credits: Profs. Kevin Zatloukal and James Wilcox

Shipping Software

- Building shippable version is ~10x harder than demo
  - demo version needs to work when *used properly*
  - shipped version needs to work properly <u>no matter what</u>
- 1m users will try millions of cases that you didn't
  - needs to work properly on all cases, even ones you didn't try
- Users are completely <u>unforgiving</u> about bugs
  - no partial credit for effort
- How is this achieved in practice?

Standard Techniques for Correctness

Standard practice uses three techniques:

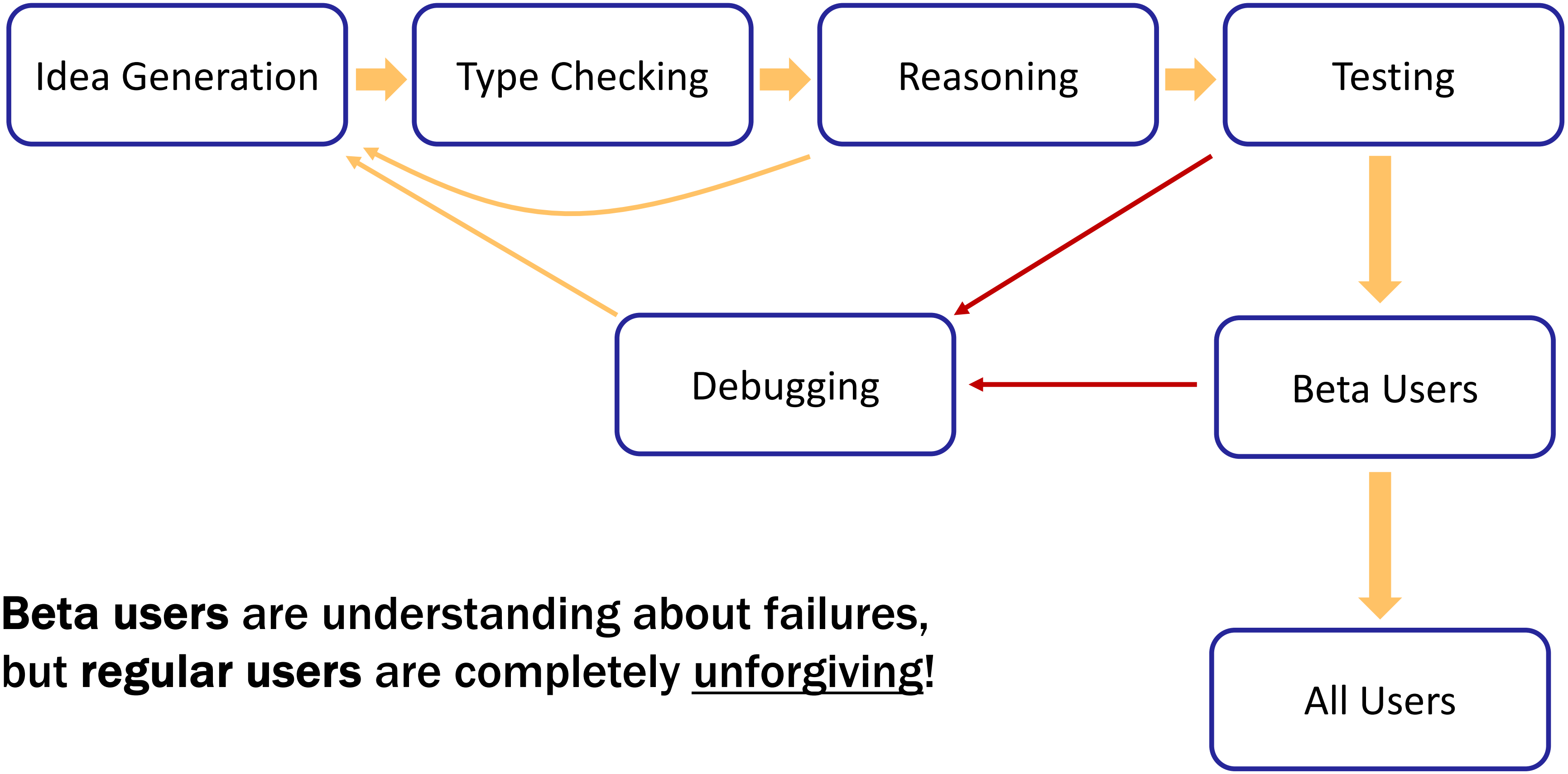
- **Testing**: try it on a well-chosen set of examples
- **Tools**: type checker, libraries, etc.
- **Reasoning**: *think through* your code on all inputs
  - have another person do the same ("code review")

Each removes ~2/3rd bugs but of different kinds

Combination removes >97% of bugs

Review: Software Development Process

Given: a problem description (in English)



Beta users are understanding about failures, but **regular users** are completely <u>unforgiving</u>!

Which Ones and How Much

- The first question to ask yourself:

  How much of each is needed for my program?

- Correctness is easier for some programs vs others

- Personally, I break this into several cases…
  - "levels" of difficulty

Correctness Levels

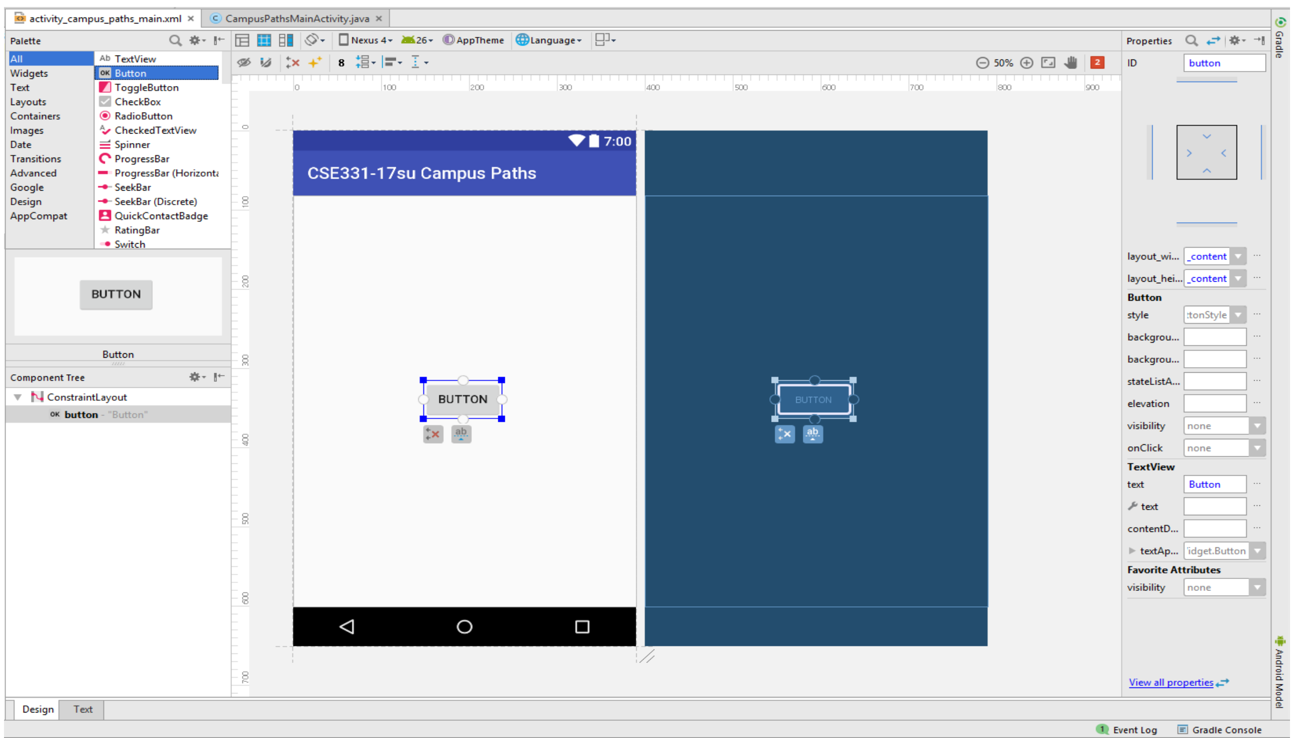
Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	?	?		
2	?	?		
3	?	?		
4	?	?		
5	?	?		

Small Number of Inputs

- Set of possible inputs / configurations is small
  - say 20 or fewer…
- Just check them all!
  - this is the right answer
- This category does not require a programmer
  - anyone can check the answer
  - programming is hard, so skip it when you can

Small Number of Inputs

- Coding is the wrong tool for this job
  - can happen in part of a larger application
- iPhone development lets you draw the UI:



Small Number of Inputs

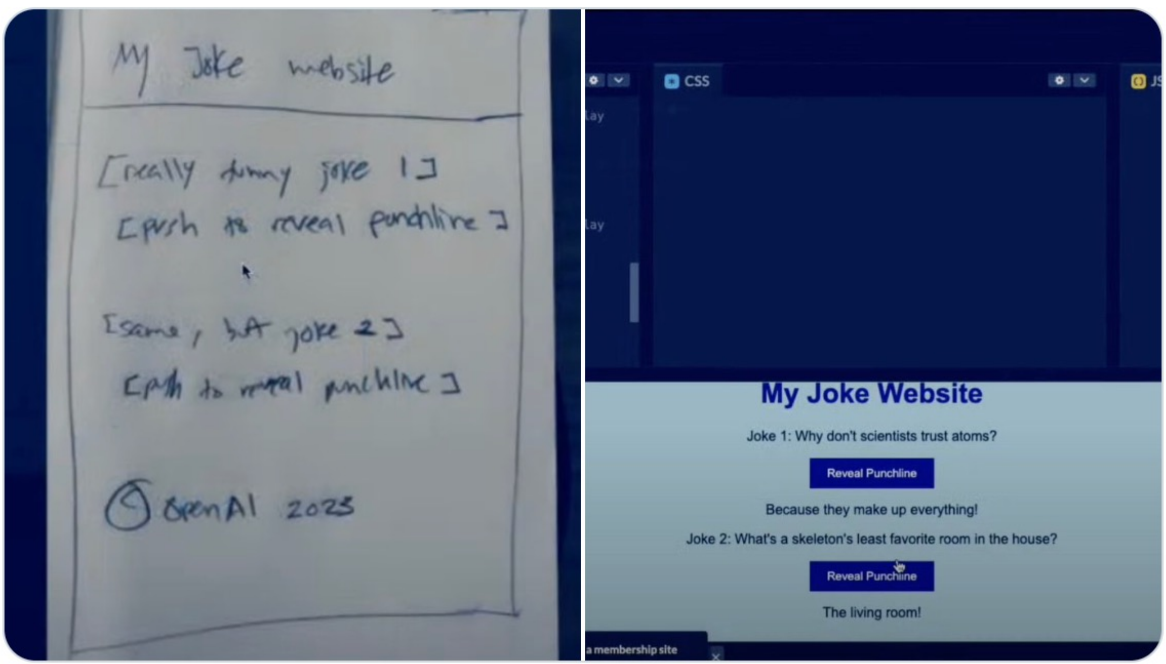
Mckay Wrigley
@mckaywrigley

Greg Brockman (@gdb) of OpenAI just demoed GPT-4 creating a working website from an image of a sketch from his notebook.

It's the coolest thing I've *ever* seen in tech.

If you extrapolate from that demo, the possibilities are endless.

A glimpse into the future of computing.



a non-programmer can check the answer

Small Number of Inputs

- Can happen as part of a larger application
  - would then require an understanding of the context
  - still a case where AI could be used (checked by a programmer)
- Happens more often than you might think
  - individual function can have a small number of inputs
    - e.g., two boolean inputs (only 4 configurations)
  - quite common with UI
    - e.g., when I click the button, it should say “hi”
- Be on the lookout for these cases
  - save yourself work by spotting them

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	?	?		
2	?	?		
3	?	?		
4	?	?		
5	?	?		

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	straight from spec	heuristics		
2	no mutation	“		
3	local variable mutation	“		
4	array mutation	“		
5	heap state mutation	“		

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	straight from spec	heuristics	type checking	
2	no mutation	“	libraries	
3	local variable mutation	“	“	
4	array mutation	“	“	
5	heap state mutation	“	“	

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	straight from spec	heuristics	type checking	code reviews
2	no mutation	“	libraries	calculation induction
3	local variable mutation	“	“	Floyd logic
4	array mutation	“	“	for-any facts
5	heap state mutation	“	“	rep invariants

Reminders

- We will set an **extremely high bar** for correctness
  - “programmers overestimate the importance of **efficiency** and underestimate the difficulty of **correctness**.”
- Now is the time to practice proper technique
  - much harder to learn technique on harder problems
- This topic: Testing and Tools
  - you have important responsibilities here
- Next topic: Reasoning (pt. 1)
  - the most important part
    everything else can (and will) be automated
  - see the other two class slogans!

Specifications

Specifications

- Correctness requires a description of the correct answer
  - true at any level of correctness

- Description must be <u>precise</u>
  - can't have disagreement about what is correct

- Informal descriptions (English) are usually imprecise
  - necessary to "formalize" the English
    - turn the English into a precise *mathematical* definition
  - professionals are *extremely* good at this
    - usually just give English definitions
    - important skill to practice
  - we will start out completely formal to make it easier

Kinds of Specifications

- **Imperative** specification says <u>how</u> to calculate the answer
  - lays out the exact steps to perform to get the answer
- **Declarative** specification says <u>what</u> the answer looks like
  - does not say how to calculate it
  - future: prove our calculation meets the spec
- Can implement a *different* imperative specification
  - future: prove ours is equivalent to the original specification

Example: Imperative Specification

- Absolute value $|x| = x$ if $x \geq 0$ and $-x$ otherwise
  - definition is an "if" statement

```
const abs = (x: bigint): bigint => {
  if (x >= 0n) {
    return x;
  } else {
    return -x;
  }
}
```

just translating math to TypeScript

straight from the spec

Example: Declarative Specification

- Square root of x is number y such that $y^2 = x$
  - not all positive integers have integer square roots, so… let's round up
  - $(y - 1)^2 \leq x \leq y^2$

    smallest integer y such that $x \leq y^2$

```
const sqrt = (x: bigint): bigint => {

    ??

}
```

we are left to figure out how to do this…

not straight from the spec

Example: Declarative Specification

- **Absolute value $|x|$ is an integer y such that**
  - $y \geq x$
  - $y \geq -x$
  - $y = x$ **or** $y = -x$

```
const abs = (x: bigint): bigint => {
  if (x >= 0) {
    return x;
  } else {
    return -x;
  }
}
```

<u>not</u> straight from the spec

requires some <u>thinking</u> to make sure this code returns a number with the properties above

correctness is inherently harder with a declarative spec

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	straight from spec	?	?	?
2				
3				
4				
5				

Straight From the Spec

- Instructions say exactly how to calculate answer
  - given an imperative specification
  - we are just translating math into code
- Still easy to make mistakes!
  - too many inputs to test them all
  - need to additional ways of checking for bugs
- Still important to get it right!

Non-programming Example

- Important to calculate grades correctly!

fx =0.6*G4+0.15*I4+0.25*J4

Homework	Extra Credit	Midterm	Final	Combined
87.5%	1	64.0%	91.6%	0.25*J2
91.4%	1	87.9%	70.8%	85.8%
86.2%	5	93.0%	62.0%	81.8%
96.5%	1	60.9%	69.0%	84.4%
98.2%	0	88.6%	91.3%	95.0%
86.3%	0	91.5%	63.0%	81.3%

- The syllabus says the formula
  - ask someone else to double-check (“code review”)
  - spot check some of them

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	straight from spec	heuristics	type checking	code reviews
2				
3				
4				
5				

Correctness when Straight From the Spec

Correctness requires these elements:

- **Code review**
  - **second set of eyes**
- **Type checker**
  - **third set of eyes (so to speak)**

    (tends to find *different* mistakes than human reviewers)
- **Good set of tests**
  - **can’t test every case… need to pick the the right ones**

    (more on this next lecture…)

Type Checkers

- The main part of "Tools" is the type checker
  - libraries are the other important part
- Type Checkers are very useful for finding bugs
  - another set of "eyes" helping us find them
  - you have probably learned this already

Type Checkers

- **TypeScript and Java have different type systems**
  - **they can catch different bugs for us**
    - TypeScript ensures references are not null (Java does not)
    - more examples coming soon…

- **Critical to understand what the tools will miss**
  - **can ignore issues the tools would catch**
  - **must carefully think about issues the tools would miss**

Important Java Examples

- **In Java, your responsibility to catch e.g.**

not all of these apply in TypeScript!

experienced programmers are always on the look out for these

1. **Using "`==`" between strings**

   Fine in principle, but not what you want

   Especially bad bug because it is likely to work in tests!

2. **Wrong order of arguments**

   Famous example: `memset(int[] A, int n, int v)`

   You call `memset(A, 0, 100)` instead of `memset(A, 100, 0)`

3. **Defining "equal(Object o)" instead of "equals"**

   Perfectly fine, but doesn't override equals method

   (`@Override` annotations added to Java to catch this!)

John Carmack @ID_AA_Carmack 2h

I spent *hours* today debugging something that turned out to be a single wrong letter in the code: a .ge() should have been .gt().

How-To For Straight From the Spec

- We will start with completely formal specs

- Translate math into our programming language
  - TypeScript here, but could also be Java

- Rest of this lecture:
  - define math for data and code
  - describe how to translate those into TypeScript

    try to make the translations as *straightforward* as possible (fewer mistakes)
  - mention new TypeScript features when related

Math Notation

Basic Data Types in Math

- In math, the basic data types are “sets”
  - sets are collections of objects called **elements**
  - write $x \in S$ to say that “x” is an element of set “S”, and $x \notin S$ to say that it is not.

- Examples:

$x \in \mathbb{Z}$	x is an integer
$x \in \mathbb{N}$	x is a non-negative integer (natural)
$x \in \mathbb{R}$	x is a real number
$x \in \mathbb{B}$	x is T or F (boolean)
$x \in \mathbb{S}$	x is a character
$x \in \mathbb{S}^*$	x is a string

non-standard names

Basic Data Types in TypeScript

Condition	Math	TypeScript	Up to Us
integer	$x \in \mathbb{Z}$	`bigint`	
natural	$x \in \mathbb{N}$	`bigint`	non-negative
real	$x \in \mathbb{R}$	`number`	
boolean	$x \in \mathbb{B}$	`boolean`	
character	$x \in \mathbb{S}$	`string`	length 1
string	$x \in \mathbb{S}^*$	`string`	

we will often write

$x : \mathbb{Z}$ instead of $x \in \mathbb{Z}$

– *only* subtraction on non-negative can produce negative

Ways to Create New Types In Math

- **Union Types** $\mathbb{S}^* \cup \mathbb{N}$
  - contains every object in either (or both) of those sets
  - e.g., all strings and natural numbers

- If $x \in \mathbb{N} \cup \mathbb{S}^*$, then x could be a natural or string

- Two sets can contain common elements
  - in this case, the sets are disjoint

Ways to Create New Types in TypeScript

- **Union Types** `string | bigint`
  - can be either one of these

- How do we work with this code?

```
const x: string | bigint = …;

// can I call isPrime(x)?
```

- We can check the type of `x` using "`typeof`"
  - TypeScript understands these expressions
  - will "narrow" the type of `x` to reflect that information

Type Narrowing With "If" Statements

- **Union Types** `string | bigint`
  - can be either one of these

- **How do we work with this code?**

```
const x: string | bigint = …;

if (typeof x === "bigint") {
  console.log(isPrime(x))    // okay! x is a bigint
} else {
  …                          // x is a string
}
```

Type Narrowing vs Casting

```
const x: string | bigint = …;

if (typeof x === "bigint") {
  console.log(isPrime(x))    // okay! x is a bigint
} else {
  …                          // x is a string
}
```

- Note that this does not require a **type cast**
  - TypeScript knows x is a `bigint` inside the "if" (narrowing)
- 331: there are **no type casts** (won't even show syntax)
  - unlike Java, TypeScript casts are unchecked at runtime
  - seem designed to create extremely painful debugging

Type Narrowing Gotcha

```
const f = (x: bigint): string | bigint => …;

if (typeof f(x) === "bigint") {
  console.log(isPrime(f(x)))    // is this okay?
}
```

- No! TypeScript will (properly) reject this
  - no guarantee that f(x) returns the same value both times!

Type Narrowing of Function Calls

```
const f = (x: bigint): string | bigint => …;

const y = f(x);
if (typeof y === "bigint") {
  console.log(isPrime(y))        // this works now
}
```

- TypeScript can see that the two values are the same
- Functions that return different values for the same inputs are confusing!
  - maybe better to avoid that

Compound Types In Math

- Compound types combine multiple data types
  - multiple ways build them

- **Record Types** $\{x : \mathbb{N},\ y : \mathbb{N}\}$
  - record with fields “x” and “y” each containing a number
  - e.g., $\{x: 3, y: 5\}$

- Note that $\{x: 3, y: 5\} = \{y: 5, x: 3\}$
  - field names matter, not order
  - (also, “=“ in math means same values)

Record Types in TypeScript

- **Record Types** `{x: bigint, y: bigint}`
  - anything with *at least* fields “x” and “y”

- Retrieve a part by name:

```
const t: {x: bigint, y: bigint} = … ;
console.log(t.x);
```

Optional Fields in TypeScript

- Records can have optional fields

```
type T = {x: bigint, y?: bigint};

const t: T = {x: 1n};
```

  – type of “ `t.y` ” is “ `bigint | undefined` ”

- Functions can have optional arguments

```
const f = (a: bigint, b?: bigint): bigint => {
  console.log(b);
};
```

  – type of “ `b` ” is “ `bigint | undefined` ”

Compound Types In Math

- **Record Types** $\{x : \mathbb{N},\ y : \mathbb{N}\}$
  - record with fields "x" and "y" each containing a number
  - e.g., $\{x{:}\ 3, y{:}\ 5\}$
- Tuple Types $\mathbb{N} \times \mathbb{N}$
  - pair of two natural numbers, e.g., $(5, 7)$
  - can do tuples of 3, 4, or more elements also
- Mostly equivalent alternatives
  - both let us put parts together into a larger object
  - record distinguishes parts by name
  - tuple distinguishes parts by order

Tuple Types in TypeScript

- Tuple Types `[bigint, bigint]`
- Must assign names to the parts to refer to them
- How would we do this in math?

Retrieving Part of a Tuple

- To refer to the parts, we must give them names

- Tuple Types $\mathbb{N} \times \mathbb{N}$

Let $(a, b) := t$.

Suppose we know that $t = (5, 7)$

":=" means a definition

Then, we have $a = 5$ and $b = 7$

- Tuple Types `[bigint, bigint]`

```
const t: [bigint, bigint] = …;
const [a, b] = t;
console.log(a);  // first part of t
```

Readonly Values

- TypeScript can ensure values aren't modified
  - extremely useful! (mutation makes everything harder)

- Tuple types should always be readonly

```
type IntPair = readonly [bigint, bigint];
```

- Individual fields of records should be marked readonly

```
type IntPair = {readonly x: bigint,
                readonly y: bigint};
```

Simple Functions in Math

- Simplest function definitions are single expressions
- Will write them in math like this:

$$\textbf{func}\ \mathrm{double}(n : \mathbb{N}) := 2n$$

$$\textbf{func}\ \mathrm{dist}(p : \{x: \mathbb{R}, y: \mathbb{R}\}) := (p.x^2 + p.y^2)^{1/2}$$

  - any normal math allowed in the expression

Simple Functions in Math

- Can define short-hand for types in math also

  type Point := $\{x: \mathbb{R}, y: \mathbb{R}\}$

  func dist(p : Point) := $(p.x^2 + p.y^2)^{1/2}$

- Can put the argument type on the right instead

  func dist(p) := $(p.x^2 + p.y^2)^{1/2}$ for any p : Point

  – needs to be described somewhere (we're not too picky)
  – will need this in some cases coming shortly…

Complex Functions in Math

- **Most interesting functions are not simple expressions**
  - **need to use different expressions in different cases**

- **Can use side-conditions to split into cases**

$$\begin{aligned} \textbf{func}\ \ & \mathrm{abs}(x : \mathbb{R}) := x && \text{if } x \geq 0 \\ & \mathrm{abs}(x : \mathbb{R}) := -x && \text{if } x < 0 \end{aligned}$$

  - **conditions must be <u>exclusive</u> and <u>exhaustive</u>**

    we do not want to require on *order* to determine which applies

  - **there is a better way to do this in many cases…**

Pattern Matching

- **Can also define functions by "pattern matching"**

$$\textbf{func} \quad \text{double}(0) := 0$$
$$\text{double}(n+1) := \text{double}(n) + 2 \qquad \text{for any } n : \mathbb{N}$$

  - **first case matches only** 0
  - **second case matches numbers** 1 **more than some** $n : \mathbb{N}$ **…**

    double(6) = double(5+1) so it matches with n = 5

    since $n \geq 0$, we have $n+1 \geq 1$, so it matches 1, 2, 3, …
  - **pattern** "$n+2$" **would match** $2, 3, 4, \ldots$

- **Simplifies the math in multiple ways…**

Pattern Matching on Natural Numbers

- **Pattern matching definition**

$$\begin{aligned} &\textbf{func}\quad \mathrm{double}(0) &&:= 0 \\ &\phantom{\textbf{func}}\quad \mathrm{double}(n+1) &&:= \mathrm{double}(n) + 2 && \text{for any } n : \mathbb{N} \end{aligned}$$

  is simpler than using side conditions

$$\begin{aligned} &\textbf{func}\quad \mathrm{double}(n) &&:= 0 && \text{if } n = 0 && \text{for any } n : \mathbb{N} \\ &\phantom{\textbf{func}}\quad \mathrm{double}(n) &&:= \mathrm{double}(n\text{-}1) + 2 && \text{if } n > 0 && \text{for any } n : \mathbb{N} \end{aligned}$$

  – **e.g., need to explain why** $\mathrm{double}(n\text{-}1)$ **is legal**

    easy in this case, but it gets harder

  – **(also makes the reasoning easier, as we will see later…)**

- **We will prefer pattern matching whenever possible**

Pattern Matching on Booleans

- Booleans have only two legal values: T and F

- Can pattern match just by listing the values:

$$\begin{aligned} \textbf{func}\ \ & \text{not}(T) := F \\ & \text{not}(F) := T \end{aligned}$$

  - negates a boolean value
  - no simpler way to define this function!

Pattern Matching on Records

- **Can pattern match on individual fields of a record**

  type Steps := {n : $\mathbb{N}$, fwd : $\mathbb{B}$}

  func change({n: n, fwd: T}) := n   for any n : $\mathbb{N}$
  change({n: n, fwd: F}) := –n   for any n : $\mathbb{N}$

  – **clear that the rules are exclusive and exhaustive**

- **Can match on multiple parameters**
  – **e.g.**, change({n: m+5, fwd: T}) := 2m   for any m : $\mathbb{N}$
  – **just make sure the rules are exclusive and exhaustive**

Pattern Matching in TypeScript

- TypeScript does not provide pattern matching
  - some other languages do! (see 341)

- We must translate into "`if`"s on our own

```
type Steps = {n: number, fwd: boolean};

const change = (s: Steps) => {
  if (s.fwd) {
    return s.n;
  } else {
    return -s.n;
  }
};
```

still straight from the spec
but easy to make mistakes

Pattern Matching in TypeScript

$$\textbf{func}\ \ \text{double}(0) \ := 0$$
$$\text{double}(n+1) := \text{double}(n) + 2 \qquad \text{for any } n : \mathbb{N}$$

- Also need to be careful with natural numbers

```
// m is non-negative
const double = (m: bigint) => {
  if (m === 0n) {
    return 0n;
  } else {
    return double(m - 1n) + 2n;
  }
};
```

straight from the spec

spec says double(m)
but code says double(m – 1)

  – pattern matching uses "n+1" but the code uses "m" (or "n")

    sadly, TypeScript will not let "n+1" be the argument value

Pattern Matching in TypeScript

func double(0) := 0

double(n+1) := double(n) + 2 for any n : $\mathbb{N}$

- **This implementation returns the same thing:**

```
// m is non-negative
const double = (m: bigint) => {
  return 2n * m;
};
```

<u>not</u> straight from the spec

  - **but that's not what the spec says!**

    spec is imperative but this is a different implementation

  - **requires reasoning tools to check that this is correct**

    (will see in post-HW Levels)

Correctness Levels

Level	Description	Testing	Tools	Reasoning
0	small # of inputs	exhaustive		
1	straight from spec	heuristics	type checking	code reviews
2				
3				
4				
5				

Testing

What Can We Learn From Testing?

“Program testing can be used to show the presence of bugs, but never to show their absence!”

Edsgar Dijkstra

Notes on Structured Programming, 1970

“Beware of bugs in the above code;
I have only proved it correct, not tried it.”

Donald Knuth, 1977

Unit vs Integration Tests

- A unit test checks one component
  - ideally, without testing anything else (not always possible)
- You will be expected to write unit tests in industry
- There are also integration tests and end-to-end tests
  - someone will write them, but maybe not you
- We will focus on unit testing in this course

“Manual” vs Programmatic Tests

- Usually possible to run the code by hand (“manually”)
  - open it in node and execute it
  - open it in the browser and look at it (UI)

- No downside… unless the code changes
  - then, you need to do the tests again

- For some code (UI especially), manual is still easier
  - if written tests are 3x as hard to create,
    then you’re better off unless you change it 3+ times
  - for UI, written tests aren’t perfect anyway

    need to see it in the browser to be sure that it looks right

Writing a Test

1. Choose an input / configuration
   – description of the inputs / configuration is the “test case”

2. **Think** through what the answer should be
   – if you run the code to get the answer, you are **not testing**

3. Write code that
   – calls the function that input
   – compares the actual answer to the expected one
   – useful libraries that do this
      we will use “mocha” in JS / TS

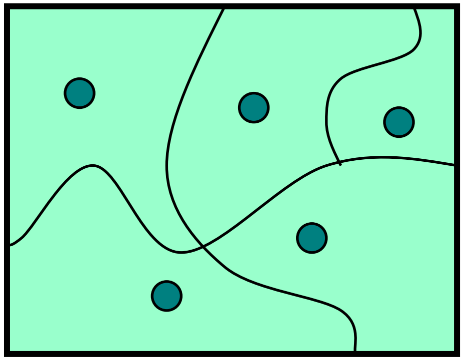
Key Idea

- Key question is what cases to test
  - small # inputs: we can test all of them
  - otherwise, we cannot

- Set of inputs can be infinite, but the code is finite
  - key idea: use knowledge of the code structure to test it

Key Idea

- Key question is what cases to test
  - small # inputs: we can test all of them
  - otherwise, we cannot



- Split the allowed inputs into **subdomains**
  - for inputs in one subdomain, code "does the same thing"

- <u>Hope</u>: code is entirely right or wrong for subdomain
  - one example in the subdomain will tell us if there is a bug
  - (note: this is not *always* true… see sec and HW Levels)

- <u>Plan</u>: Look at the code. See when it "does the same thing"

Need to Look At the Code

```
// Returns true iff n is a prime number
const isPrime = (n: bigint): boolean => { … }
```

- How about if we test 2, 3, 4, 7, 12, 97, 99?
  - seems okay?

Need to Look At the Code

```
// Returns true iff n is a prime number
const isPrime = (n: bigint): boolean => {
  if (n < 100n) {
    return PRIME_CACHE[n];  // precomputed answers
  } else {
    let k = 2n;
    while (k*k <= n) {
      if (n % k === 0n)
        return false;
      k = k + 1n;
    }
    return true;
  }
};
```

Need to Look At the Code

```
// Returns true iff n is a prime number
const isPrime = (n: bigint): boolean => {
  if (n < 100n) {
    return PRIME_CACHE[n];
  } else {
    …
  }
}
```

- Cases 2 … 100 are just table lookups!

Primary Heuristic: Clear-Box Testing

- We need to look at the code to know what to test
  - this will be our **primary** heuristic

- In this class, I want a clear rule for how many tests
  - want homework and tests to have clear right/wrong answers

- Outside of class, these tests are also good
  - but other programmers may not use the same rules

Testing Straight-Line Code

Straight-line Code looks like

```
return 2 * (n - 1n) + 1n;
```

Or, more generally, like this

```
const m = n - 1n;
return 2n * m + 1n;
```

- Any number of constant values allowed
  - often makes the code easier to read, but no different
- Inputs where it executes the same straight-line code are "***doing the same thing***"

Testing Straight-Line Code

<u>Rule</u>: same straight-line code is one subdomain

Straight-line Code looks like

```
return 2n * (n - 1n) + 1n;
```

Or, more generally, like this

```
const m = n - 1n;
return 2n * m + 1n;
```

Testing Subdomains

Rule: at least two test cases per subdomain

(assuming subdomain contains at least two inputs)

- Real bugs will be missed if only one test case
  - common bug: copy-and-paste a stub and forget to change it!
  - will happen to you eventually if you don't test two cases
- Still doesn't guarantee the code is right! (see HW Levels)
- More is obviously also okay
  - not a contest to write the fewest tests

Testing Function Calls

In general, function calls are still straight-line code

```
const m = n - 1n;
return Math.sin(2n * m + 1n);
```

- **All inputs are still are “the same”**
  - **two cases is still enough**
  - **not your job to test the other function!**

    test each function on its own assuming the others work

    <u>exception</u>: recursive calls (more later)

Testing Conditionals

Conditionals look like this

```
if (n > 0n) {
  return 2n * (n - 1n) + 1n;
} else {
  return 0n;
}
```

Two branches ("then" and "else")

- in this case, both branches are straight-line code
- note: the ternary operator creates branches

Testing Conditionals

Rule: branches are in separate subdomains

- Would be **negligent** not to test both branches
- If both are straight-line code, then 4 tests
- With if/else if/else, we'd need 6 tests
  - 3 branches x 2 per straight-line block = 6 cases

Other Heuristics

Some other heuristics are also useful

- **<u>Boundary Cases</u>**: if 10 and 11 are separated, then make sure you test 10 and 11
  - easy to have “off by one” bugs
  - happens if you use “< n” instead of “<= n”

    behavior changes between n-1 and n instead

    (see John Carmack’s tweet!)

- Often doesn’t require any more tests
  - can be one of two cases for straight-line code

Testing Conditionals

Conditionals look like this (with `n` an integer)

```
if (n > 0n) {
  return 2n * (n - 1n) + 1n;
} else {
  return 0n;
}
```

- Boundary cases are 0 and 1
  - cases for “then” block could be 1 and 10 (say)
  - cases for “else” block could be 0 and -1 (say)

Testing Subdomains

Another rule for subdomains with *obvious* boundaries

<u>Rule</u>: test each boundary case and
at least one non-boundary case

- If there are no boundaries, test two non-boundary
- If there is one boundary, then test it and one non-boundary
- If there are two boundaries, then test both and one non-boundary
  - e.g., if branch is executed for x between 3 and 10
  - 3 tests are now necessary (e.g., 3, 6, and 10)

Testing Recursion

Recursive calls are more complicated

```
const f = (n: bigint): bigint => {
  if (n >= 2n) {
    return 2n * f(n / 2n) + 1n;
  } else {
    return 0n;
  }
}
```

- Heuristics thus far would allow 0, 1, 2, 3
  - only tests 0 or 1 recursive calls
  - not enough! (see Section Levels)

Testing Recursion

Clear-box Testing for recursive calls:

Rule: inputs that cause 0, 1, and 2+ recursive calls are in separate subdomains

- Call this the "0–1–many" heuristic
- Split into 3 subdomains, then apply other rules
  - if subdomains run the same straight-line code, then 6 tests
  - if "0 recursive calls" has two branches, then 8 tests
  - if a subdomain has only one input, then just one test

    e.g., "0" is in its own subdomain, that's just one test

Testing Recursion

Clear-box Testing for recursive calls:

Rule: inputs that cause 0, 1, and 2+ recursive calls are in separate subdomains

- Call this the "0–1–many" heuristic
- Note the savings of using a **library** (part of "tools")
  - still one subdomain if you call someone else's function
  - but usually, 5-6 test cases at *minimum* if you call your own

Summary of Heuristics

- Split into subdomains where code is different
  - branches of conditionals
  - 0, 1, many recursive calls

- At least two tests per subdomain

  (unless subdomain is only 1 input)
  - include all boundaries and a non-boundary

- Not a contest to write the fewest tests!

Summary of Heuristics

- Continue splitting until no more splits needed
  - e.g., two inputs that both make 0 recursive calls BUT are in separate branches… are in separate subdomains
- For "2+ recursive calls", look at first two calls
  - different paths are split into separate subdomains
  - e.g., same branch on first call but different on second
- Complete summary in the notes on website

What Else?

- We only have rules for:
  - straight-line code
  - conditionals (“if” statements)
  - recursion
- What about everything else?
- Without mutation, this is all we need
  - loops require mutation

Typical Numbers of Tests

- Typical function should have ~10-30 tests
- Should be **embarrassed** if you have <10 tests
  - better have a good explanation in terms of subdomains
- More than 30 starts to feel too time consuming
  - not bad and may be necessary
  - but we will adjust our rules to try to avoid this

    see the later slide…

Example 1

```
// n must be a non-negative integer
const f = (n: bigint): number => {
  if (n === 0n) {
    return 0;
  } else {
    return Math.sin(Math.PI*(Number(n) + 0.5));
  }
}
```

How many tests? Which ones?

– 0 (top branch) and 1, 5 (bottom branch)

Example 2

```
// n must be a non-negative integer
const f = (n: bigint): bigint => {
  if (n < 3n) {
    return 0n;
  } else if (n < 10n) {
    return (n - 3n) / 10n;
  } else {
    return 1n;
  }
}
```

How many tests? Which ones?

- 0, 1, 2 (top) and 3, 6, 9 (middle) and 10, 12 (bottom)
- note that 0 is also a boundary case

Example 3

```
// n must be a positive integer
const f = (n: bigint): number => {
    if (n === 1n) {
      return 0;
    } else {
      return 1 + f(1n + (n - 2n) / 2n);
    }
  }
```

How many tests? Which ones?

- 1 (0 recursive calls)
- 2, 3 (1 recursive call)
- 4, 10 (many recursive calls)

Example 4

```
// n must be an integer between 1 and 10
const f = (n: bigint): bigint => {
   if (n === 1n) {
     return 0n;
   } else {
     return 1n + 2n * f(n - 1n);
   }
 }
```

How many tests? Which ones?

– small # of inputs, so... all of them

Example 5 (Section Levels Q4c)

```
const f = (n: bigint): bigint => {
    if (n === 0n) {
      return 0n;
    } else if (n === 1n) {
      return 1n;
    } else if (n % 2 === 1n) {  // n is > 1 and odd
      return f(n - 2n) + 1n;
    } else {                    // n is > 1 and even
      return f(n - 2n) + 3n;
    }
}
```

How many tests? Which ones?

Example 5 (Section Levels Q4c)

```
const f = (n: bigint): bigint => {
    if (n === 0n) {
      return 0n;
    } else if (n === 1n) {
      return 1n;
    } else if (n % 2 === 1n) {   // n is > 1 and odd
      return f(n - 2n) + 1n;
    } else {                     // n is > 1 and even
      return f(n - 2n) + 3n;
    }
}
```

0 recursive calls:
Look for base case

How many tests? Which ones?

- 0, 1 (0 recursive calls)

Example 5 (Section Levels Q4c)

```
const f = (n: bigint): bigint => {
    if (n === 0n) {
      return 0n;
    } else if (n === 1n) {
      return 1n;
    } else if (n % 2 === 1n) {   // n is > 1 and odd
      return f(n - 2n) + 1n;
    } else {                     // n is > 1 and even
      return f(n - 2n) + 3n;
    }
}
```

← enters (n === 1n) base case (for the odd branch)

← enters (n === 0n) base case (for the even branch)

How many tests? Which ones?

- 0, 1 (0 recursive calls)
- 2, 3 (1 recursive call)

1 recursive call:
For each recursive branch, determine which base case(s) it can enter (this is a subdomain)

Example 5 (Section Levels Q4c)

```
const f = (n: bigint): bigint => {
    if (n === 0n) {
      return 0n;
    } else if (n === 1n) {
      return 1n;
    } else if (n % 2 === 1n) {   // n is > 1 and odd
      return f(n - 2n) + 1n;
    } else {                     // n is > 1 and even
      return f(n - 2n) + 3n;
    }
}
```

← enters odd recursive case

← enters even recursive case

How many tests? Which ones?

- 0, 1 (0 recursive calls)
- 2, 3 (1 recursive call)
- 4, 5, 10, 11 (many recursive calls)

many recursive calls:
For each recursive branch, determine which recursive branch(es) it can enter (this is a subdomain)

Escape Hatches

- If the previous rules give >20 tests…

  1. Look only at the outermost recursive call
     (rather than the outmost *two* calls)

- If there are still >20 tests…

  2. Pick only one example per subdomain (shudders)

- If there are still >20 tests…

  3. Maybe rewrite your function to be simpler?

Other Heuristics

Not required for 331 but useful in practice:

- Make sure every argument value is changed
- Look at special values
  - null, undefined, NaN, empty array, etc. often have bugs
- Look at the specification for branches
  - maybe the code doesn't split inputs where it should!
  - e.g., spec splits into "if $x \geq 0$" but code is "`if (x > 0)`"