
CSE 331

Software Design & Implementation

Topic: Introduction

 **Discussion:** What are you excited for this summer?

Reminders

- Read the welcome email
- Check your access to Ed, Gradescope, and Canvas
- Should see email about Gitlab repositories soon

Upcoming Deadlines

- Syllabus Quiz due Thursday (6/22)
- HW1 due Thursday (6/22)

Last Time...

- Welcome email
- Syllabus Overview

Today's Agenda

- Upcoming Assignments
- Motivation
- Reasoning

Upcoming Assignments

Syllabus Quiz

- Due on Thursday night
 - read the syllabus in depth
 - answer a few multiple choice/select questions
 - infinite attempts before deadline
- Why?
 - had a lot of confusion in past quarters
 - make student requests manageable for course staff

HW1

- Due on Thursday night
 - practice interview question
 - **write** an algorithm to rearrange array elements as described
 - **argue** in concise, convincing English that it is correct
 - don't just explain *what the code does!*
 - **do not run** your code! (pretend it's on a whiteboard)
 - know that is correct *without* running it (a necessary skill)
- This is expected to be difficult (esp. the "argue" part)
 - graded on effort, not correctness
 - do not spend more than 90 minutes on it
 - want you to see that it is tricky... *without the tools coming next*

Motivation

What are the goals of CSE 331?

Learn the skills to be able to contribute to a modern software project

- move from CSE 143 problems toward what you'll see in industry and in upper-level courses

Specifically, how to write code of

- higher **quality**
- increased **complexity**

We will discuss *tools* and *techniques* to help with this and the *concepts* and *ideas* behind them

- there are *timeless principles* to both
- widely used across the industry

What is high quality?

Code is high quality when it is

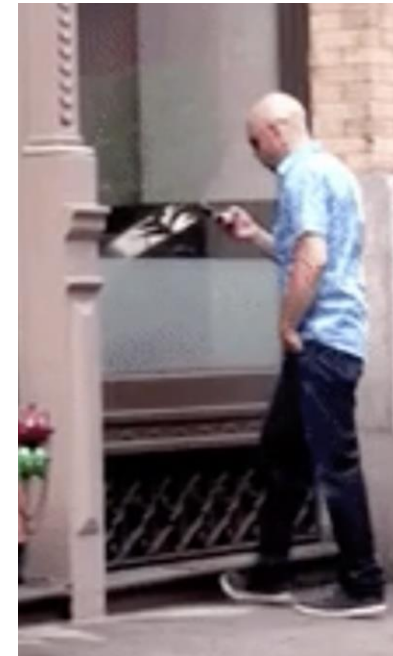
1. **Correct**
Everything else is of secondary importance
2. Easy to **change**
Most work is making changes to existing systems
3. Easy to **understand**
Needed for 1 & 2 above

How do we ensure correctness...

... when **people** are involved?

People have been known to

- walk into windows
- drive away with a coffee cup on the roof
- drive away still tied to gas pump
- lecture wearing one brown shoe and one black shoe



Key Insight

1. Can't stop people from making mistakes

How do we ensure correctness?

Best practice: use three techniques (we'll study each)

1. **Tools**

- type checkers, test runners, etc.

2. **Inspection**

- think through your code carefully
- have another person review your code

technical interviews focus on this
(a.k.a. "reasoning")

3. **Testing**

- usually >50% of the work in building software

Together can catch >97% of bugs.

Scale makes everything harder

Many studies showing scale makes quality harder to achieve

- Time to write N-line program grows faster than linear
 - Good estimate is $O(N^{1.05})$ [Boehm, '81]
- Bugs grow like $\Theta(N \log N)$ [Jones, '12]
 - 10% of errors are between modules [Seaman, '08]
- Communication costs dominate schedules [Brooks, '75]
- Small probability cases become high probability cases
 - Corner cases are more important with more users

Corollary: quality must be even higher, per line, in order to achieve overall quality in a *large* program

How do we cope with scale?

We tackle increased software scale with **modularity**

- Split code into pieces that can be built independently
- Each must be documented so others can use it
- Also helps understandability and changeability

What are the goals of CSE 331?

In summary, we want our to support code of:

Higher Quality:

- Correct
- Easy to change
- Easy to understand

Increased Complexity:

- Modular

Reasoning

Our Approach

- We will learn a set of **formal tools** for proving correctness
 - math can seem daunting – it will connect back!
 - later, this will also allow us to generate the code
- Most professionals can do reasoning like this in their head
 - most do an *informal* version of what we will see
 - with practice, it will be the same for you
- Formal version has key advantages
 - teachable
 - mechanical (no intuition or creativity required)
 - necessary for hard problems
 - we turn to formal tools when problems get too hard

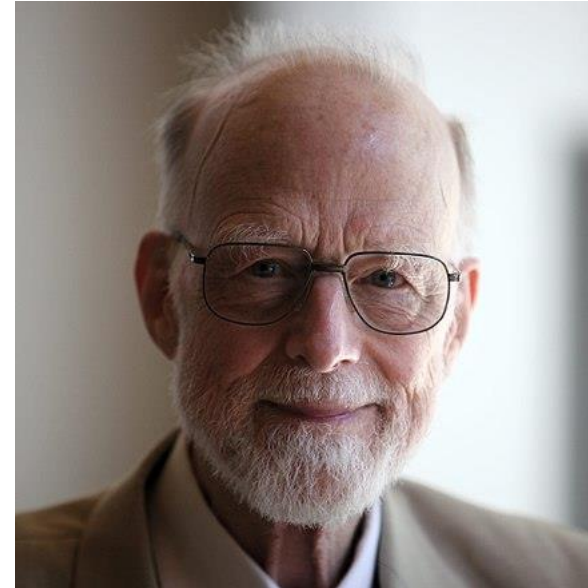
Formal Reasoning

- Invented by Robert Floyd and Sir Anthony Hoare
 - Floyd won the Turing award in 1978
 - Hoare won the Turing award in 1980



Robert Floyd

picture from [Wikipedia](#)



Tony Hoare

Terminology of Floyd Logic

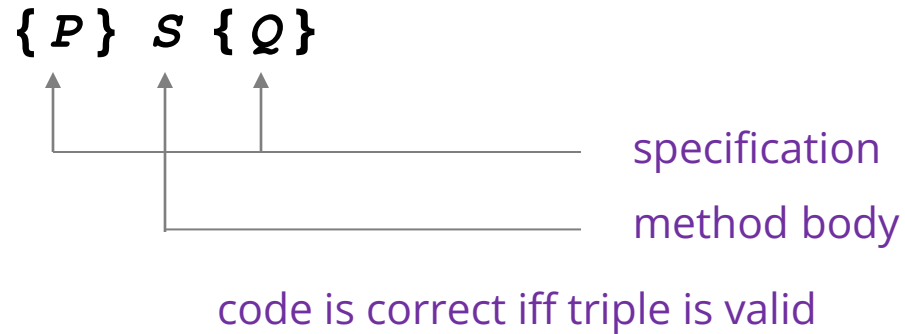
- The *program state* is the values of all the (relevant) variables
- An *assertion* is a true / false claim (proposition) about the state at a given point during execution (e.g., on line 39)
- An assertion *holds* for a program state if the claim is true when the variables have those values

- An assertion before the code is a *precondition*
 - these represent assumptions about when that code is used
- An assertion after the code is a *postcondition*
 - these represent what we want the code to accomplish

Hoare Triples

- A **Hoare triple** is two assertions and one piece of code:

- P the precondition
- S the code
- Q the postcondition



- A Hoare triple $\{P\} S \{Q\}$ is called **valid** if:
 - in any state where P holds, executing S produces a state where Q holds
 - i.e., if P is true before S , then Q must be true after it
 - otherwise, the triple is called **invalid**

Notation

- Floyd logic writes assertions in $\{..\}$
 - since Java code also has $\{..\}$, we will use $\{\{...\}\}$
 - e.g., $\{\{ \mathbf{w} \geq 1 \}\} \mathbf{x} = 2 * \mathbf{w}; \{\{ \mathbf{x} \geq 2 \}\}$
- Assertions are math, not Java
 - you should use the usual math notation
 - (e.g., $=$ instead of $==$ for equals)
 - purpose is communication with humans (not computers)
 - we will need **and**, **or**, **not** as well
 - can also write use \wedge (and) \vee (or) etc.
- The Java language also has assertions (**assert** statements)
 - throws an exception if the condition does not evaluate true
 - we will discuss these more later in the course

Example 1

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{x \neq 0\}\} y = x * x; \{\{y > 0\}\}$

Example 1

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{x \neq 0\}\} y = x*x; \{\{y > 0\}\}$

Valid

- y could only be zero if x were zero (which it isn't)

Example 2

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{z \neq 1\}\} y = z * z; \{\{y \neq z\}\}$

Example 2

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{z \neq 1\}\} y = z * z; \{\{y \neq z\}\}$

Invalid

- counterexample: $z = 0$

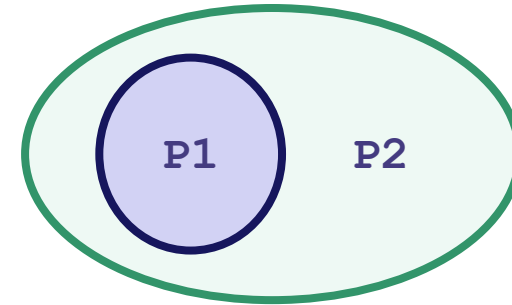
Checking Validity

- So far:
 - code is correct iff Hoare triple valid
 - decided if a Hoare triple is valid by ... **hard** thinking
- Soon: mechanical process for reasoning about
 - assignment statements
 - [next section] conditionals
 - [next lecture] loops
 - (all code can be understood in terms of those 3 elements)
- Next: terminology for comparing different assertions
 - useful, e.g., to compare possible preconditions

Weaker vs. Stronger Assertions

If P1 implies P2 (written $P1 \Rightarrow P2$), then:

- P1 is **stronger** than P2
- P2 is **weaker** than P1



Whenever P1 holds, P2 also holds

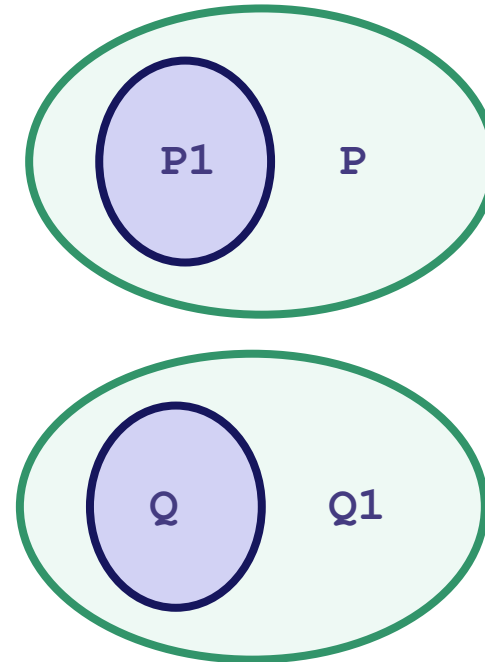
- So it is more (or at least as) “difficult” to satisfy P1
 - the program states where P1 holds are a subset of the program states where P2 holds
- So P1 puts more constraints on program states
- So it is a stronger set of requirements on the program state
 - P1 gives you more information about the state than P2

Examples

- $x = 17$ is stronger than $x > 0$
- x is prime is neither stronger nor weaker than x is odd
- x is prime and $x > 2$ is stronger than x is odd

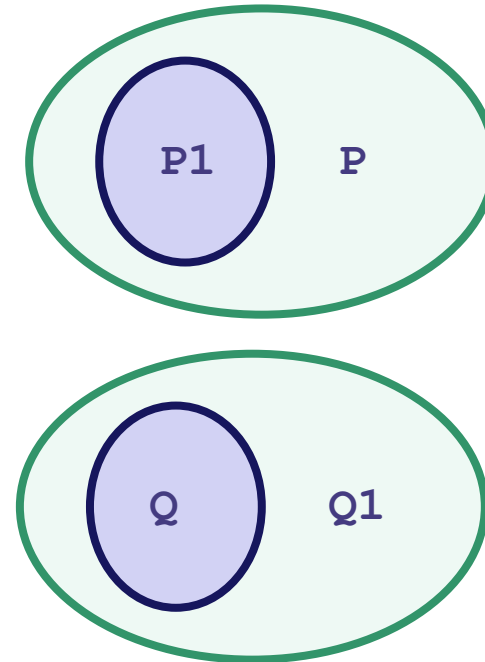
Floyd Logic Facts

- Suppose $\{P\} S \{Q\}$ is valid.
- If $P1$ is stronger than P , then $\{P1\} S \{Q\}$ is valid.
- If $Q1$ is weaker than Q , then $\{P\} S \{Q1\}$ is valid.
- Example:
 - Suppose P is $x \geq 0$ and $P1$ is $x > 0$
 - Suppose Q is $y > 0$ and $Q1$ is $y \geq 0$
 - Since $\{\{x \geq 0\}\} y = x+1 \{\{y > 0\}\}$ is valid, $\{\{x > 0\}\} y = x+1 \{\{y \geq 0\}\}$ is also valid



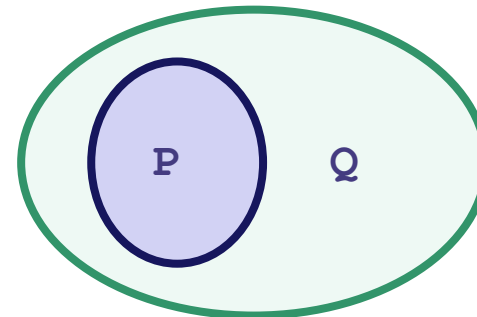
Floyd Logic Facts

- Suppose $\{P\} S \{Q\}$ is valid.
- If $P1$ is stronger than P , then $\{P1\} S \{Q\}$ is valid.
- If $Q1$ is weaker than Q , then $\{P\} S \{Q1\}$ is valid.
- **Key points:**
 - always okay to **strengthen** a **precondition**
 - always okay to **weaken** a **postcondition**



Floyd Logic Facts

- When is $\{P\} ; \{Q\}$ is valid?
 - with no code in between
- Valid if any state satisfying P also satisfies Q
- I.e., if P is **stronger** than Q



Forward & Backward Reasoning

Forward Reasoning

- Start with the **given** precondition
- Fill in the **strongest** postcondition

- For an assignment, $\mathbf{x} = \mathbf{y}$...
 - add the fact “ $x = y$ ” to what is known
 - important subtleties here... (more on those later)

- Later: if statements and loops...

Example of Forward Reasoning

Work forward from the precondition

{{ w > 0 }}

x = 17;

{{ _____ }}

y = 42;

{{ _____ }}

z = w + x + y;

{{ _____ }}

Example of Forward Reasoning

Work forward from the precondition

$\{\{ w > 0 \}\}$

$\mathbf{x = 17;}$

$\{\{ w > 0 \text{ and } x = 17 \}\}$

$\mathbf{y = 42;}$

$\{\{ \text{_____} \}\}$

$\mathbf{z = w + x + y;}$

$\{\{ \text{_____} \}\}$

Example of Forward Reasoning

Work forward from the precondition

$\{ \{ w > 0 \} \}$

$x = 17;$

$\{ \{ w > 0 \text{ and } x = 17 \} \}$

$y = 42;$

$\{ \{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \} \}$

$z = w + x + y;$

$\{ \{ \text{_____} \} \}$

Example of Forward Reasoning

Work forward from the precondition

$\{ \{ w > 0 \} \}$

$\mathbf{x = 17;}$

$\{ \{ w > 0 \text{ and } x = 17 \} \}$

$\mathbf{y = 42;}$

$\{ \{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \} \}$

$\mathbf{z = w + x + y;}$

$\{ \{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \text{ and } z = w + x + y \} \}$



Example of Forward Reasoning

Work forward from the precondition

$\{\{ w > 0 \}\}$

$\mathbf{x = 17;}$

$\{\{ w > 0 \text{ and } x = 17 \}\}$

$\mathbf{y = 42;}$

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \}\}$

$\mathbf{z = w + x + y;}$

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \text{ and } z = w + 59 \}\}$

Forward Reasoning

- Start with the **given** precondition
- Fill in the **strongest** postcondition
- For an assignment, $\mathbf{x} = \mathbf{y}$...
 - add the fact “ $x = y$ ” to what is known
 - important subtleties here... (more on those later)
- Later: if statements and loops...

Backward Reasoning

- Start with the **required** postcondition
- Fill in the **weakest** precondition
- For an assignment, $\mathbf{x = y}$:
 - just replace “x” with “y” in the postcondition
 - if the condition using “y” holds beforehand, then the condition with “x” will afterward since $x = y$ then
- Later: if statements and loops...

Example of Backward Reasoning

Work backward from the desired postcondition

`{{ _____ }}`

`x = 17;`

`{{ _____ }}`

`y = 42;`

`{{ _____ }}`

`z = w + x + y;`

`{{ z < 0 }}`

Example of Backward Reasoning

Work backward from the desired postcondition

`{{ _____ }}`

`x = 17;`

`{{ _____ }}`

`y = 42;`

`{{ w + x + y < 0 }}`

`z = w + x + y;`

`{{ z < 0 }}`



Example of Backward Reasoning

Work backward from the desired postcondition

$\{ \underline{\hspace{10em}} \}$

$\mathbf{x = 17;}$

$\{ \{ w + x + 42 < 0 \} \}$

$\mathbf{y = 42;}$

$\{ \{ w + x + y < 0 \} \}$

$\mathbf{z = w + x + y;}$

$\{ \{ z < 0 \} \}$

Example of Backward Reasoning

Work backward from the desired postcondition

$\{\{ w + 17 + 42 < 0 \}\}$

$\mathbf{x = 17;}$

$\{\{ w + x + 42 < 0 \}\}$

$\mathbf{y = 42;}$

$\{\{ w + x + y < 0 \}\}$

$\mathbf{z = w + x + y;}$

$\{\{ z < 0 \}\}$



Backward Reasoning

- Start with the **required** postcondition
- Fill in the **weakest** precondition
- For an assignment, $\mathbf{x = y}$:
 - just replace “x” with “y” in the postcondition
 - if the condition using “y” holds beforehand, then the condition with “x” will afterward since $x = y$ then
- Later: if statements and loops...

Correctness by Forward Reasoning

Use forward reasoning to determine if this code is correct:

`{{ w > 0 }}`

`x = 17;`

`y = 42;`

`z = w + x + y;`

`{{ z > 50 }}`

Example of Forward Reasoning

`{{ w > 0 }}`

`x = 17;`

`{{ w > 0 and x = 17 }}`

`y = 42;`

`{{ w > 0 and x = 17 and y = 42 }}`

`z = w + x + y;`

`{{ w > 0 and x = 17 and y = 42 and z = w + 59 }}`

`{{ z > 50 }}`

Do the facts that are always true imply the facts we need?
I.e., is the bottom statement **weaker** than the top one?

(Recall that weakening the postcondition is always okay.)

Correctness by Backward Reasoning

Use backward reasoning to determine if this code is correct:

```
{{ w < -60 }}
```

```
  x = 17;
```

```
  y = 42;
```

```
  z = w + x + y;
```

```
{{ z < 0 }}
```

Correctness by Backward Reasoning

Use backward reasoning to determine if this code is correct:

$\{\{ w < -60 \}\}$

$\{\{ w + 17 + 42 < 0 \}\} \iff \{\{ w < -59 \}\}$

$x = 17;$

$\{\{ w + x + 42 < 0 \}\}$

$y = 42;$

$\{\{ w + x + y < 0 \}\}$

$z = w + x + y;$

$\{\{ z < 0 \}\}$

Do the facts that are always true imply the facts we need?

I.e., is the top statement **stronger** than the bottom one?

(Recall that strengthening the precondition is always okay.)

Combining Forward & Backward

It is okay to use both types of reasoning

- Reason forward from precondition
- Reason backward from postcondition

Will meet in the middle:

`{{ P }}`

`S1`

`S2`

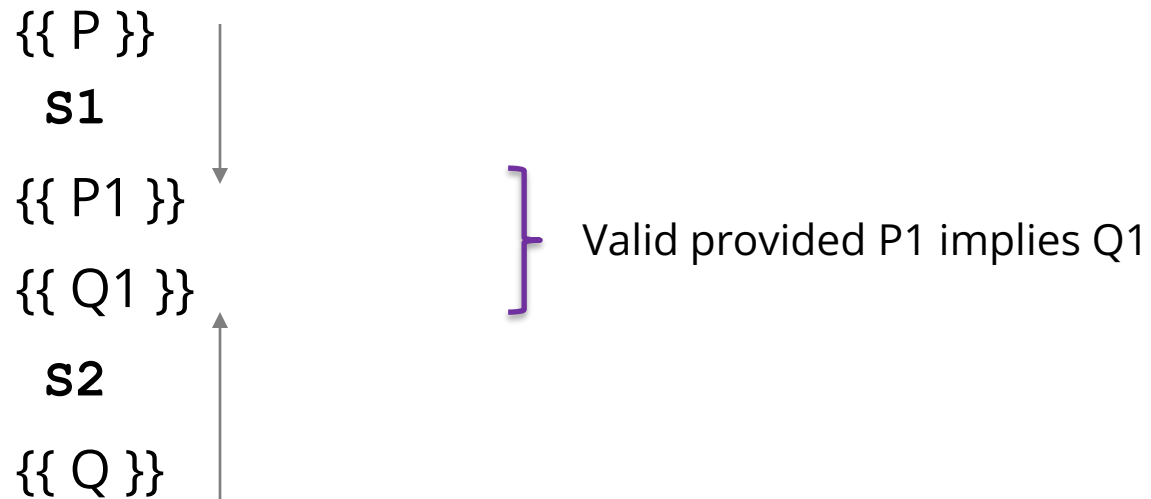
`{{ Q }}`

Combining Forward & Backward

It is okay to use both types of reasoning

- Reason forward from precondition
- Reason backward from postcondition

Will meet in the middle:

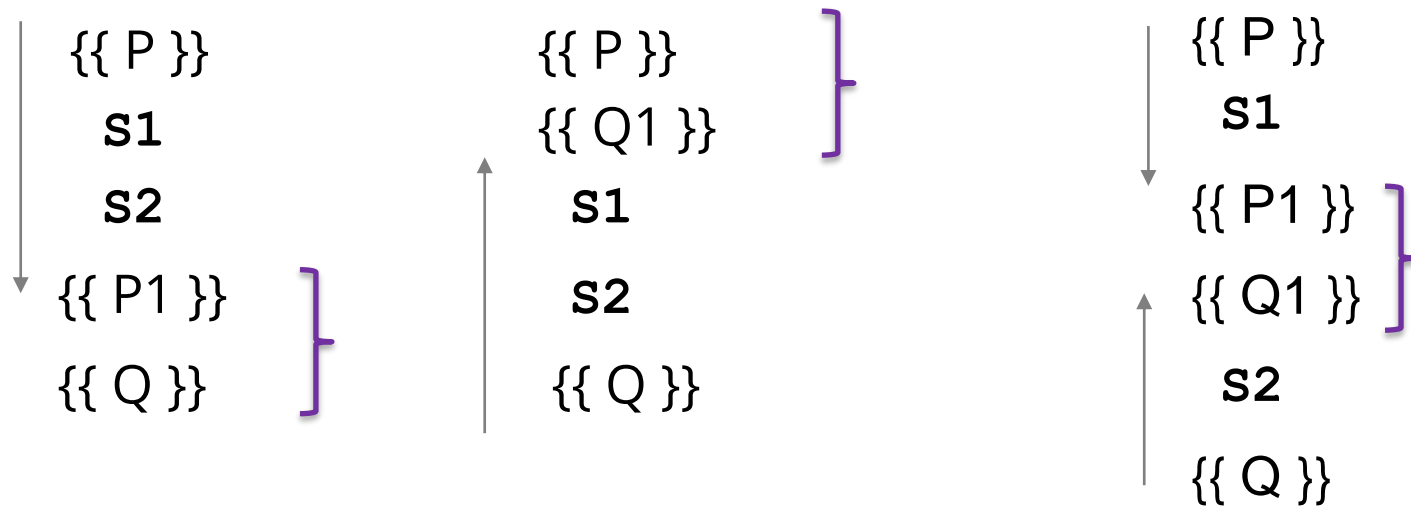


Combining Forward & Backward

Reasoning in either direction gives valid assertions

Just need to check adjacent assertions:

- top assertion must imply bottom one



Subtleties in Forward Reasoning...

- Forward reasoning can **fail** if applied blindly...

$\{\{\}\}$

$w = x + y;$

$\{\{ w = x + y \}\}$

$x = 4;$

$\{\{ w = x + y \text{ and } x = 4 \}\}$

$y = 3;$

$\{\{ w = x + y \text{ and } x = 4 \text{ and } y = 3 \}\}$

This implies that $w = 7$, but that is not true!

- w equals whatever $x + y$ was **before** they were changed

Fix 1

- Use **subscripts** to refer to old values of the variables
- Un-subscripted variables should always mean **current** value

$\{\{\}\}$

$w = x + y;$

$\{\{ w = x + y \}\}$

$x = 4;$

$\{\{ w = x_1 + y \text{ and } x = 4 \}\}$

$y = 3;$

$\{\{ w = x_1 + y_1 \text{ and } x = 4 \text{ and } y = 3 \}\}$

Fix 2 (better, when possible)

- Express prior values in terms of the current value

$\{\{\}\}$

$\mathbf{w} = \mathbf{x} + \mathbf{y};$

$\{\{ \mathbf{w} = \mathbf{x} + \mathbf{y} \}\}$

$\mathbf{x} = \mathbf{x} + 4;$

$\{\{ \mathbf{w} = \mathbf{x}_1 + \mathbf{y} \text{ and } \mathbf{x} = \mathbf{x}_1 + 4 \}\}$

Now, $\mathbf{x}_1 = \mathbf{x} - 4$

so $\mathbf{w} = \mathbf{x}_1 + \mathbf{y} \Leftrightarrow \mathbf{w} = \mathbf{x} - 4 + \mathbf{y}$

$\Rightarrow \{\{ \mathbf{w} = \mathbf{x} - 4 + \mathbf{y} \}\}$

Note for updating variables, e.g., $\mathbf{x} = \mathbf{x} + 4$:

- Backward reasoning just substitutes new value (no change)
- Forward reasoning requires you to invert the “+” operation

Forward vs. Backward

- Forward reasoning:
 - Find strongest postcondition
 - Intuitive: “simulate” the code in your head
 - BUT you need to change facts to refer to *prior values*
 - Inefficient: Introduces many irrelevant facts
 - usually need to weaken as you go to keep things sane
- Backward reasoning
 - Find weakest precondition
 - Formally simpler, but (initially) unintuitive
 - Efficient

Before next class...

1. Familiarize yourself with website:

<http://courses.cs.washington.edu/courses/cse331/23su/>

- read the welcome email
- read the syllabus

2. Try to do [HW1](#) and [syllabus quiz](#) before section tomorrow!

- submit a PDF on Gradescope
- limit this to at most 60 min
- do not use formal reasoning