CSE 331
Software Design & Implementation

Spring 2020

Section 1 – Code Reasoning

Administrivia

- HW1 due next Monday.

- Any questions before we dive in?
  – What are the most interesting/confusing/puzzling things so far in the course?

Agenda

- Introductions?
- Review logical reasoning about code with Hoare Logic
- Practice both forward and backward modes
  - Just assignment, conditional (“if-then-else”), and sequence
  - Logical rules from yesterday’s lecture/notes
- Review logical strength of assertions (weaker *vs.* stronger)
- Practice determining stronger/weaker assertions

Why reason about code?

- Prove that code is correct
- Understand *why* code is correct
- Diagnose why/how code is *not* correct
- Specify code behavior

Logical reasoning about code

- Determine facts that hold of program state between statements
  - "Fact" ~ assertion (logical formula over program state, informally "value(s) of some/all program variables)
  - Driven by assumption (precondition) or goal (postconditon)

- Forward reasoning
  - What facts follow from initial assumptions?
  - Go from <u>pre</u>condition to <u>post</u>condition

- Backward reasoning
  - What facts need to be true to reach a goal?
  - Go from <u>post</u>condition to <u>pre</u>condition

Hoare Logic: Validity by Reasoning

- Checking validity of **`{P} S {Q}`**
  - Valid iff, starting from any state satisfying ***P***, executing ***S*** results in a state satisfying ***Q***

- Forward reasoning:
  - Reason from ***P*** to strongest postcondition **`{P} S {R}`**
  - Check that ***R*** implies ***Q*** (i.e., ***Q*** is weaker)

- Backward reasoning:
  - Reason from ***Q*** to get weakest precondition **`{R} S {Q}`**
  - Check that ***P*** implies ***R*** (i.e., ***P*** is stronger)

Implication (=>)

- Logic formulas with *and* (&, &&, or ∧), *or* (|, ||, or ∨) and *not* (! or ¬) have the same meaning they do in programs

- Implication might be a bit new, but the basic idea is pretty simple. Implication p=>q is true as long as q is always true whenever p is

p	q	p => q
T	T	T
T	F	F
F	T	T
F	F	T

Assignment Statements

- Reasoning about **`x = y;`**
- Forward reasoning:
  - add "x = y" as a new fact
  - (also rewrite any existing references to "x" to use new value)
- Backward reasoning:
  - replace all instances of "x" in the postcondition with "y"

Conditionals, more closely

Forward reasoning

$\{P\}$
if (b)
    $\{P \wedge b\}$
    S_1
    $\{Q_1\}$
else
    $\{P \wedge !b\}$
    S_2
    $\{Q_2\}$
$\{Q_1 \vee Q_2\}$

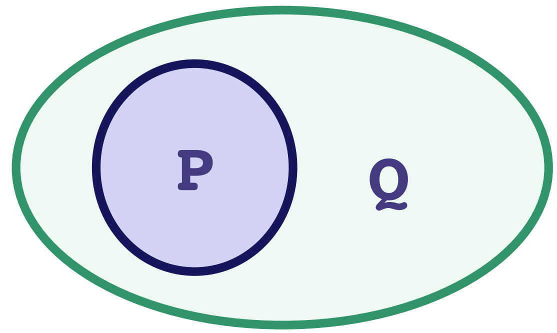
Backward reasoning

$\{ (b \wedge P_1) \vee (!b \wedge P_2) \}$
if (b)
    $\{P_1\}$
    S_1
    $\{Q\}$
else
    $\{P_2\}$
    S_2
    $\{Q\}$
$\{Q\}$

Weaker vs. stronger

Formal definition:

- If $P \Rightarrow Q$, then
  - *Q* is weaker than *P*
  - *P* is stronger than *Q*



Intuitive definition:

- "Weak" means unrestrictive; a weaker assertion has a larger set of possible program states (*e.g.*, **`x != 0`**)
- "Strong" means restrictive; a stronger assertion has a smaller set of possible program states (*e.g.*, **`x = 1`** or **`x > 0`** are both stronger than **`x != 0`**).

Worksheet

- Take ~10 minutes to get where you can
- Find a partner and work with them
- Let me know if you feel stuck
- We'll walk through some solutions afterwards

Worksheet – problem 2

```
{ true }
if (x>0) {
  { x > 0 }
  y = 2*x;
  { x > 0 ∧ y = 2x }
} else {
  { x <= 0 }
  y = -2*x;
  { x <= 0 ∧ y = -2x }
}
{ (x > 0 ∧ y = 2x) ∨ (x <= 0 ∧ y = -2x) }
⇒ { y = 2|x| }
```

Worksheet – problem 4

```
{ y > 15 ∨ (y <= 5 ∧ y + z > 17) }
if (y > 5) {
  { y > 15 }
  x = y + 2
  { x > 17 }
} else {
  { y + z > 17 }
  x = y + z;
  { x > 17 }
}
{ x > 17 }
```

Worksheet – problem 6 (forward)

```
{ true }
if (x < y) {
  { true ∧ x < y }
  m = x;
  { x < y ∧ m = x }
} else {
  { true ∧ x >= y }
  m = y;
  { x >= y ∧ m = y }
}
{ (x < y ∧ m = x) ∨ (x >= y ∧ m = y) }
⇒ { m = min(x, y) }
```

Worksheet – problem 6 (backward)

```
{ true } ⇔
{ (x <= y ∧ x < y) ∨ (y <= x ∧ x >= y) }
if (x < y) {
  { x = min(x, y) } ⇔ { x <= y }
  m = x;
  { m = min(x, y) }
} else {
  { y = min(x, y) } ⇔ { x >= y }
  m = y;
  { m = min(x, y) }
}
{ m = min(x, y) }
```

Worksheet – problem 7

```
{ y > 23 }                    { y >= 23 }

{ y = 23 }                    { y >= 23 }

{ y < 0.23 }                  { y < 0.00023 }

{ x = y * z }                 { y = x / z }

{ is_prime(y) }               { is_odd(y) }
```

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`		`{ y >= 23 }`
`{ y < 0.23 }`		`{ y < 0.00023 }`
`{ x = y * z }`		`{ y = x / z }`
`{ is_prime(y) }`		`{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`	is stronger than	`{ y >= 23 }`
`{ y < 0.23 }`		`{ y < 0.00023 }`
`{ x = y * z }`		`{ y = x / z }`
`{ is_prime(y) }`		`{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }` is stronger than `{ y >= 23 }`

`{ y = 23 }` is stronger than `{ y >= 23 }`

`{ y < 0.23 }` is weaker than `{ y < 0.00023 }`

`{ x = y * z }` `{ y = x / z }`

`{ is_prime(y) }` `{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }` is stronger than `{ y >= 23 }`

`{ y = 23 }` is stronger than `{ y >= 23 }`

`{ y < 0.23 }` is weaker than `{ y < 0.00023 }`

`{ x = y * z }` is **incomparable** with `{ y = x / z }`

`{ is_prime(y) }` `{ is_odd(y) }`

Worksheet – problem 7

`{ y > 23 }`	is stronger than	`{ y >= 23 }`
`{ y = 23 }`	is stronger than	`{ y >= 23 }`
`{ y < 0.23 }`	is weaker than	`{ y < 0.00023 }`
`{ x = y * z }`	is **incomparable** with	`{ y = x / z }`
`{ is_prime(y) }`	is **incomparable** with	`{ is_odd(y) }`

Questions?

- What is the most surprising thing about this?
- What is the most confusing thing?
- What will need a bit more thinking to digest?