Warmup

A programmer's roommate tells him, "Would you mind going to the store and picking up a loaf of bread. Also, if they have eggs, get a dozen."

The programmer returns with 12 loaves of bread.

Section 3:
HW4, ADTs, and more

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Agenda

Announcements

- HW3: due tonight at 11pm

Polynomial arithmetic

Abstract data types (ADT)

Representation invariants (RI)

Abstraction Functions

HW4: Polynomial Graphing Calculator

Problem 0: Write pseudocode algorithms for polynomial operations

Problem 1: Answer questions about RatNum

Problem 2: Implement RatTerm

Problem 3: Implement RatPoly

Problem 4: Implement RatPolyStack

Problem 5: Try out the calculator

RatThings

RatNum
- ADT for a Rational Number
- Has NaN

RatTerm
- Single polynomial term
- Coefficient (RatNum) & degree

RatPoly
- Sum of RatTerms

RatPolyStack
- Ordered collection of RatPolys

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

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$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrr} & 5x^4 & + 4x^3 & - x^2 & & + 5 \\ + & 3x^5 & - 2x^3 & + x & - 5 & \end{array}$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr} & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\ + & 3x^5 & 0x^4 & - 2x^3 & 0x^2 + x & - 5 \end{array}$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrrrr} & 5x^4 & + & 4x^3 & - & x^2 & 0x & + & 5 \\ + & 3x^5 & 0x^4 & - & 2x^3 & 0x^2 & + x & - & 5 \\ \hline & 3x^5 & + & 5x^4 & + & 2x^3 & - x^2 & + x & + 0 \end{array}$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$5x^4 + 4x^3 - x^2 + 5$$

$$- 3x^5 - 2x^3 + x - 5$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrrrr} & 5x^4 & + & 4x^3 & - & x^2 & 0x & + & 5 \\ - & 3x^5 & 0x^4 & - & 2x^3 & 0x^2 & + & x & - 5 \end{array}$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrrrr} & 5x^4 & + & 4x^3 & - & x^2 & 0x & + & 5 \\ - & 3x^5 & 0x^4 & - & 2x^3 & 0x^2 & + x & - & 5 \\ \hline -3x^5 & + & 5x^4 & + & 6x^3 & - & x^2 & - x & + 10 \end{array}$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$4x^3 - x^2 + 5$

$*$ $x - 5$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rr}
 & 4x^3 - x^2 + 5 \\
* & x - 5 \\
\hline
 & -20x^3 + 5x^2 \qquad - 25
\end{array}
$$

Polynomial Multiplication

$$(4x^3 - x^2 + 5) * (x - 5)$$

$$4x^3 - x^2 + 5$$

$$* \qquad x - 5$$

$$-20x^3 + 5x^2 - 25$$

$$4x^4 - x^3 + 5x$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rr}
 & 4x^3 - x^2 + 5 \\
* & x - 5 \\
\hline
 & -20x^3 + 5x^2 \quad - 25 \\
+ & 4x^4 \quad -x^3 \quad + 5x \\
\hline
 & \mathbf{4x^4 - 21x^3 + 5x^2 + 5x - 25}
\end{array}
$$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

$x^3 - 2x - 5 \overline{)\ 5x^6 + 4x^4 - x^3 + 5}$

Poly Division

1	0	-2	-5	5	0	4	-1	0	0	5

Poly Division

$$\begin{array}{r|l} & 5 \\ \hline 1 \quad 0 \quad -2 \quad -5 & 5 \quad 0 \quad 4 \quad -1 \quad 0 \quad 0 \quad 5 \end{array}$$

Poly Division

$$
\begin{array}{r|l}
1\ \ 0\ -2\ -5 & \ \ 5\ \ 0\ \ 4\ \ -1\ \ 0\ \ 0\ \ 5 \\
 & \ \ 5\ \ 0{-}10\ -25 \\
\hline
 & 5
\end{array}
$$

Poly Division

```
                                  5
 1  0 -2 -5 |  5   0   4   -1   0   0   5
               5   0-10  -25
               0   0  14   24
```

Poly Division

```
                              5
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              ----------------
              0  0 14  24
                   14  24 0
```

Poly Division

```
                  5  0
1  0 -2 -5 | 5  0  4  -1  0  0  5
           | 5  0-10 -25
           ---------------------
           | 0  0 14  24
           |       14  24 0
```

Poly Division

```
                               5  0
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              0  0 14  24
                   14  24 0
                   14  24 0  0
```

Poly Division

```
                  5  0 14
              ┌──────────────────
1  0 -2 -5    │ 5  0  4  -1  0  0  5
              │ 5  0-10 -25
              ├──────────────────
              0  0 14  24
                   14  24 0
                   14  24 0  0
```

Poly Division

```
                              5   0 14
1  0 -2 -5  |  5   0   4   -1   0   0   5
               5   0-10 -25
               ----------------
               0   0 14   24
                     14   24 0
                     14   24 0   0
                     14   0 -28 -70
```

Poly Division

```
                         5   0 14
1  0 -2 -5  |  5   0   4   -1   0   0   5
               5   0-10 -25
               0   0 14   24
                     14   24 0
                     14   24 0   0
                     14   0 -28 -70
                     0   24   28   70
```

Poly Division

```
                                      5   0  14
1   0  -2  -5   |   5   0   4   -1   0   0   5
                    5   0 -10  -25
                    -------------------
                    0   0  14   24
                           14   24  0
                           14   24  0   0
                           14    0 -28 -70
                           ----------------------
                           0   24   28   70
                               24   28   70  5
```

Poly Division

```
                    5   0 14 24
          ┌──────────────────────
1  0 -2 -5 │ 5  0  4  -1  0  0  5
           5  0-10 -25
          ──────────────
           0  0 14  24
              14  24 0
              14  24 0  0
              14  0 -28 -70
             ──────────────
              0  24  28  70
                 24  28  70 5
                 24  0  -48 -120
```

Poly Division

5 0 14 24

1 0 -2 -5 | 5 0 4 -1 0 0 5

5 0-10 -25

0 0 14 24

14 24 0

14 24 0 0

14 0 -28 -70

0 24 28 70

24 28 70 5

24 0 -48 -120

0 28 118 125

Poly Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

$5x^3 + 14x + 24$

Poly Division

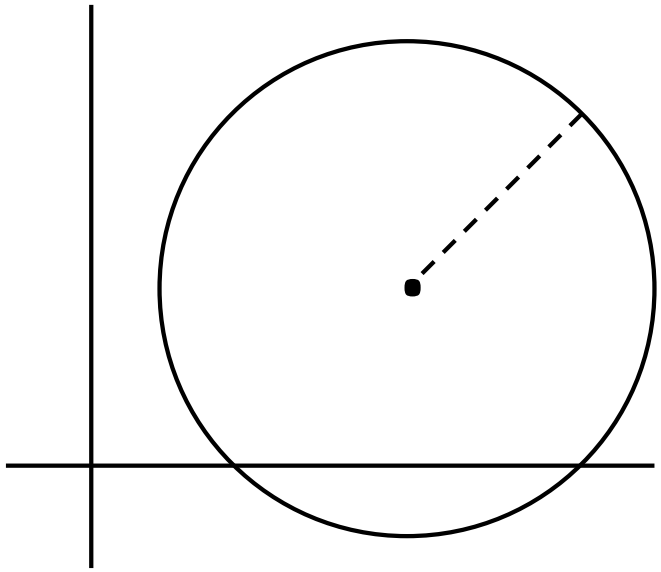
$$(5x^6 + 4x^4 - x^3 + 5) \;/\; (x^3 - 2x - 5)$$

$$5x^3 + 14x + 24 + \frac{28x^2 + 118x + 125}{x^3 - 2x - 5}$$

If none of that made sense…

ADT Example: Circle

Circle on the Cartesian coordinate plane



Circle: Class Specification

What represents the abstract state of a Circle?

What are some properties of a circle we can determine?

How can we implement this?

What are some ways to “break” a circle?

Representation Invariants

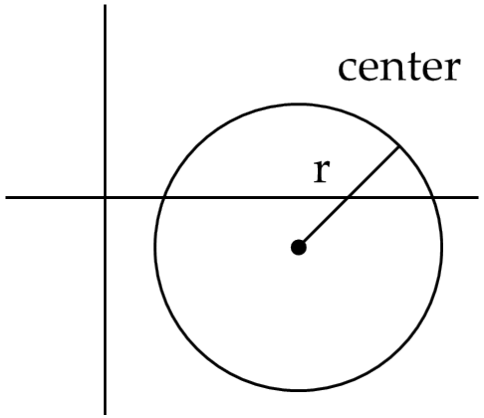
Constrains an object's internal state

Maps concrete representation of object to a boolean

If representation invariant is false/violated, the object is "broken" – doesn't map to any abstract value

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Rep invariant:
        //

        //  ...
}
```

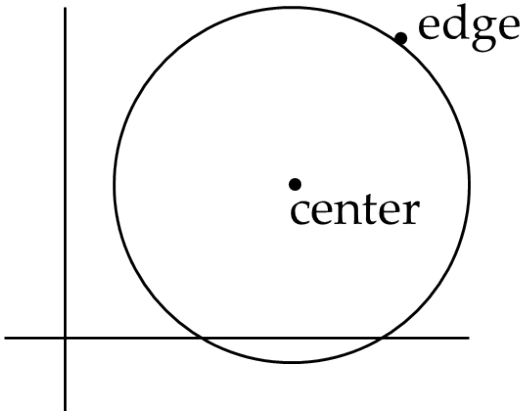


Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Rep invariant:
        //

        //    ...
}
```

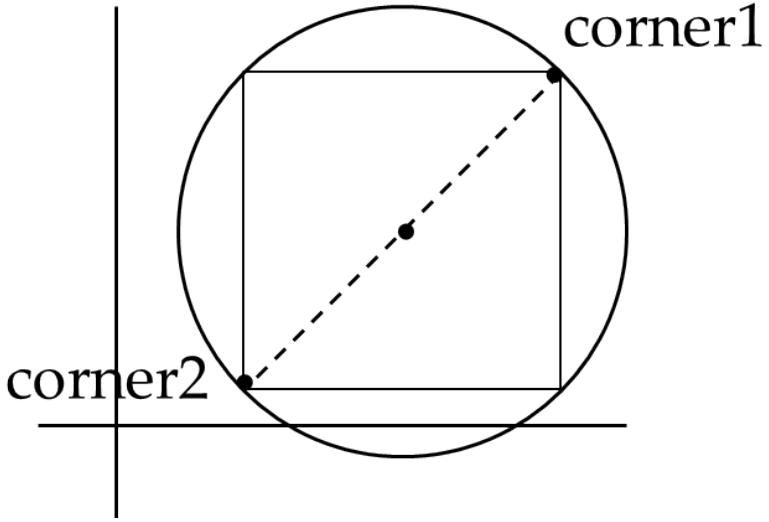


Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Rep invariant:
        // center != null &&
       // edge != null &&
       // !center.equals(edge)
        //     ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Rep invariant:
        //


        //     ...
}
```



Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Rep invariant:
        // corner1 != null &&
       // corner2 != null &&
       // !corner1.equals(corner2)
        //     ...
}
```

Checking Rep Invariants

- Representation invariant should hold before and after every public method

Write and use `checkRep()`

- Call before and after public methods
- Make use of Java's assert syntax!
- OK that it adds extra code
  - Asserts won't be included on release builds
  - Important for finding bugs

checkRep() Example with Asserts

```
public class Circle1 {

        private Point center;

        private double rad;


        private void checkRep() {

                assert center != null : “This does not have a
                                        center”;

                assert radius > 0 : “This circle has a negative
                                    radius”;

        }

}
```

Using Asserts

To enable asserts: Go to Run->Run Configurations…->Arguments. Then put **–ea** in VM arguments section

- Do this for every main class

RatNum…

Abstraction Function

Abstraction function: a **mapping** from **internal state** to **abstract value**

Abstract fields may not map directly to representation fields

- Circle has **radius** but not necessarily

```
private int radius;
```

Internal representation can be anything as long as it somehow encodes the abstract value

Representation Invariant excludes values for which the abstraction function has no meaning

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center =
        //     c.radius =

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center = this.center
        //     c.radius = this.rad

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center =
        //      c.radius =

        // Rep invariant:
        // center != null && edge ! null &&
                                              !center.equals(edge)
       //    ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = this.center
        //      c.radius = sqrt((center.x-edge.x)^2 +
                                    (center.y-edge.y)^2)

        // Rep invariant:
        // center != null && edge ! null &&
                                     !center.equals(edge)
        //     ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center =
        //      c.radius =

        // Rep invariant:
        // corner1 != null && corner2 != null &&
                                        !corner1.equals(corner2)

        //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = <(corner1.x + corner2.x) / 2,
                                (corner.y + corner2.y) / 2>

        //      c.radius = (1/2)*sqrt((corner1.x-corner2.x)^2 +
                                (corner1.y-corner2.y)^2)

        // Rep invariant:
        // corner1 != null && corner2 != null &&
                                !corner1.equals(corner2)

        //    ...
}
```