CSE 331
Software Design & Implementation

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Lecture 1 – Reasoning About Straight-Line Code

Motivation for Reasoning

- Want a way to determine correctness without running the code
- Most important part of the correctness techniques
  - tools, **inspection**, testing
- You need a way to do this in interviews
  - key reason why coding interviews are done without computers
- This is not easy

Our Approach

- We will learn a set of **formal tools** for proving correctness
  - (later, this will also allow us to generate the code)

- Most professionals can do reasoning like this in their head
  - most do an *informal* version of what we will see
  - eventually, it will be the same for you

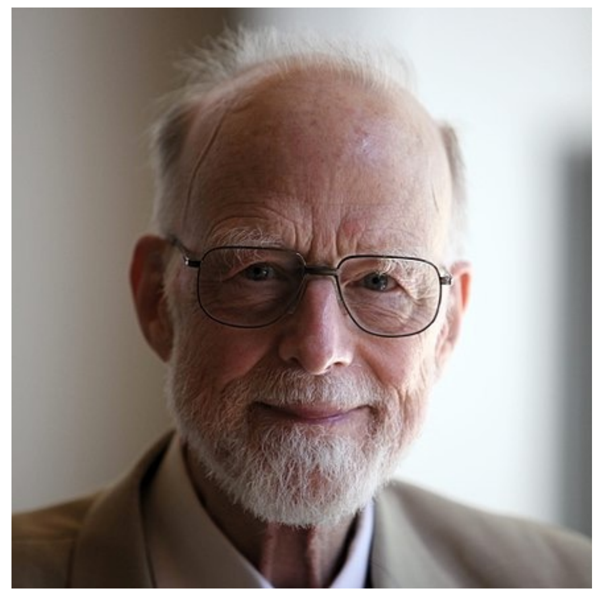
- Formal version has key advantages
  - teachable
  - mechanical (no intuition or creativity required)
  - necessary for hard problems
    - we turn to formal tools when problems get too hard

Formal Reasoning

- Invented by Robert Floyd and Sir Anthony Hoare
  - Floyd won the Turing award in 1978
  - Hoare won the Turing award in 1980

Robert Floyd

picture from [Wikipedia]



Tony Hoare

Terminology of Floyd Logic

- The *program state* is the values of all the (relevant) variables
- An *assertion* is a true / false claim (proposition) about the state at a given point during execution (e.g., on line 39)
- An assertion *holds* for a program state if the claim is true when the variables have those values
- An assertion before the code is a *precondition*
  - these represent assumptions about when that code is used
- An assertion after the code is a *postcondition*
  - these represent what we want the code to accomplish

Hoare Triples

- A Hoare triple is two assertions and one piece of code:

  {*P*} *S* {*Q*}

  - *P* the precondition
  - *S* the code
  - *Q* the postcondition

  specification

  method body

  code is correct iff triple is valid

- A Hoare triple **{*P*} *S* {*Q*}** is called valid if:
  - in any state where P holds,
    executing S produces a state where Q holds
  - i.e., if *P* is true before *S*, then *Q* must be true after it
  - otherwise, the triple is called invalid

Notation

- Floyd logic writes assertions in {..}
  - since Java code also has {..}, I will use {{…}}
  - e.g., {{ w >= 1 }} **`x = 2 * w;`** {{ x >= 2 }}
- Assertions are math / logic not Java
  - you can use the usual math notation
    - (e.g., **=** instead of **==** for equals)
  - purpose is communication with other humans (not computers)
  - we will need **`and`**, **`or`**, **`not`** as well
    - can also write use ∧ (and) ∨ (or) etc.
- The Java language also has assertions (**`assert`** statements)
  - throws an exception if the condition does not evaluate true
  - we will discuss these more later in the course

Example 1

Is the following Hoare triple valid or invalid?

– assume all variables are integers and there is no overflow

```
{{ x != 0 }} y = x*x; {{ y > 0 }}
```

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Is the following Hoare triple valid or invalid?

– assume all variables are integers and there is no overflow

```
{{ x != 0 }} y = x*x; {{ y > 0 }}
```

Valid

- **`y`** could only be zero if **`x`** were zero (which it isn't)

Example 2

Is the following Hoare triple valid or invalid?

– assume all variables are integers and there is no overflow

```
{{ z != 1 }} y = z*z; {{ y != z }}
```

Example 2

Is the following Hoare triple valid or invalid?

– assume all variables are integers and there is no overflow

```
{{ z != 1 }} y = z*z; {{ y != z }}
```

Invalid

- counterexample: `z = 0`

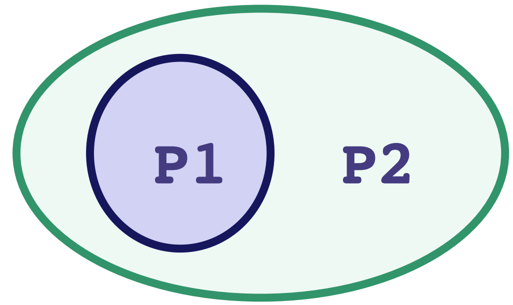
Checking Validity

- So far: decided if a Hoare triple is valid by ... **hard** thinking
- Soon: mechanical process for reasoning about
  - assignment statements
  - conditionals
  - [next lecture] loops
  - (all code can be understood in terms of those 3 elements)
- Can use those to check correctness in a “turn the crank” manner
- Next: a way to compare different assertions
  - useful, e.g., to compare possible preconditions

Weaker vs. Stronger Assertions

If P1 implies P2 (written P1 ⇒ P2), then:

- P1 is stronger than P2
- P2 is weaker than P1



Whenever P1 holds, P2 also holds

- So it is more (or at least as) “difficult” to satisfy P1
  - the program states where P1 holds are a subset of the program states where P2 holds
- So P1 puts more constraints on program states
- So it is a stronger set of requirements on the program state
  - P1 gives you more information about the state than P2

Examples

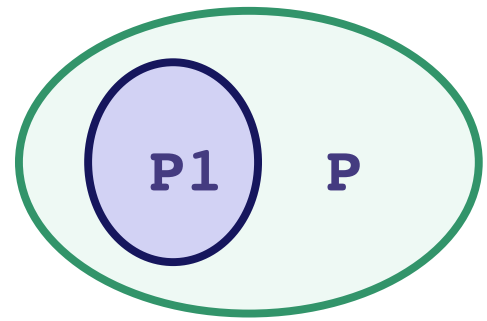
- **`x = 17`** is stronger than **`x > 0`**
- **`x is prime`** is neither stronger nor weaker than **`x is odd`**
- **`x is prime and x > 2`** is stronger than **`x is odd`**

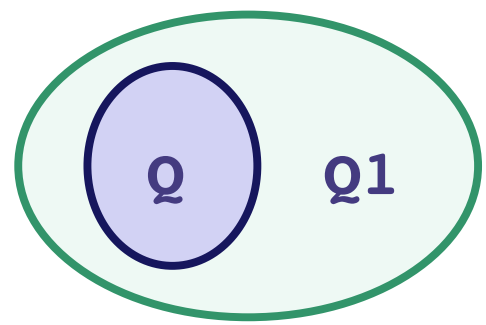
Floyd Logic Facts

- Suppose `{P} S {Q}` is valid.

- If `P1` is stronger than `P`,
  then `{P1} S {Q}` is valid.

- If `Q1` is weaker than `Q`,
  then `{P} S {Q1}` is valid.

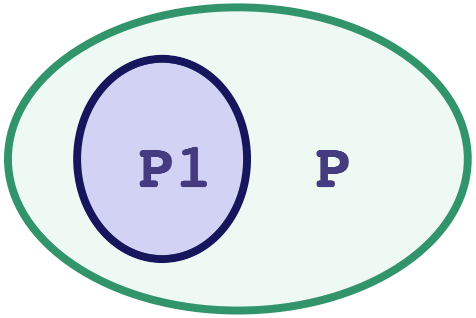


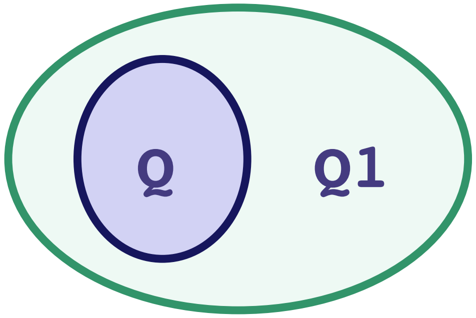


- Example:
  – Suppose `P` is x >= 0 and `P1` is x > 0
  – Suppose `Q` is y > 0 and `Q1` is y >= 0
  – Since {{ x >= 0 }} `y = x+1` {{ y > 0 }} is valid,
  {{ x > 0 }} `y = x+1` {{ y >= 0 }} is also valid

Floyd Logic Facts

- Suppose `{P} S {Q}` is valid.
- If `P1` is stronger than `P`,
  then `{P1} S {Q}` is valid.
- If `Q1` is weaker than `Q`,
  then `{P} S {Q1}` is valid.
- **Key points**:
  - always okay to **strengthen** a **precondition**
  - always okay to **weaken** a **postcondition**

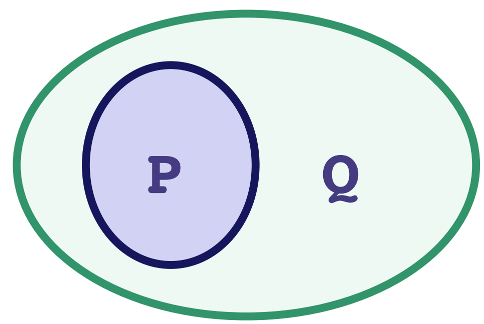




Floyd Logic Facts

- When is **`{P} ; {Q}`** is valid?
  - with no code in between

- Valid if any state satisfying P also satisfies Q
- I.e., if P is **stronger** than Q



Forward & Backward Reasoning

Example of Forward Reasoning

Work forward from the precondition

{{ w > 0 }}

```
x = 17;
```

{{ ________________________________ }}

```
y = 42;
```

{{ ________________________________ }}

```
z = w + x + y;
```

{{ ________________________________ }}

Example of Forward Reasoning

Work forward from the precondition

{{ w > 0 }}

```
x = 17;
```

{{ w > 0 and x = 17 }}

```
y = 42;
```

{{ ________________________________ }}

```
z = w + x + y;
```

{{ ________________________________ }}

Example of Forward Reasoning

Work forward from the precondition

{{ w > 0 }}

```
x = 17;
```

{{ w > 0 and x = 17 }}

```
y = 42;
```

{{ w > 0 and x = 17 and y = 42 }}

```
z = w + x + y;
```

{{ ________________________________ }}

Example of Forward Reasoning

Work forward from the precondition

{{ w > 0 }}

```
x = 17;
```

{{ w > 0 and x = 17 }}

```
y = 42;
```

{{ w > 0 and x = 17 and y = 42 }}

```
z = w + x + y;
```

{{ w > 0 and x = 17 and y = 42 and z = w + x + y }}

Example of Forward Reasoning

Work forward from the precondition

{{ w > 0 }}

```
x = 17;
```

{{ w > 0 and x = 17 }}

```
y = 42;
```

{{ w > 0 and x = 17 and y = 42 }}

```
z = w + x + y;
```

{{ w > 0 and x = 17 and y = 42 and z = w + 59 }}

Forward Reasoning

- Start with the **given** precondition
- Fill in the **strongest** postcondition

- For an assignment, **`x = y`**...
  - add the fact “x = y” to what is known
  - important <u>subtleties</u> here... (more on those later)

- Later: if statements and loops...

Example of Backward Reasoning

Work backward from the desired postcondition

{{ ____ }}

```
x = 17;
```

{{ ____ }}

```
y = 42;
```

{{ ____ }}

```
z = w + x + y;
```

{{ $z < 0$ }}

Example of Backward Reasoning

Work backward from the desired postcondition

{{ ________________________________ }}

```
x = 17;
```

{{ ________________________________ }}

```
y = 42;
```

{{ w + x + y < 0 }}

```
z = w + x + y;
```

{{ z < 0 }}

Example of Backward Reasoning

Work backward from the desired postcondition

{{ ________________________________ }}

```
x = 17;
```

{{ w + x + 42 < 0 }}

```
y = 42;
```

{{ w + x + y < 0 }}

```
z = w + x + y;
```

{{ z < 0 }}

Example of Backward Reasoning

Work backward from the desired postcondition

{{ w + 17 + 42 < 0 }}

```
x = 17;
```

{{ w + x + 42 < 0 }}

```
y = 42;
```

{{ w + x + y < 0 }}

```
z = w + x + y;
```

{{ z < 0 }}

Backward Reasoning

- Start with the **required** postcondition
- Fill in the **weakest** precondition

- For an assignment, **`x = y`**:
  - just replace “x” with “y” in the postcondition
  - if the condition using “y” holds beforehand, then the condition with “x” will afterward since x = y then

- Later: if statements and loops...

Correctness by Forward Reasoning

Use forward reasoning to determine if this code is correct:

{{ w > 0 }}

```
x = 17;
y = 42;
z = w + x + y;
```

{{ z > 50 }}

Example of Forward Reasoning

{{ w > 0 }}

```
x = 17;
```

{{ w > 0 and x=17 }}

```
y = 42;
```

{{ w > 0 and x=17 and y=42 }}

```
z = w + x + y;
```

{{ w > 0 and x=17 and y=42 and z = w + 59 }}

{{ z > 50 }}

Do the facts that are always true imply the facts we need?

I.e., is the bottom statement **weaker** than the top one?

(Recall that weakening the postcondition is always okay.)

Correctness by Backward Reasoning

Use backward reasoning to determine if this code is correct:

{{ w < -60 }}

```
x = 17;
y = 42;
z = w + x + y;
```

{{ z < 0 }}

Correctness by Backward Reasoning

Use backward reasoning to determine if this code is correct:

{{ w < -60 }}

{{ w + 17 + 42 < 0 }} ⟺ {{ w < -59 }}

```
x = 17;
```

{{ w + x + 42 < 0 }}

```
y = 42;
```

{{ w + x + y < 0 }}

```
z = w + x + y;
```

{{ z < 0 }}

Do the facts that are always true imply the facts we need?

I.e., is the top statement **stronger** than the bottom one?

(Recall that strengthening the precondition is always okay.)

Combining Forward & Backward

It is okay to use both types of reasoning

- Reason forward from precondition
- Reason backward from postcondition

Will meet in the middle:

{{ P }}
```
S1
S2
```
{{ Q }}

Combining Forward & Backward

It is okay to use both types of reasoning

- Reason forward from precondition
- Reason backward from postcondition

Will meet in the middle:

{{ P }}
  `S1`
{{ P1 }}
                    Valid provided P1 implies Q1
{{ Q1 }}
  `S2`
{{ Q }}

Combining Forward & Backward

Reasoning in either direction gives valid assertions

Just need to check adjacent assertions:

- top assertion must imply bottom one

{{ P }}	{{ P }}	{{ P }}
`S1`	{{ Q1 }}	`S1`
`S2`	`S1`	{{ P1 }}
{{ P1 }}	`S2`	{{ Q1 }}
{{ Q }}	{{ Q }}	`S2`
		{{ Q }}