CSE 331: Software Design & Implementation

Homework 3 (due Wednesday, October 18th at 11:00 PM)

The problems in this assignment contain a mix of written and coding parts. When you have completed them, submit your solutions in Gradescope. The written problems should be clearly labeled and submitted as a pdf to the "HW3 Written" assignment. The following completed files should be directly submitted for the coding portion to the "HW3 Coding" assignment:

```
index.tsx                   quilt_draw_table.tsx          quilt_ops_test.ts
patterns.ts                 quilt_draw_table_test.tsx
patterns_test.ts            quilt_ops.ts
```

For complete instructions for how to submit assignments in this course see the Homework Turn-in Guide.

Before we can get to the problems, however, we need the following mathematical definitions.

Squares

In this assignment, we will write some programs that display quilt patterns. Each quilt is made up of squares. Mathematically, each square is a record of the following type:

$$\textbf{type}\ \text{Square}\ :=\ \{\text{shape} : \text{Shape},\ \text{color} : \text{Color},\ \text{corner} : \text{Corner}\}$$

The properties of individual squares include shape and color, which are elements of the following types:

$$\textbf{type}\ \text{Shape}\ :=\ \text{STRAIGHT} \mid \text{ROUND}$$

$$\textbf{type}\ \text{Color}\ :=\ \text{GREEN} \mid \text{RED}$$

The last property, corner, describes the orientation of the square. Both square shapes have drawing primarily in one corner, so specifying which corner that is indicates the orientation, i.e., how the square is rotated.

$$\textbf{type}\ \text{Corner}\ :=\ \text{NW} \mid \text{NE} \mid \text{SW} \mid \text{SE}$$

Quilts

A quilt is a 2D table of squares. We will represent each quilt as a list of lists of squares. We will call a list of squares a "row", and then a quilt is a list of rows. We define these two types inductively as follows:

$$\begin{aligned}\textbf{type}\ \text{Row} :=&\ \ \text{rnil} \ \mid\ \text{rcons}(\text{hd} : \text{Square},\ \text{tl} : \text{Row})\\ \textbf{type}\ \text{Quilt} :=&\ \ \text{qnil} \ \mid\ \text{qcons}(\text{hd} : \text{Row},\ \text{tl} : \text{Quilt})\end{aligned}$$

All rows in a quilt should have the same length. Mathematically, we can define a row's length recursively:

$$\begin{aligned}\textbf{func}\ \text{rlen}(\text{rnil}) &:= 0\\ \text{rlen}(\text{rcons}(a, L)) &:= 1 + \text{rlen}(L)\end{aligned}$$

Note, however, that our type definitions allow the quilt to contain rows of different lengths! It is an additional *invariant* of the Quilt type that all rows in each quilt must have the same length.

We can also define concatenation of rows as follows:

$$\begin{aligned}\textbf{func}\ \text{rconcat}(\text{rnil}, R) &:= R\\ \text{rconcat}(\text{rcons}(s, L), R) &:= \text{rcons}(s, \text{rconcat}(L, R))\end{aligned}$$

These two functions, whose names start with "r", are defined on lists of squares (rows). There are analogous definitions of functions, qlen and qconcat, whose names start with "q", that operate on lists of rows (quilts).

1. We Find the Defendant Quilty (16 points)

The following parts consist of a mix of written and coding work: parts (a,c,e,g) are coding and should be submitted with "HW3 Coding", while parts (b,d,f) are written and should be submitted with "HW3 Written".

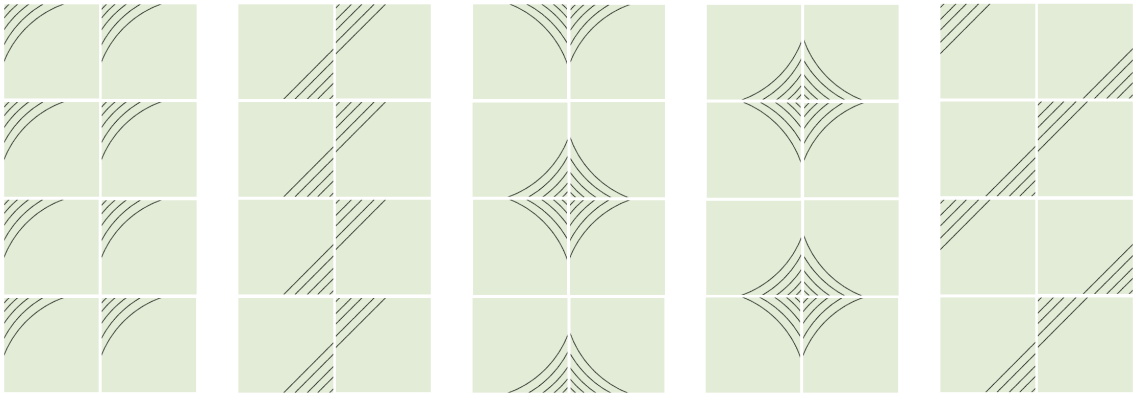
Start by checking out the starter code using the command

```
git clone https://gitlab.cs.washington.edu/cse331-23au-materials/hw-quilt.git
```

Then, install the modules using `npm install --no-audit`. You can run the tests using `npm run test`.

The provided TypeScript code includes the functions from the previous page in `quilt.ts`. It also includes a variety of constant variables to use. Have a look at those. To use these functions and types in different files, you will need to `import` them, for example, `import { Quilt, Square } from ./quilt;`.

(a) Implement the functions `PatternA`, . . . , `PatternE` in `patterns.ts` to return quilts displaying each of the 2×4 pattern shown below. (Implement them in the most straightforward manner—no loops or recursion! Our staff solutions for the functions at this step each have 3-7 lines of code.)



You can see the pattern returned by your code by doing the following: start the application with `npm run start`, open a browser to `http://localhost:8080?pattern=A`, and change the `pattern=` query parameter in the URL to the pattern you want to see.

(b) What level of the correctness does the programming in part (a) fall in? What steps are required to ensure that that your answer is correct? Check that your code in part (a) is correct by doing those steps.

(c) Change each of the pattern functions to support an optional "color" argument of type `Color`. If the argument is not provided, default to green. Modify `index.tsx` to pass in the color argument. It is already parsed in the `GetColor` function which is called on line 72, but the result is not used in the following calls to the `Pattern` functions.

You can change the color to red by adding the `color=red` query parameter in the URL: `http://localhost:8080?pattern=A&color=red`

(d) What level of the correctness does the programming in part (c) fall in? What steps are required to ensure that that your answer is correct? Check that your code in part (c) is correct by doing those steps.

(e) Next, we will change each of the pattern functions to be able to produce quilts with different numbers of rows. We'll do so by giving them a (required) "rows" argument of type number indicating the required number. Before we can do that, however, we need to specify what their behavior should be for different choices of the numbers of rows. We will do so by defining a mathematical function that returns the quilt we want. Mathematical definitions of patterns A–E are given on page 13.

Change the code for each pattern to match these definitions, writing it as closely as possible to the mathematical description. For example, the functions should be written recursively. Our staff solutions for the functions at this step each have 10-20 lines of code.

Note that the patterns are not defined for negative numbers of rows. In those cases, your code should throw an `Error`. (The syntax for this is the same as in Java. To throw an `Error`, you write `throw new Error(".. some explanation ..")`.) Furthermore, patterns C and D are only defined for even numbers of rows, so they should throw an `Error` for odd numbers as well.

In Typescript, required parameters must be listed before optional parameters, so you'll need to place your "rows" argument before your "color" argument. Modify the calls to PatternA in `patterns_test.ts` (for now) to pass in 4 as the number of rows (you won't need to pass in a color since it is optional).

Modify `index.tsx` to pass in the row query argument. It is already parsed in the `GetRows` function which is called on line 73, but the result is not used in the following calls to the `Pattern` functions. Then, you can change the number of rows to 8, for example, by adding a `rows=8` query parameter in the URL: `http://localhost:8080?pattern=A&color=red&rows=8`.

For now, just confirm that each of your methods produces the same patterns as before when we set `rows=4` but that they return only the top half when we set `rows=2`. (We will do more testing below.)

(f) What level of the correctness does the programming in part (e) fall in? What steps are actually required to ensure that that your answer is correct? (Hint: the tests above are not enough.) Also, why is it important that we write the code to be as similar to the mathematical description as possible?

(g) Following the heuristics we've learned, write test cases for each of the pattern functions in the file `patterns_test.ts`. An example test case is already provided for PatternA, but you will need more cases to cover all the important subdomains.

Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Note that you can check that a function throws an exception using `assert.throws`. An example of how to do this is included in the comments of the provided test. Just uncomment that.

Confirm that your tests all pass by running `npm run test`.

2. Twice to Meet You (16 points)

The following parts consist entirely of written work. It should be submitted with "HW3 Written".

Suppose that you see the following snippet of TypeScript code in some large TypeScript program. The code in the snippet uses `len`, sum, `twice_evens`, and twice_odds, all of which are TypeScript implementations of the mathematical functions of the same names defined in the section solutions.

```
const x = sum(twice_evens(L));
const y = sum(twice_odds(L));

if (len(L) === 2) {
  return x + y;  // = 3 * sum(L)
}
```

The comment shows the definition of what should be returned, but the code is not a direct translation of that. Below, we will use reasoning to prove that the code is correct.

Note that, if $\text{len}(L) = 2$, then $L = \text{cons}(a, \text{cons}(b, \text{nil}))$ for some integers a and b, as we have previously proven in section. We will use that below.

(a) Using the fact that $L = \text{cons}(a, \text{cons}(b, \text{nil}))$, prove by calculation that $\text{sum}(L) = a + b$.

(b) Using the fact that $L = \text{cons}(a, \text{cons}(b, \text{nil}))$, prove by calculation that $\text{sum}(\text{twice-evens}(L)) = 2a + b$.

(c) Using the fact that $L = \text{cons}(a, \text{cons}(b, \text{nil}))$, prove by calculation that $\text{sum}(\text{twice-odds}(L)) = a + 2b$.

(d) Prove that the code is correct by showing that $x + y = 3\,\text{sum}(L)$, i.e., that

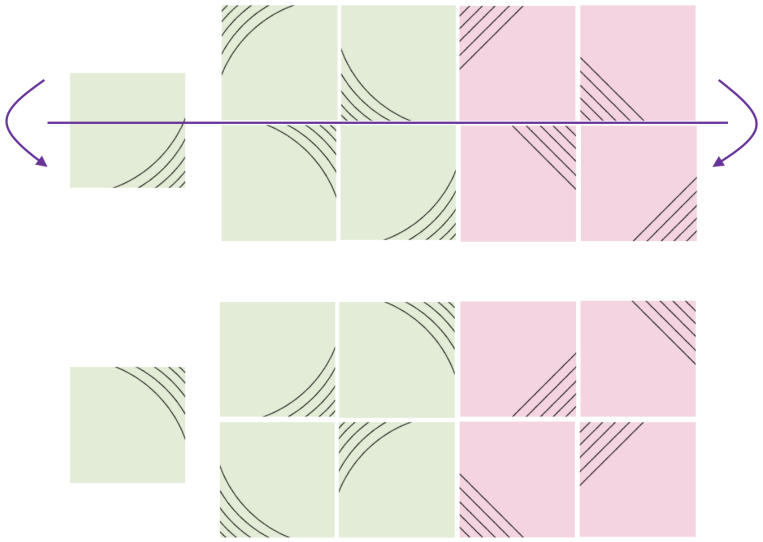
$$\text{sum}(\text{twice-evens}(L)) + \text{sum}(\text{twice-odds}(L)) = 3\,\text{sum}(L)$$

You are free to cite parts (a-c) in your calculation since we know that $L = \text{cons}(a, \text{cons}(b, \text{nil}))$ holds on the line with the `return` statement.

3. Skinny Flipping (16 points)

The following parts consist of a mix of written and coding work: parts (b,e,h) are coding and are submitted with "HW3 Coding", while parts (a,c,d,f,g) are written and are submitted with "HW3 Written".

In this problem, we will write a function that "flips a quilt vertically, as if spun around a horizontal line through the center". Here is an example:



The quilt on the bottom is the result of vertically flipping the quilt on the top.

(a) The problem definition was in English, so our first step is to formalize it.

Start by writing a mathematical definition of a function "sflip-vert" that flips a single **square** vertically.

(b) Translate your mathematical definition into TypeScript code in the `quilt_ops.ts` file.

Write tests for your function in `quilt_ops_test.ts`.

Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Confirm that your tests all pass by running `npm run test`.

(c) Next, we will define a mathematical function "rflip-vert" that flips a **row** vertically.

Let's start by writing this out in more detail. Let a, b, and c be squares. Fill in the blanks showing the result of applying rflip-vert to different rows, which we will write as lists of squares.

Feel free to abbreviate sflip-vert in your answer as "s".

rnil _____

rcons(a, rnil) _____

rcons(a, rcons(b, rnil)) _____

rcons(a, rcons(b, rcons(c, rnil))) _____

...

(d) Write a mathematical definition of a function rflip-vert using recursion.

(e) Translate your mathematical definition into TypeScript code in the `quilt_ops.ts` file.

Write tests for your function in `quilt_ops_test.ts`.

Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Confirm that your tests all pass by running `npm run test`.

(f) Now, we are ready to define a function "qflip-vert" that flips a **quilt** vertically. Note that this operation flips individual rows vertically, but also reverses their order!

Again, let's start by writing this out in more detail. Let u, v, and w be rows. Fill in the blanks showing the result of applying qflip-vert to different quilts, which we will write as lists of rows.

Your answers should use rflip-vert (not sflip-vert), which you can abbreviate as just "r".

qnil _____

qcons(u, qnil) _____

qcons(u, qcons(v, qnil)) _____

qcons(u, qcons(v, qcons(w, qnil))) _____

...

(g) Write a mathematical definition of a function "qflip-vert".

Hint: it may be useful to review definition of the function rev, from lecture. Also, remember that the function qconcat, which concatenates two quilts, is already provided for you.

(h) Translate your mathematical definition into TypeScript code in the `quilt_ops.ts` file.

Write tests for your function in `quilt_ops_test.ts`.

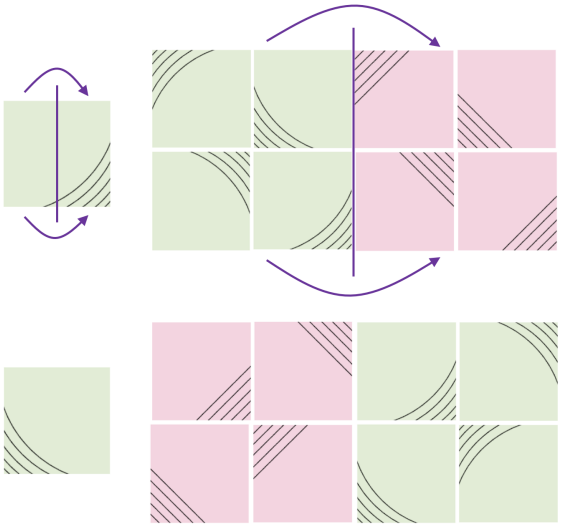
Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Confirm that your tests all pass by running `npm run test`.

4. Flip Service (16 points)

The following parts consist of a mix of written and coding work: parts (b,d,f,g) are coding and are submitted with "HW3 Coding", while parts (a,c,e,h) are written and are submitted with "HW3 Written".

In this problem, we will write a function that "flips a quilt horizontally, as if spun around a vertical line through the center". Here is an example:



The quilt on the bottom is the result of horizontally flipping the quilt on the top.

(a) The problem definition was in English, so our first step is to formalize it.

Let's start by writing a mathematical definition of a function "sflip-horz" that flips an individual square horizontally.

(b) Translate your mathematical definition into TypeScript code in the `quilt_ops.ts` file.

Write tests for your function in `quilt_ops_test.ts`.

Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Confirm that your tests all pass by running `npm run test`.

(c) Next, write a mathematical definition of a function "rflip-horz" that flips a row horizontally. You should use sflip-horz in your definition.

Note that this operation not only flips the individual squares horizontally but also reverses their order. Remember that the function rconcat, which concatenates two rows, is already provided for you.

(d) Translate your mathematical definition into TypeScript code in the `quilt_ops.ts` file.

Write tests for your function in `quilt_ops_test.ts`.

Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Confirm that your tests all pass by running `npm run test`.

(e) Now, write a mathematical definition of a function "qflip-horz" that flips a quilt horizontally. You should use rflip-horz in your definition.

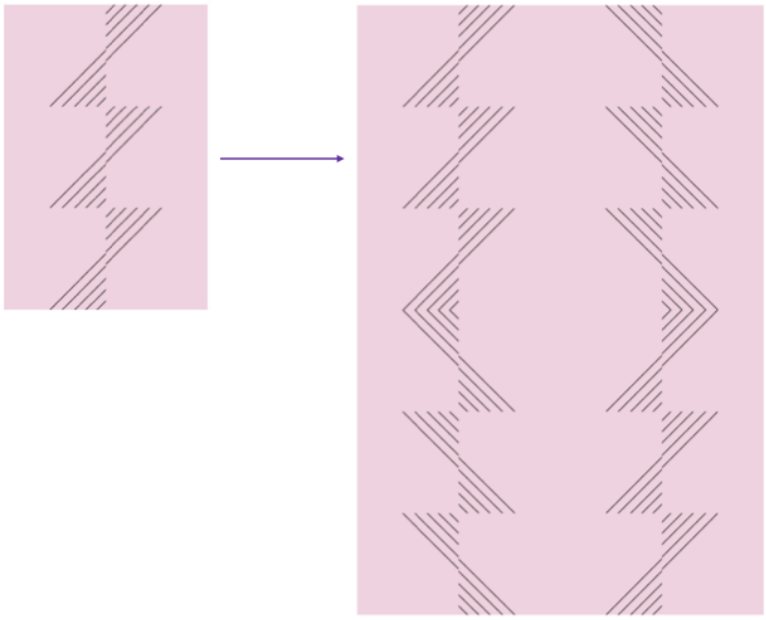
(f) Translate your mathematical definition into TypeScript code in the `quilt_ops.ts` file.

Write tests for your function in `quilt_ops_test.ts`.

Include a brief comment above each test case justifying why you chose it. See page 15 for complete instructions and examples.

Confirm that your tests all pass by running `npm run test`.

(g) The code provided in `quilt_ops.ts` includes a function called "`symmetrize`" that takes a quilt as an argument and returns one that is symmetric, both horizontally and vertically. It does this by sewing the given quilt and its horizontally-flipped version and then sewing the result together with a vertically flipped version of that. (The code is just two lines. Have a look at it if this description was unclear.)



Modify `index.tsx` to look for a query parameter called "`symmetrize`" and, if it is present, call the `symmetrize` function on the quilt before displaying it. You can see if a parameter is present by calling the "`has`" method on the `URLSearchParams` object. In this case, we call `params.has("symmetrize")`.

You should then be able to see the symmetrized version by adding this parameter to the URL, e.g.: `http://localhost:8080?pattern=A&color=red&rows=4&symmetrize`.

(h) What steps are required to ensure that that this code in `index.tsx` is correct? (Remember that `symmetrize` is already tested.) Check that your code in part (g) is correct by doing those steps.

5. Swap, Drop, and Roll (16 points)

The following parts consist entirely of written work. It should be submitted with "HW3 Written".

In this problem, we will use the following function, which swaps adjacent pairs of elements in a list:

$$
\begin{array}{lll}
\textbf{func}\ \mathsf{swap}(\mathsf{nil}) & := \mathsf{nil} & \\
\quad \mathsf{swap}(\mathsf{cons}(a, \mathsf{nil})) & := \mathsf{cons}(a, \mathsf{nil}) & \text{for any } a : \mathbb{Z} \\
\quad \mathsf{swap}(\mathsf{cons}(a, \mathsf{cons}(b, L))) & := \mathsf{cons}(b, \mathsf{cons}(a, \mathsf{swap}(L))) & \text{for any } a, b : \mathbb{Z} \text{ and } L : \mathsf{List}
\end{array}
$$

Lists of length 0 and 1 are left as is, whereas if the list has length 2 or more, the order of the first two elements are swapped before we recurse on the rest of the list after those two elements.

Suppose you see the following snippet in some TypeScript code. It uses `len` and swap, which are TypeScript implementations of the mathematical functions "len" and "swap".

```
if (len(L) === 3) {
  return cons(1, cons(2, L));  // = swap(swap(cons(1, cons(2, L))))
}
```

The comment shows the definition of what should be returned, but the code is not a direct translation of those. Below, we will use reasoning prove that the code is correct.

(a) Let x be any integer. Prove that $\mathsf{swap}(\mathsf{swap}(\mathsf{cons}(x, \mathsf{nil}))) = \mathsf{cons}(x, \mathsf{nil})$.

(b) Let x and y be any integers and R be any list. Prove that

$$\mathsf{swap}(\mathsf{swap}(\mathsf{cons}(x, \mathsf{cons}(y, R)))) = \mathsf{cons}(x, \mathsf{cons}(y, \mathsf{swap}(\mathsf{swap}(R))))$$

(c) Let $L = \mathsf{cons}(s, \mathsf{cons}(t, \mathsf{cons}(u, \mathsf{cons}(v, \mathsf{cons}(w, \mathsf{nil})))))$ for some integers $s, t, u, v, w : \mathbb{Z}$, i.e., L is some list of length 5. Prove that $\mathsf{swap}(\mathsf{swap}(L)) = L$.

You should apply part (a) once and part (b) multiple times (with different choices of x and y) rather than performing the same calculation again here.

(d) Prove that the code is correct by showing that $\mathsf{swap}(\mathsf{swap}(\mathsf{cons}(1, \mathsf{cons}(2, L)))) = \mathsf{cons}(1, \mathsf{cons}(2, L))$, using the fact that L has length 3, i.e., that $L = \mathsf{cons}(a, \mathsf{cons}(b, \mathsf{cons}(c, \mathsf{nil})))$ for some integers a, b, c.

Feel free to apply prior parts, if useful, rather than performing calculations again.

6. Quilt To Last (20 points)

The following parts consist of a mix of written and coding work: parts (b,c,d,e,g) are coding and are submitted with "HW3 Coding", while parts (a,f) are written and are submitted with "HW3 Written".

We will skip writing the formal function definitions on paper for this problem as they are all simple or very similar to previous definitions you wrote. We will also provide all the tests cases for you as the syntax for these tests can be tricky. However, you should look at the test cases for each function when you test them to make sure the subdomains tested are clear.

In this last problem, we will add a new option to the UI that displays a simpler, textual visualization of the quilt. This might be useful for a screen reader application used by visually impaired users. We will start by creating such a UI for individual rows.

NW	NW	SE	SW
NW	NW	NE	NW
NW	NW	SE	SW
NW	NW	NE	NW

The file `quilt_draw_table.tsx` contains the following function that displays a square as a cell of an HTML table (a `<TD>` element):

```
export function SquareTableElem(props: {square: Square, key: number}): JSX.Element
```

Note that this function takes its arguments as a record so that it can be used in an HTML literal as in `<SquareTableElem square={s} key={0}/>` and that it returns HTML, which has type `JSX.Element`.

The `key` attribute is an identifier used in React (a library helping us run these functions in the browser). It is a **unique** value given to each element of a list of JSX.elements when rendered that helps React identify the elements and keep track of things. `<SquareTableElem>`s will be put into a list (part b), so each element must be given a unique key. If you forget to include keys when working with lists in React, you will get warnings about "missing unique key props".

(a) If we had not looked at the code of `SquareTableElem`, how many tests should we need? Now that we have seen the code, why is it possible to only have 4 tests, not more?

(b) Fill in the body of the function with the following declaration in the code:

```
export function RowTableElems(props: {row: Row, key: number}): JsxList
```

This recursive function turns a row of squares into a list of HTML elements by calling the function `SquareTableElem` on each one of them.

The return type `JsxList` is a list of `JSX.Element` objects. It is defined in `jsx_list.ts`, check it out as well as the helpful functions defined there!

Make sure that the `JSX.Element` created for each square has a different `key` attribute by incrementing the `key` parameter on each recursive call of `RowTableElems`.

Confirm that the provided tests for `RowTableElems` and `SquareTableElem` in `quilt_draw_table_test.tsx` now pass by running `npm run test`.

The code provided in quilt_draw_table.tsx already implements a function with the following declaration:

```
export function RowTableElem(props: {row: Row, key: number}): JSX.Element
```

It turns a row of squares into a single HTML <TR> element whose body is an array of elements, one for each of the squares in the row. It gets the square <TD> elements by calling RowTableElems on the list and then turns the list into an array by calling jcompact from jsx_list.ts.

(c) Fill in the body of the function with the following declaration in the code:

```
export function QuiltTableElems(props: {quilt: Quilt, key: number}): JsxList
```

The recursive function turns a quilt into a list of HTML elements by calling RowTableElem on each one of them.

Make sure that the JSX.Element created for each row has a different key attribute by incrementing the key parameter on each recursive call QuiltTableElems.

Confirm that the provided tests for QuiltTableElems in quilt_draw_table_test.tsx pass by running npm run test.

(d) Fill in the body of the function with the following declaration In the code:

```
export function QuiltTableElem(props: {quilt: Quilt}): JSX.Element
```

This should return a single HTML <TABLE> element containing a <TBODY> element whose body is an array of <TR> elements, one for each of the row in the quilt. You should get the latter by calling QuiltTableElems and turning the list it returns into an array by calling jcompact from jsx_list.ts (as we did in RowTableElem).

Note that this does not take a key parameter. None is necessary because we will create only a single table, not a list of them next to each other (which React would need a way to distinguish).

Confirm that the provided tests for QuiltTableElem in quilt_draw_table_test.tsx pass by running npm run test.

(e) Modify index.tsx to look for a query parameter called "table" and, if it is present, render the quilt with a QuiltTableElem instead of QuiltElem.

The quilt is rendered at the bottom of the index.tsx file using the method root.render. You will want to pass a different argument to that method when "table" is present. (Here, React replaces the custom tags with the HTML returned by the function with that name when passed those attributes as an argument.)

You should then be able to see the table version by adding this parameter to the URL, e.g.: http://localhost:8080?pattern=A&color=red&rows=8&table.

(f) What steps are required to ensure that that this code in index.tsx is correct? Check that your code in part (e) is correct by doing those steps.

(g) As a last step to polish off our app and further improve the experience for our users, we will add a <form> (like we used in HW2) to allow users to display different quilts without modifying the URL directly.

In quilt_form.tsx, we provided a function, QuiltForm(), that returns an HTML form with selectors for each quilt customization. In index.tsx, use this function to display the form when the "pattern" parameter is missing from the URL (previously, we redirected the page to display a default quilt with pattern A in this case). Like other functions that produce HTML, QuiltForm takes a "_props" parameter, but in this case we don't need any props, so when you call the function you can pass in an empty record: {}.

Congratulations! You have a functioning app.

7. Extra Credit: Cycle-babble (0 points)

The following problem consist entirely of written work. If you complete it, submit it with "HW3 Written".

For this last problem, we define "cycle", which returns a list with each element moved forward one index (e.g., the element at index n moves to index $n-1$), except for the first element, which is moved to the end.

$$\begin{aligned} \textbf{func } \text{cycle}(\text{nil}) &:= \text{nil} \\ \text{cycle}(\text{cons}(a, L)) &:= \text{concat}(L, \text{cons}(a, \text{nil})) \quad \text{for any } a : \mathbb{Z} \text{ and } L : \text{List} \end{aligned}$$

where concat is recursively on its first argument by

$$\begin{aligned} \textbf{func } \text{concat}(\text{nil}, R) &:= R \quad \text{for any } R : \text{List} \\ \text{concat}(\text{cons}(a, L), R) &:= \text{cons}(a, \text{concat}(L, R)) \quad \text{for any } a : \mathbb{Z} \text{ and } L, R : \text{List} \end{aligned}$$

For example, we have $\text{cycle}(\text{cons}(1, \text{cons}(2, \text{cons}(3, \text{nil})))) = \text{cons}(2, \text{cons}(3, \text{cons}(1, \text{nil})))$.

Let a be any integer and L be any list. In this problem, you will prove, **without induction**, that we have $\text{len}(\text{cycle}(\text{cons}(a, L))) \geq 1$. Do so in the following steps.

(a) Prove by cases that $\text{cycle}(\text{cons}(a, L)) \neq \text{nil}$.

(b) Use part (a) to prove that $\text{len}(\text{cycle}(\text{cons}(a, L))) \geq 1$. You may need to use the following facts:

- If a list $L \neq \text{nil}$, then we know that $L = \text{cons}(b, R)$ for some $b : \mathbb{Z}$ and $R : \text{List}$.
- We have $\text{len}(L) \geq 0$ for any list L since, by definition, len returns a natural number.

Quilt Patterns

The following mathematical functions PatternA, ..., PatternE, formally define the correct quilt to display with that pattern for different numbers of rows. Specifically, the function call PatternX(n, c) returns the correct quilt with pattern X having n rows with squares in color c. Note that these patterns are defined only for $n \geq 0$ and, in cases C and D, only for even n.

Patterns A & B

We define the A pattern, for any **natural number** of rows, recursively as follows:

$$\begin{aligned} \textbf{func}\ \text{PatternA}(0, c) &:= \text{qnil} \\ \text{PatternA}(n+1, c) &:= \text{qcons}([s_c, s_c], \text{PatternA}(n, c)) \end{aligned}$$

where c is the color parameter and s_c is the square $\{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{NW}\}$. Remember that $[s_c, s_c]$ is shorthand for $\text{rcons}(s_c, \text{rcons}(s_c, \text{rnil}))$. The function call PatternA(n, c) will return a quilt with n rows, with each row having exactly 2 squares.

We define the B pattern, for any **natural number** of rows, recursively as follows:

$$\begin{aligned} \textbf{func}\ \text{PatternB}(0, c) &:= \text{qnil} \\ \text{PatternB}(n+1, c) &:= \text{qcons}([s_c, t_c], \text{PatternB}(n, c)) \end{aligned}$$

where c is the color parameter and s_c is the square $\{\text{shape} : \text{STRAIGHT},\ \text{color} : c,\ \text{corner} : \text{SE}\}$ and t_c is the square $\{\text{shape} : \text{STRAIGHT},\ \text{color} : c,\ \text{corner} : \text{NW}\}$.

Patterns C & D

We define the C pattern, for any **even number** of rows, recursively as follows:

$$\begin{aligned} \textbf{func}\ \text{PatternC}(0, c) &:= \text{qnil} \\ \text{PatternC}(n+2, c) &:= \text{qcons}([s_c, t_c], \text{qcons}([u_c, v_c], \text{PatternC}(n, c))) \end{aligned}$$

where the four squares mentioned above are

$$\begin{aligned} s_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{NE}\} \\ t_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{NW}\} \\ u_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{SE}\} \\ v_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{SW}\} \end{aligned}$$

Note that PatternC(n, c) also returns a quilt with n rows, with each row containing 2 squares.

We define the D pattern, for any **even number** of rows, the same way:

$$\begin{aligned} \textbf{func}\ \text{PatternD}(0, c) &:= \text{qnil} \\ \text{PatternD}(n+2, c) &:= \text{qcons}([s_c, t_c], \text{qcons}([u_c, v_c], \text{PatternD}(n, c))) \end{aligned}$$

except that, now, the four squares mentioned above are

$$\begin{aligned} s_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{SE}\} \\ t_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{SW}\} \\ u_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{NE}\} \\ v_c &:= \{\text{shape} : \text{ROUND},\ \text{color} : c,\ \text{corner} : \text{NW}\} \end{aligned}$$

Pattern E

We define the E pattern, for any **natural number** of rows, recursively as follows:

$$\begin{aligned}
\textbf{func}\ \mathsf{PatternE}(0, c) &:= \mathsf{qnil} \\
\mathsf{PatternE}(1, c) &:= \mathsf{qcons}([s_c, t_c], \mathsf{qnil}) \\
\mathsf{PatternE}(n+2, c) &:= \mathsf{qcons}([s_c, t_c], \mathsf{qcons}([u_c, v_c], \mathsf{PatternE}(n, c)))
\end{aligned}$$

where the four squares mentioned above are

$$\begin{aligned}
s_c &:= \{\mathsf{shape} : \mathsf{STRAIGHT},\ \mathsf{color} : c,\ \mathsf{corner} : \mathsf{NW}\} \\
t_c &:= \{\mathsf{shape} : \mathsf{STRAIGHT},\ \mathsf{color} : c,\ \mathsf{corner} : \mathsf{SE}\} \\
u_c &:= \{\mathsf{shape} : \mathsf{STRAIGHT},\ \mathsf{color} : c,\ \mathsf{corner} : \mathsf{SE}\} \\
v_c &:= \{\mathsf{shape} : \mathsf{STRAIGHT},\ \mathsf{color} : c,\ \mathsf{corner} : \mathsf{NW}\}
\end{aligned}$$

Note that we have defined a pattern for any natural number of rows, even though, as in patterns C and D, the odd and even rows are different!

Writing Test Cases

In this homework, we ask you to write test cases for functions based on the heuristics we have learned in class. For more details about how to use assert statements and the format of test files, see the week 3 section slides.

For each test case you write, we ask that you include a comment above the case in your code justifying why you chose to include that case. This justification should cite a testing heuristic or explain what plausible bug could have been missed without the case. This comment can be brief. Certainly not more than a sentence, and likely just a few words.

Below is an example function with tests written according to our heuristics and comment justifications that are the length and level of detail that we expect from you. You don't have to follow this format exactly, but this is the right idea.

example.ts:

```
function f(n: number): number {
    if (n === 0) {
        return 0;
    } else {
        return 2 * f(n - 1) + 1;
    }
}
```

example_test.ts:

```
import * as assert from 'assert';
import { f } from './example'

describe('example', function() {
    it('testf', function() {
        // 0-1-many heuristic, base case
        assert.deepStrictEqual(f(0), 0);

        // 0-1-many heuristic, 1 recursive call (only 1 possible)
        assert.deepStrictEqual(f(1), 1);

        // 0-1-many heuristic, more than 1 recursive call (1st)
        assert.deepStrictEqual(f(2), 3);

        // 0-1-many heuristic, more than 1 recursive call (2nd)
        assert.deepStrictEqual(f(3), 7);
    });
});
```