CSE 331
Software Design & Implementation

Winter 2022

Section 1 – Code Reasoning

Administrivia

- HW1 due next Tuesday, January 1/11.
- Any questions before we dive in?
  - What are the most interesting/confusing/puzzling things so far in the course?

Agenda

- Introductions?
- Review logical reasoning about code with Hoare Logic
- Practice both forward and backward modes
  - Just assignment, conditional ("if-then-else"), and sequence
  - Logical rules from yesterday's lecture/notes
- Review logical strength of assertions (weaker *vs.* stronger)
- Practice determining stronger/weaker assertions

Introductions

Why reason about code?

- Prove that code is correct
- Understand *why* code is correct
- Diagnose why/how code is *not* correct
- Specify code behavior

Logical reasoning about code

- Determine facts that hold of program state between statements
  - "Fact" ~ assertion (logical formula over program state, informally "value(s) of some/all program variables)
  - Driven by assumption (precondition) or goal (postconditon)

- Forward reasoning
  - What facts follow from initial assumptions?
  - Go from precondition to postcondition

- Backward reasoning
  - What facts need to be true to reach a goal?
  - Go from postcondition to precondition

Hoare Logic: Validity by Reasoning

- Checking validity of **{*P*} S {*Q*}**
  - Valid iff, starting from any state satisfying ***P***, executing ***S*** results in a state satisfying ***Q***

- Forward reasoning:
  - Reason from ***P*** to strongest postcondition **{*P*} *S* {*R*}**
  - Check that ***R*** implies ***Q*** (i.e., ***Q*** is weaker)

- Backward reasoning:
  - Reason from ***Q*** to get weakest precondition **{*R*} *S* {*Q*}**
  - Check that ***P*** implies ***R*** (i.e., ***P*** is stronger)

Implication (=>)

- Logic formulas with *and* (&, &&, or ∧), *or* (|, ||, or ∨) and *not* (! or ¬) have the same meaning they do in programs

- Implication might be a bit new, but the basic idea is pretty simple. Implication p=>q is true as long as q is always true whenever p is

p	q	p => q
T	T	T
T	F	F
F	T	T
F	F	T

Assignment Statements

- Reasoning about `x = y;`

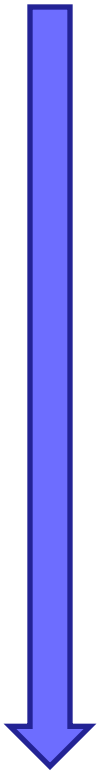
- Forward reasoning:
  - add “x = y” as a new fact
  - (also rewrite any existing references to “x” to use new value)

- Backward reasoning:
  - replace all instances of “x” in the postcondition with “y”

Conditionals, more closely

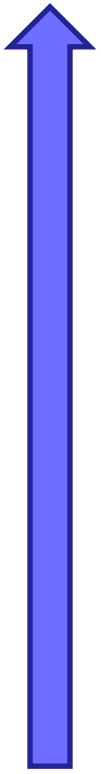
Forward reasoning

```
{P}
if (b)
     {P ∧ b}
     S1
     {Q1}
else
     {P ∧ !b}
     S2
     {Q2}
{Q1 ∨ Q2}
```



Backward reasoning

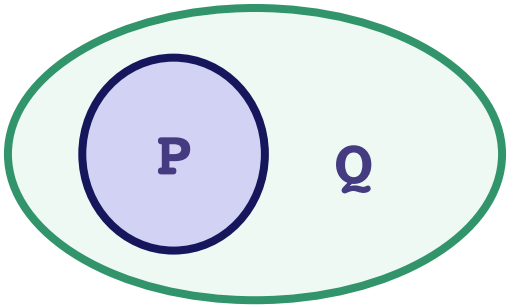
```
{ (b ∧ P1) ∨ (!b ∧ P2) }
if (b)
  {P1}
   S1
  {Q}
else
   {P2}
   S2
   {Q}
{Q}
```



Weaker vs. stronger

Formal definition:

- If $P \Rightarrow Q$, then
  - Q is weaker than P
  - P is stronger than Q



Intuitive definition:

- "Weak" means unrestrictive; a weaker assertion has a larger set of possible program states (*e.g.*, `x != 0`)
- "Strong" means restrictive; a stronger assertion has a smaller set of possible program states (*e.g.*, `x = 1` or `x > 0` are both stronger than `x != 0`).

Let's do Q1

Worksheet – problem 1

```
{ x >= 0 ∧ y >= 0 }
y = 16;
{ x >= 0 ∧ y = 16 }
x = x + y;
{ x >= 16 ∧ y = 16 }
X = sqrt(x);
{ x >= 4 ∧ y = 16 }
y = y - x;
{ x >= 4 ∧ y = 16 - x }
⇒ { x >= 4 ∧ y <= 12 }
```

Let's do Q3

Worksheet – problem 3 (backward)

```
{ x + 3 * b - 4 > 0 }
a = x + b;
{ a + 2 * b - 4 > 0 }
c = 2 * b - 4;
{ a + c > 0 }
x = a + c;
{ x > 0 }
```

Let's do Q6

Worksheet – problem 6 (forward)

```
{ true }
if (x < y) {
  { true ∧ x < y }
  m = x;
  { x < y ∧ m = x }
} else {
  { true ∧ x >= y }
  m = y;
  { x >= y ∧ m = y }
}
{ (x < y ∧ m = x) ∨ (x >= y ∧ m = y) }
⇒ { m = min(x, y) }
```

Worksheet – problem 6 (backward)

```
{ true } ⇔
{ (x <= y ∧ x < y) ∨ (y <= x ∧ x >= y) }
if (x < y) {
  { x = min(x, y) } ⇔ { x <= y }
  m = x;
  { m = min(x, y) }
} else {
  { y = min(x, y) } ⇔ { x >= y }
  m = y;
  { m = min(x, y) }
}
{ m = min(x, y) }
```

Let's do Q7

Worksheet – problem 7

```
{ y > 23 }
```

```
{ y >= 23 }
```

```
{ y = 23 }
```

```
{ y >= 23 }
```

```
{ y < 0.23 }
```

```
{ y < 0.00023 }
```

```
{ x = y * z }
```

```
{ y = x / z }
```

```
{ is_prime(y) }
```

```
{ is_odd(y) }
```

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`		`{ y >= 23 }`
`{ y < 0.23 }`		`{ y < 0.00023 }`
`{ x = y * z }`		`{ y = x / z }`
`{ is_prime(y) }`		`{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`	is stronger than	`{ y >= 23 }`
`{ y < 0.23 }`		`{ y < 0.00023 }`
`{ x = y * z }`		`{ y = x / z }`
`{ is_prime(y) }`		`{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`	is stronger than	`{ y >= 23 }`
`{ y < 0.23 }`	is weaker than	`{ y < 0.00023 }`
`{ x = y * z }`		`{ y = x / z }`
`{ is_prime(y) }`		`{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`	is stronger than	`{ y >= 23 }`
`{ y < 0.23 }`	is weaker than	`{ y < 0.00023 }`
`{ x = y * z }`	is **incomparable** with	`{ y = x / z }`
`{ is_prime(y) }`		`{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`	is stronger than	`{ y >= 23 }`
`{ y < 0.23 }`	is weaker than	`{ y < 0.00023 }`
`{ x = y * z }`	is **incomparable** with	`{ y = x / z }`
`{ is_prime(y) }`	is **incomparable** with	`{ is_odd(y) }`

Questions?

- What is the most surprising thing about this?
- What is the most confusing thing?
- What will need a bit more thinking to digest?