CSE 331
Software Design and Implementation

Lecture 7
Abstraction Functions

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Announcements

Announcements

- HW2 due tonight 10 pm
- Wednesday, July 4 is Independence Day
  - No lecture

- Section Thursday, July 5
- HW3 due Thursday, July 5 at 10 pm
  - Seek HW3 help on Tuesday; no office hours Wednesday!
- Reading 3 posted on website
  - Quiz 3 (coming soon!) due Thursday, July 5 at 10 pm

Motivation

Review

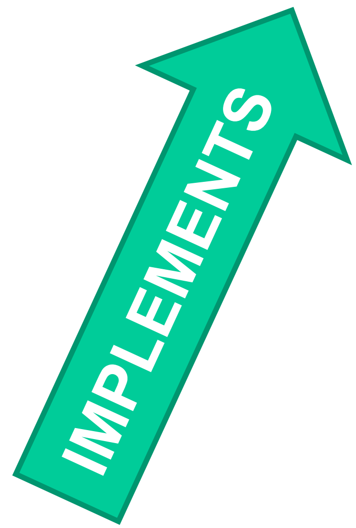
lec04

Method Specification (abstraction)

Method Body
`(concrete code)`

lec05

Abstract Data Type (abstraction)



Data Structure
`(concrete code)`

Example: CharSet Abstraction

```
// Overview: A CharSet is a finite mutable set of Characters

// @effects: creates a fresh, empty CharSet
public CharSet() {…}

// @modifies: this
// @effects: this_post = this_pre + {c}
public void insert(Character c) {…}

// @modifies: this
// @effects: this_post = this_pre - {c}
public void delete(Character c) {…}

// @return: (c ∈ this)
public boolean member(Character c) {…}

// @return: cardinality of this
public int size() {…}
```

set – see Wolfram Alpha definition

set union

set difference

Informal notation warning

Charset Representation Invariant

```
class CharSet {
    // Rep invariant:
    //   this.elts has no nulls and no duplicates
    private List<Character> elts = …
    …
}
```

Rep inv. constrains structure, not meaning

An implementation of **`insert`** that preserves the rep invariant:

```
public void insert(Character c) {
  Character cc = new Character(encrypt(c));
  if (!elts.contains(cc))
    elts.addElement(cc);
}
public boolean member(Char
  return elts.contains(c)
}
```

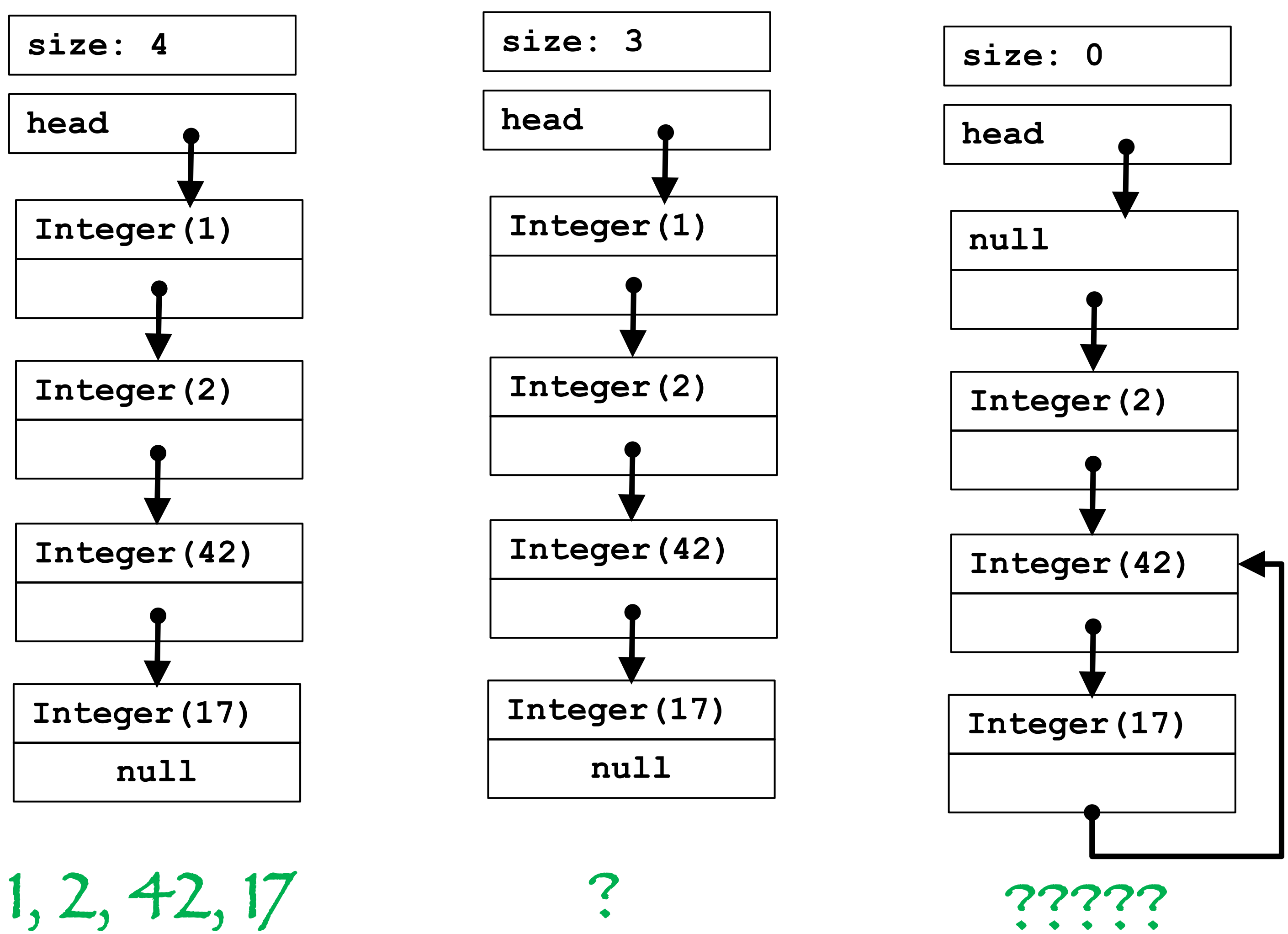
```
CharSet s = new CharSet();
s.insert('a');
if (s.member('a'))
    …
```

Program is wrong

- Clients observe incorrect behavior
- What client code exposes the error?
- Where is the error?
- We must consider the *meaning*
- The *abstraction function* helps us

An ADT has an abstract value

Abstract Value: An Int List is a finite sequence of integer values



Connecting implementations to specs

Representation Invariant: maps Object → boolean

lec06

- Indicates if an instance is *well-formed*
- Defines the set of valid concrete values
- Only values in the valid set make sense as implementations of an abstract value
- **For implementors/debuggers/maintainers of the abstraction: no object should *ever* violate the rep invariant**
  - Such an object has no useful meaning

Abstraction Function: maps Object → abstract value

lec07 (today)

- What the data structure *means* as an abstract value
- How the data structure is to be interpreted
- Only defined on objects meeting the rep invariant
- **For implementors/debuggers/maintainers of the abstraction:** Each procedure should meet its spec (abstract values) by "doing the right thing" with the concrete representation

Functions

Set

- An unordered collection of objects

  S = {3, 1, 2, mouse}

- An object can be in the set or not

  $3 \in S$ $-1 \notin S$

∈ = "elelment of"

- Set builder notation

  T = {x | x ∈ S and x is an integer}

  = {2, 1, 3}

| = "such that"

- Some familiar sets

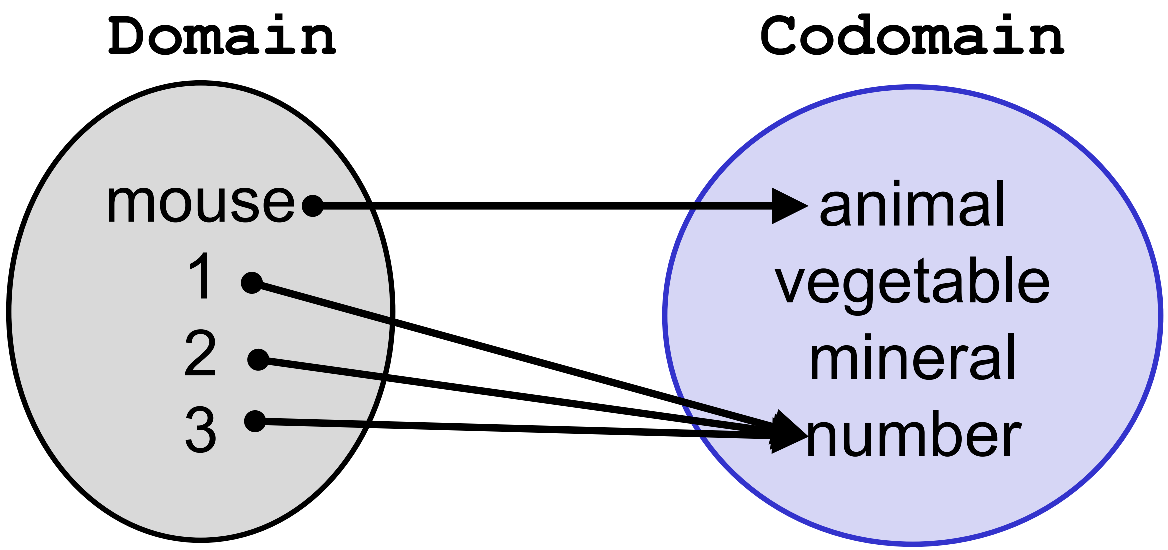
  $\mathbb{Z}$ **= {...-1, 0, 1, 2, ...}** "the integers"

  $\mathbb{Q}$ **= {p/q | p, q ∈** $\mathbb{Z}$ **}** "the rational numbers"

Function

- A relation that uniquely associates members of one set with members of another set [Wolfram]

  F : S → Y "F maps S to Y"

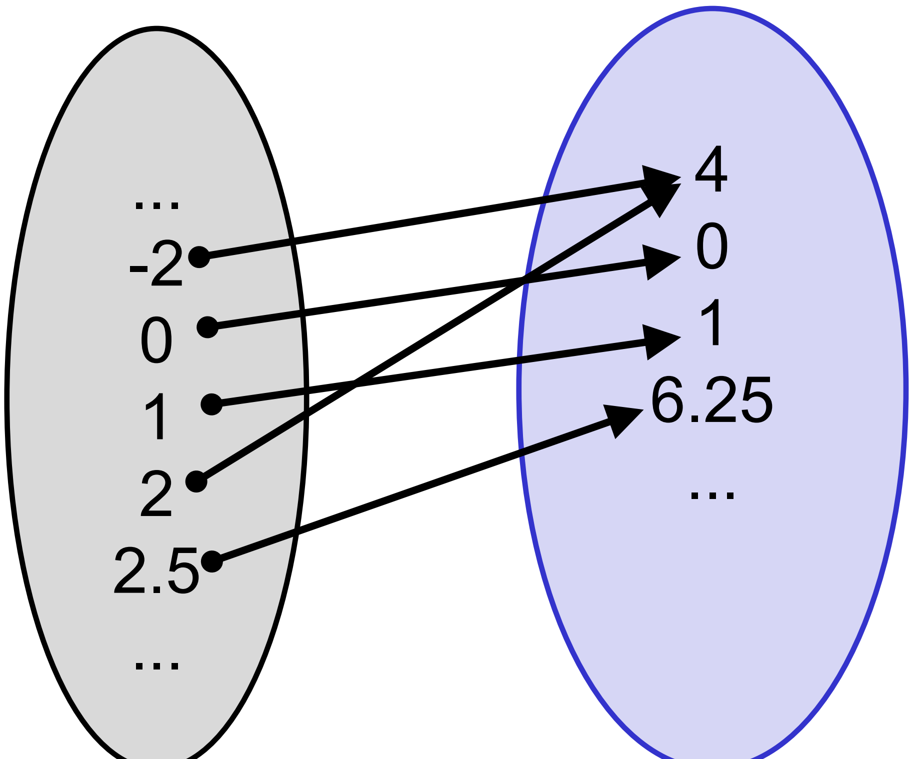


Range: {animal, number}

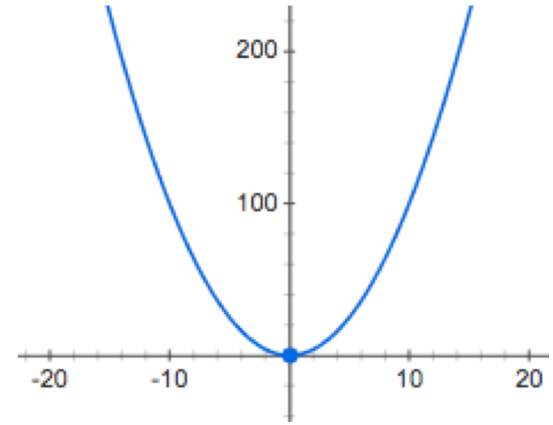
Example Function

$F : \mathbb{R} \rightarrow \mathbb{R}$

$F(x) = x^2$



$F(x) = x^2$

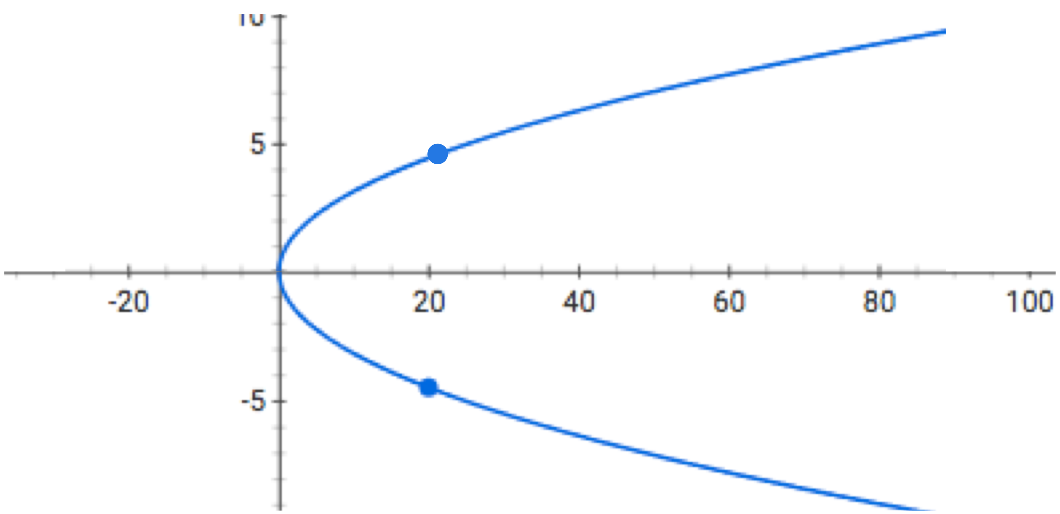


passes vertical line test

Example NOT Function

Inverse of $F(x) = x^2$

`y = ± sqrt(x)`



`sqrt(25) = 5`
`sqrt(25) = -5`

Does not pass vertical line test – Not a function!

Functions in Math and Programming

- In programming, the term “function” is often loosely used
- Related to the concepts of “method” and “subroutine”

```
float square(float x) {
  return x * x;
}
```

This method implements a mathematical function

```
void greet(String name) {
  System.out.println("Hello, " + name);
}
```

This method does not implement a mathematical function

Abstraction Functions

Abstraction Function

The abstraction function maps concrete representations to the abstract values they represent

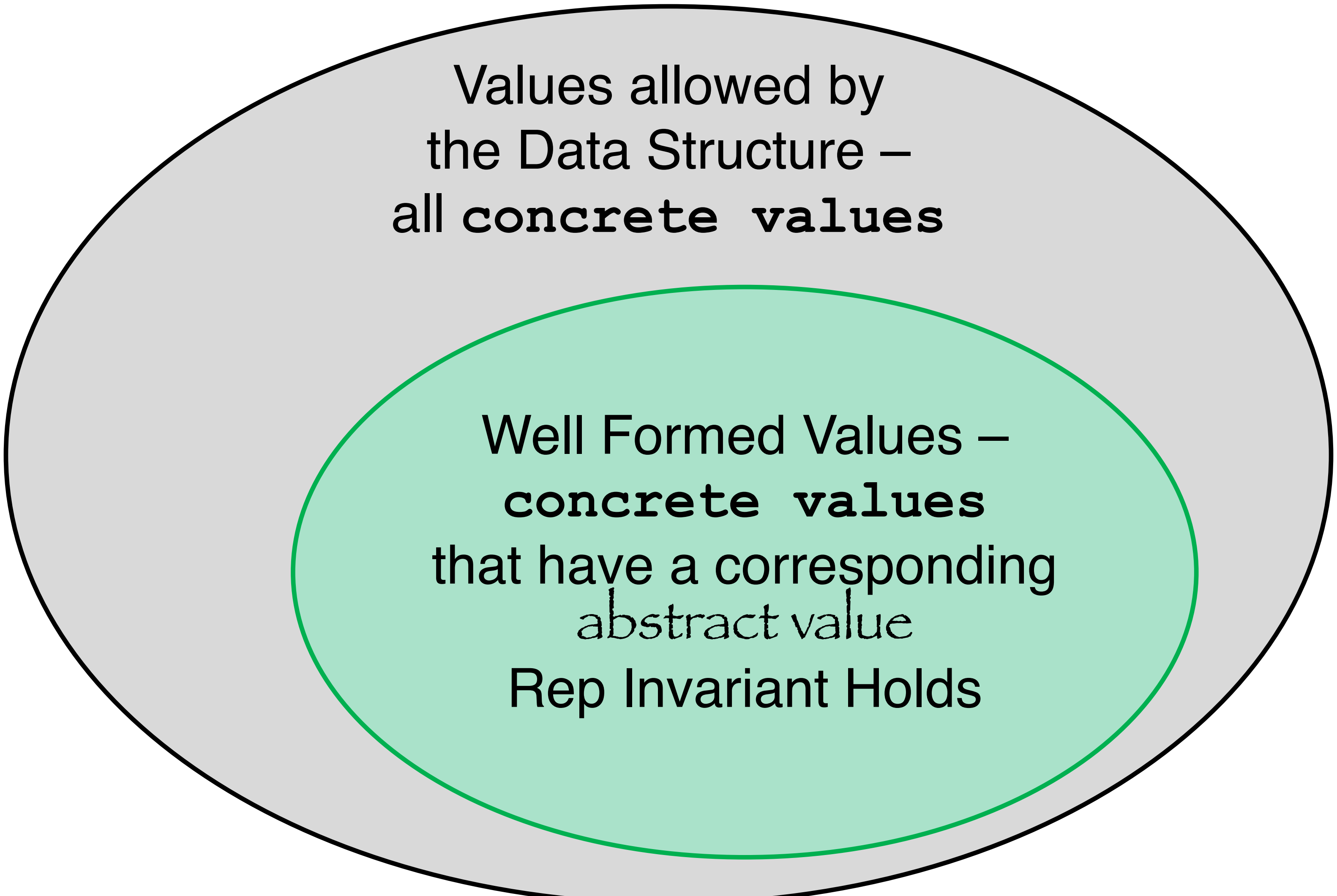
$$AF: \text{concrete rep} \rightarrow \text{abstract value}$$

AF(CharSet this) = { c | c is contained in this.elts }

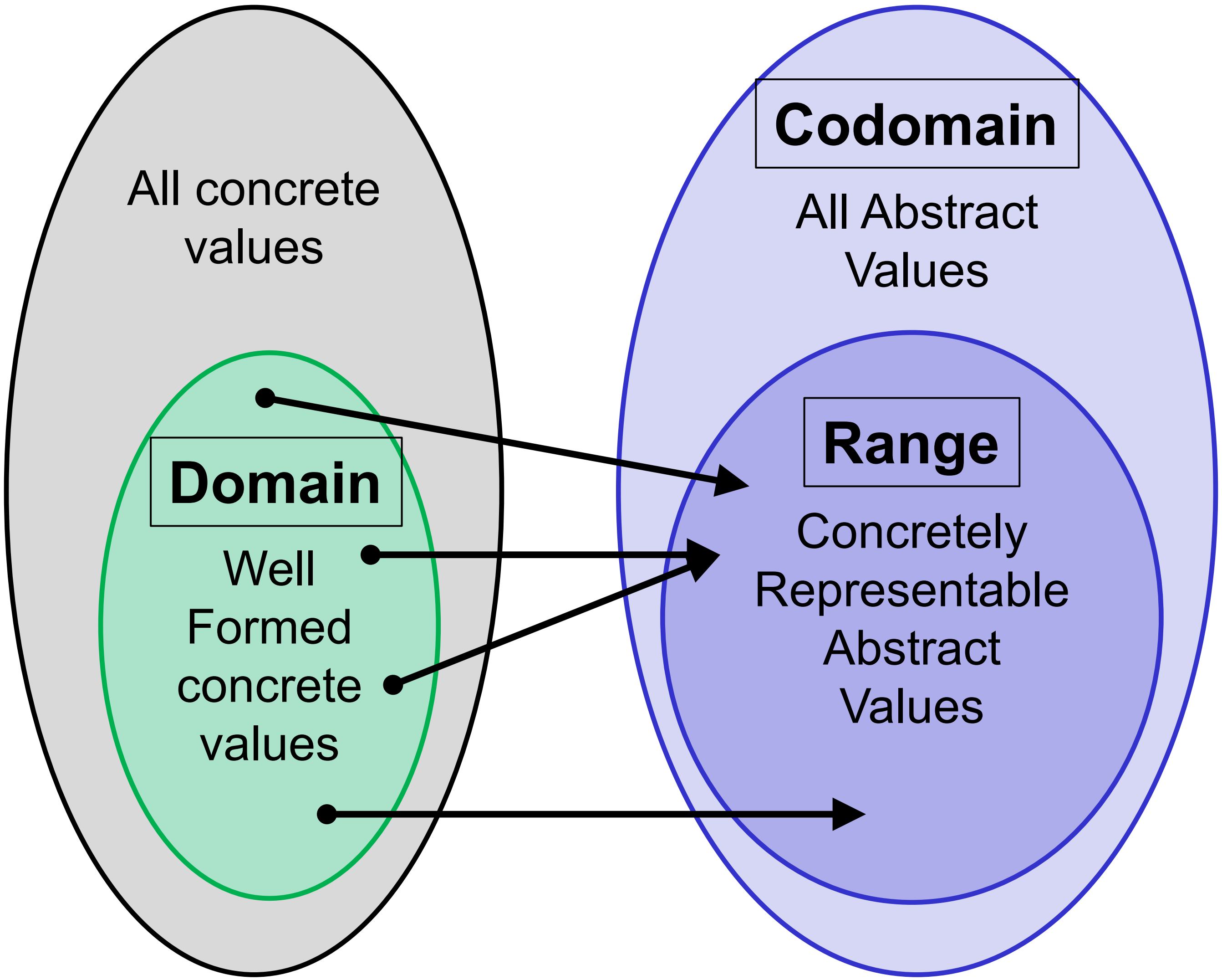
“set of Characters contained in this.elts”

- The abstraction function lets us reason about what [concrete] methods do in terms of the clients’ [abstract] view
  - Makes sure that all methods use the rep in the same way
- Math concept of function, not programming concept of function
  - AF not implementable in code since range is abstract values

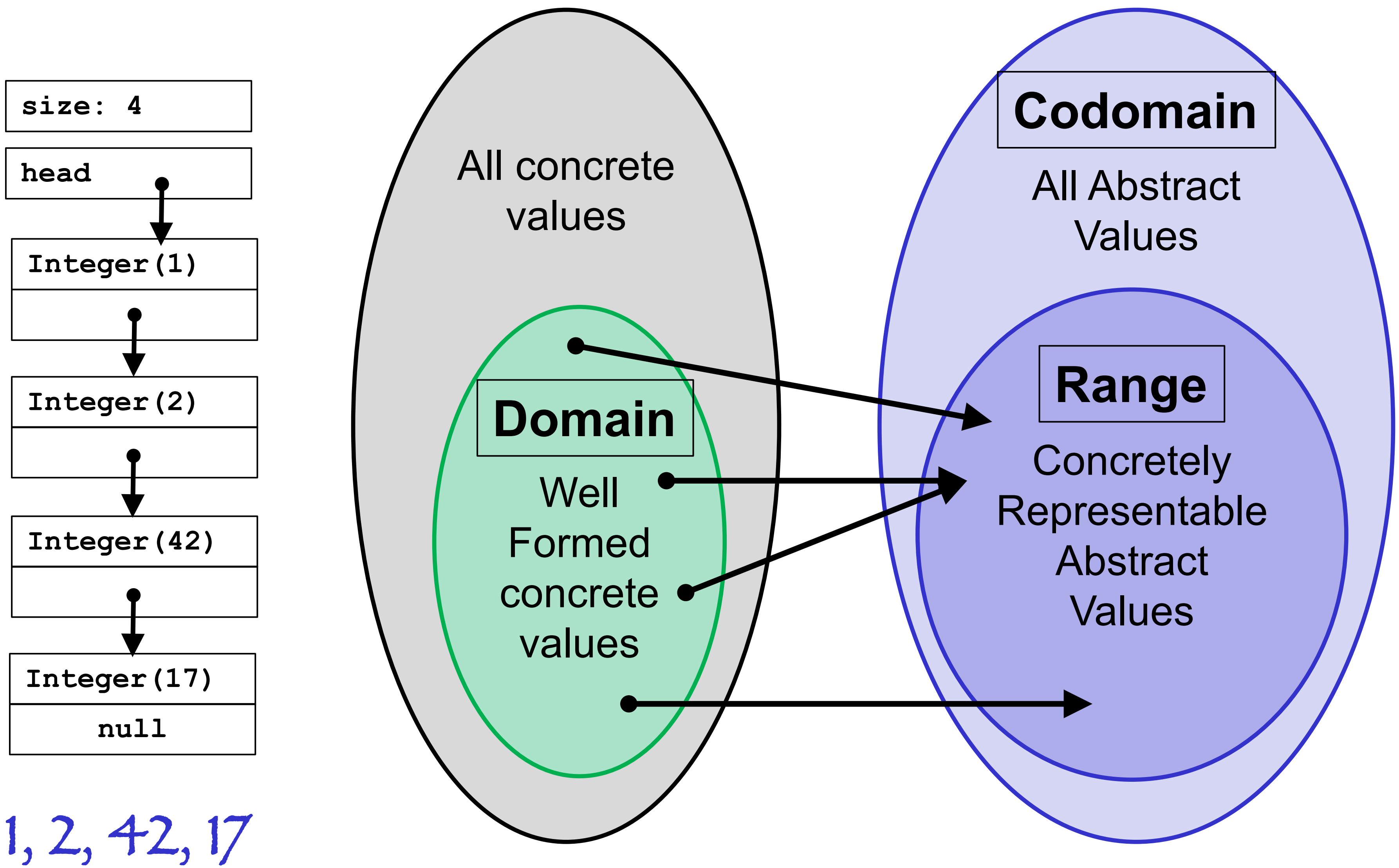
Abstraction Function



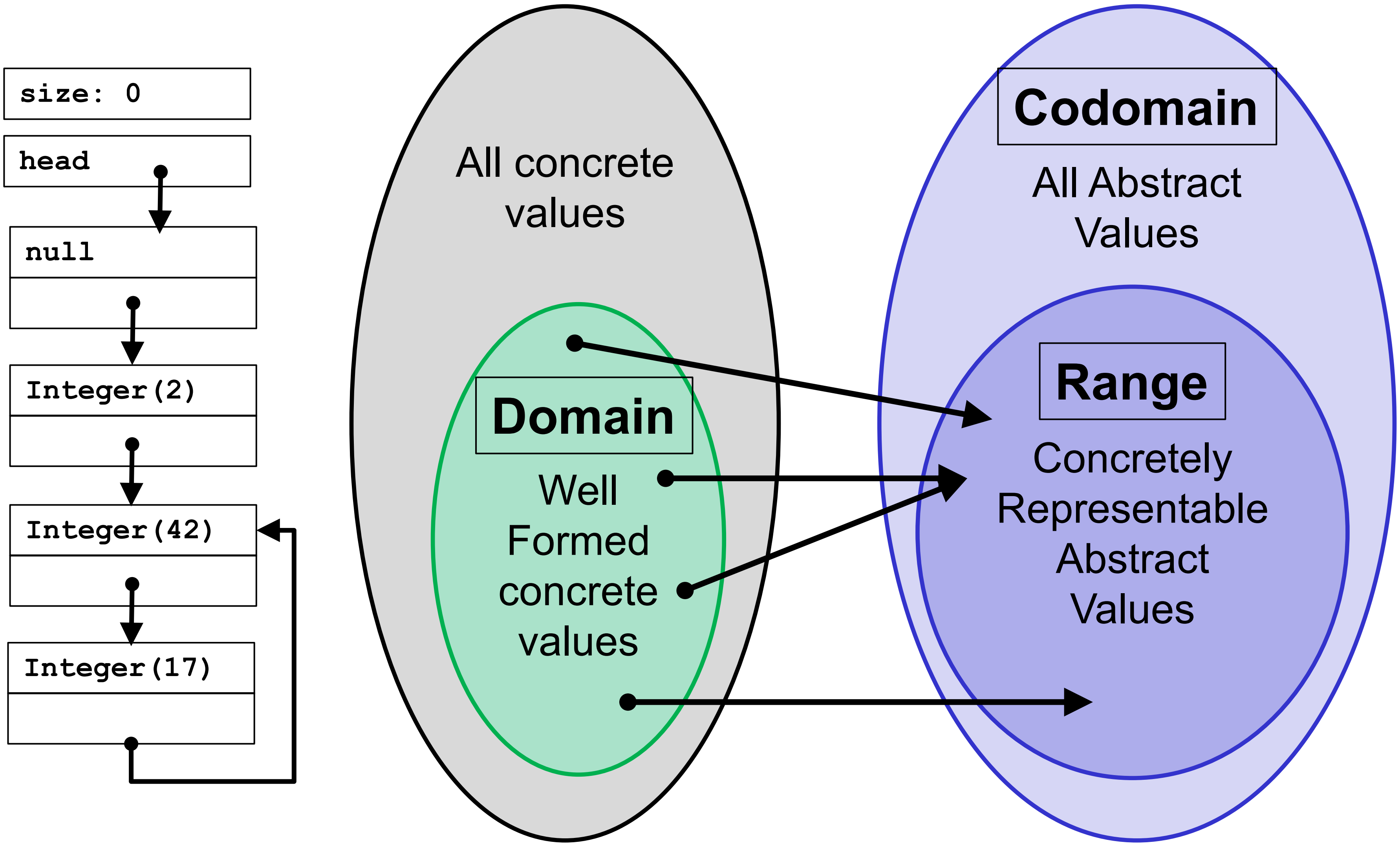
Abstraction Function



Abstraction Function

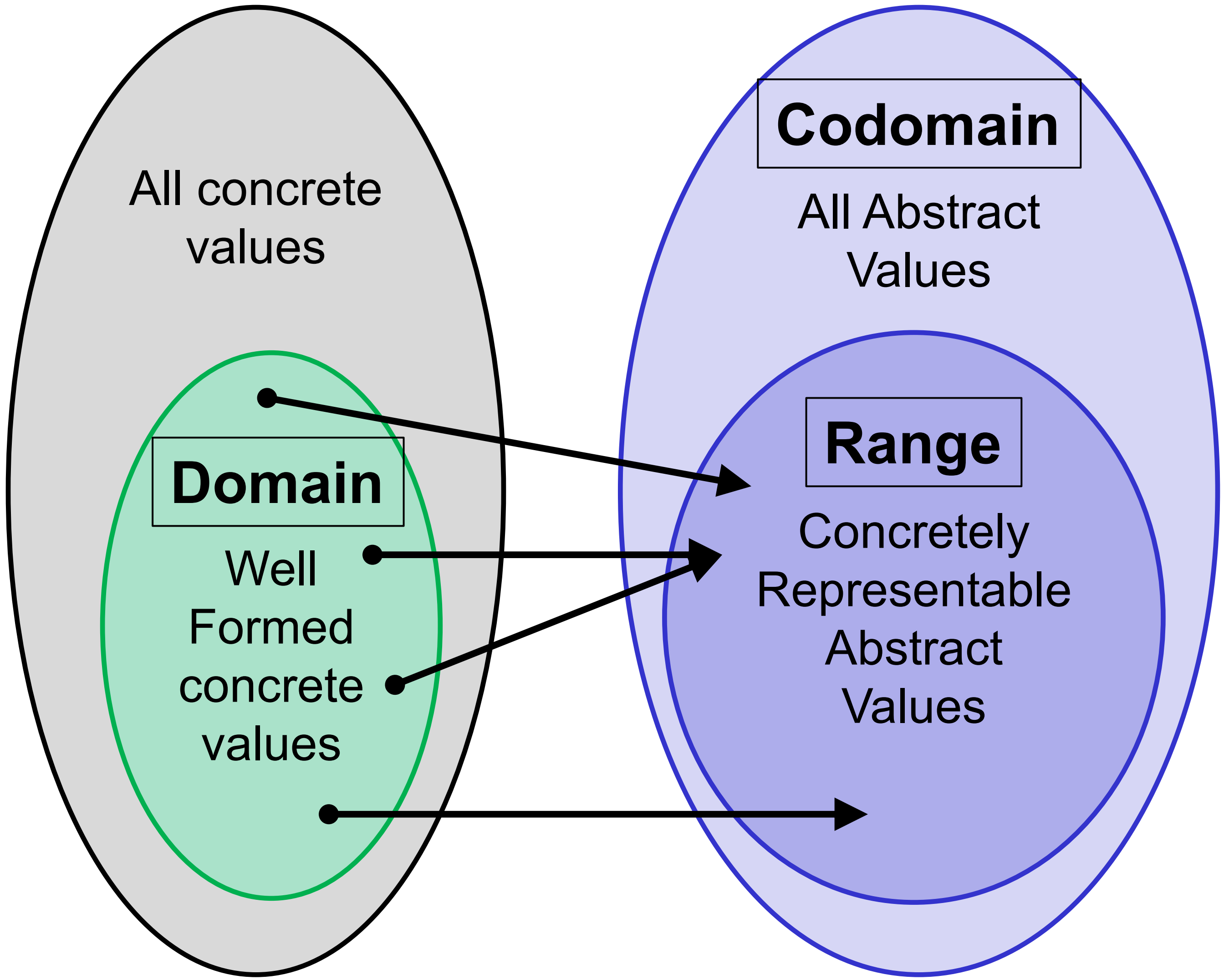


Abstraction Function



Abstraction Function

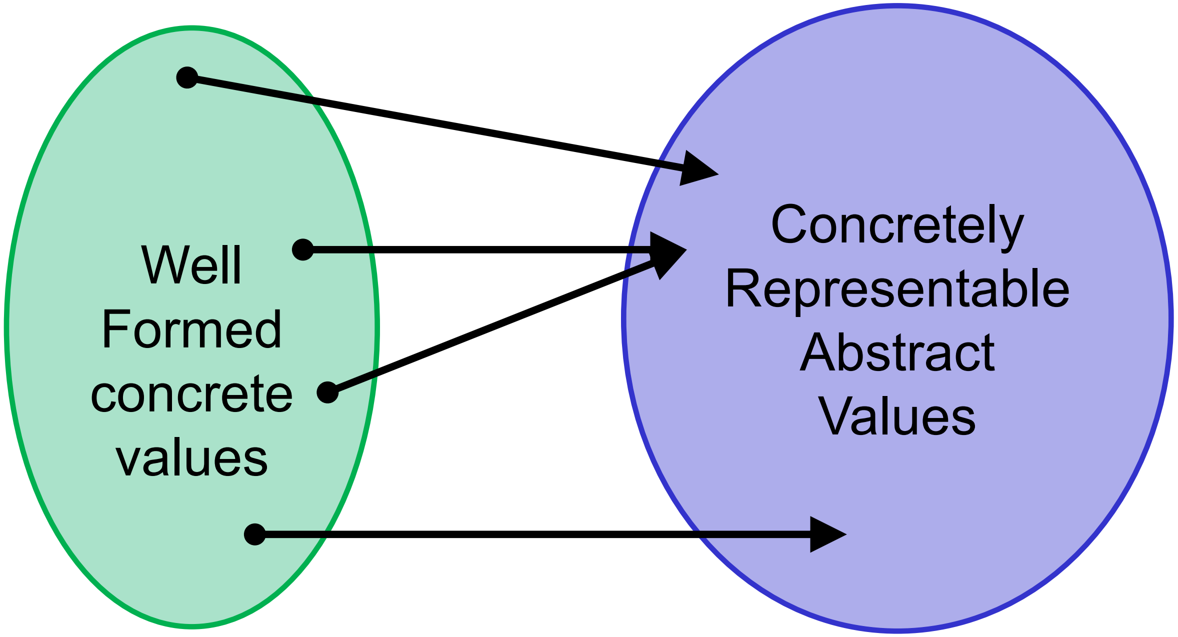
0,
1,
$2^{10,000}$



Summary so far:

The abstraction function maps concrete representations to the abstract values they represent

AF: concrete rep → abstract value



The abstraction function is a function

Why do we map concrete to abstract and not vice versa?

- It's not a function in the other direction
  - Example: lists **`[a,b]`** and **`[b,a]`** might each represent the set **`{a, b}`**
- It's not as useful in the other direction
  - Purpose is to reason about whether our methods are manipulating concrete representations correctly in terms of the abstract specifications

Writing an abstraction function

Domain: all representations that satisfy the rep invariant

Range: concretely representable abstract values

Overview section of the specification should provide a notation of writing abstract values

- Could implement a method for printing in this notation
  - Useful for debugging
  - Often a good choice for **`toString`**

Abstraction Function and Stack

```
/** A last-in, first-out stack. A typical stack is
              e0, e1, ... en
where en is the top element of the stack and is most
recently pushed and first available to be popped. */
public class Stack {
  // Rep invariant:
  //   0 <= this.top <= this.a.length
  //   this.a != null
  // Abstraction Function:
  // AF(this) = A last-in, first-out stack
  //   defined by an ordered sequence of integers
  //            this.a[0] ... this.a[this.top-1]
  //   where the rightmost integer in the
  //   sequence is at the top of the stack
  private int[] a;
  private int top;
  ...
}
```

implicit: the number of elements in the stack is **`top`**

implicit: **`top`** points to the array element just “after” the top of the stack

Stack AF example

recall: **`top`** points to the array element just after the top of the stack

new()	0	0	0

top=0

empty stack

push(17)	17	0	0

top=1

17

push(-9)	17	-9	0

top=2

17, -9

pop()	17	-9	0

top=1

17

Abstract states are the same

17 = 17

Concrete states are different

`<[17,0,0], top=1>`

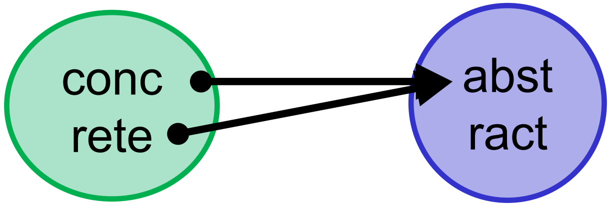
≠

`<[17,-9,0], top=1>`

AF is a function

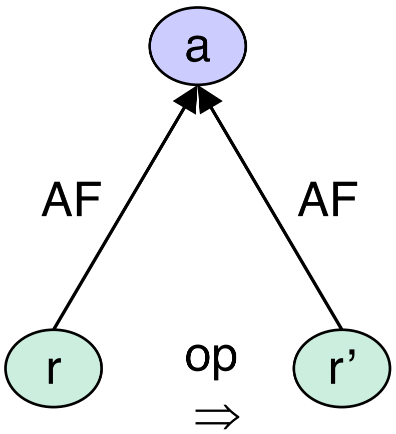
Inverse of AF is not a function

Benevolent side effects



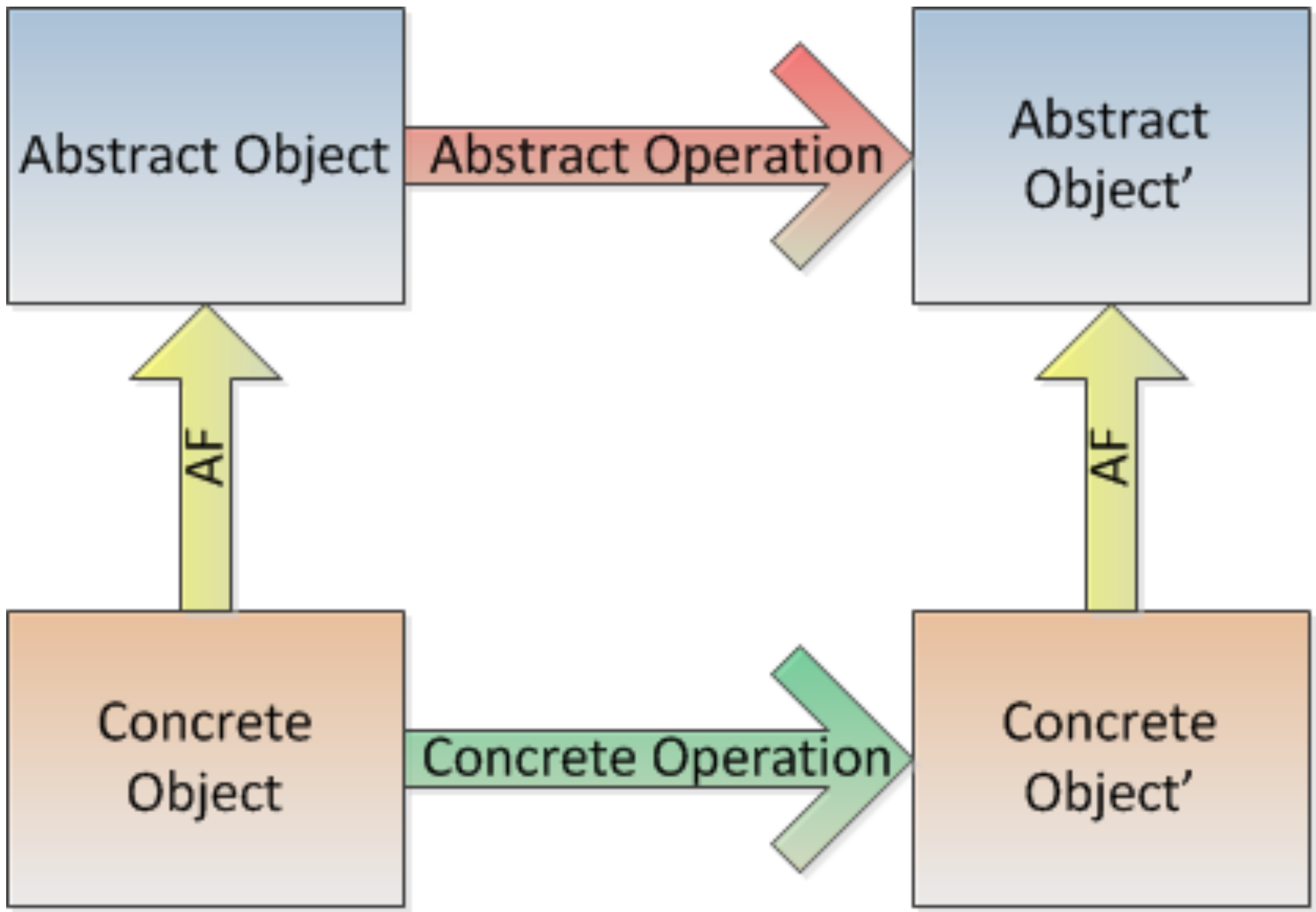
Different implementation of **`member`**:

```
boolean member(Character c1) {
   int i = elts.indexOf(c1);
   if (i == -1)
     return false;
   // move-to-front optimization
   Character c2 = elts.elementAt(0);
   elts.set(0, c1);
   elts.set(i, c2);
   return true;
}
```



- Move-to-front speeds up repeated membership tests
- Mutates rep, but does not change *abstract* value
  - *AF maps both reps to the same abstract value*
    - Precise reasoning/explanation for "clients can't tell"

Abstract and Concrete operations



Abstraction Function and Charset

The AF tells us what the rep means...

```
public void insert(Character c) {
  Character cc = new Character(encrypt(c));
  if (!elts.contains(cc))
    elts.addElement(cc);
}
```

AF(this) = { c | encrypt(c) is contained in this.elts }

```
public boolean member(Character c) {
  return elts.contains(c);
}
```

AF(this) = { c | c is contained in this.elts }

The two methods assume different abstraction functions! BAD!!!

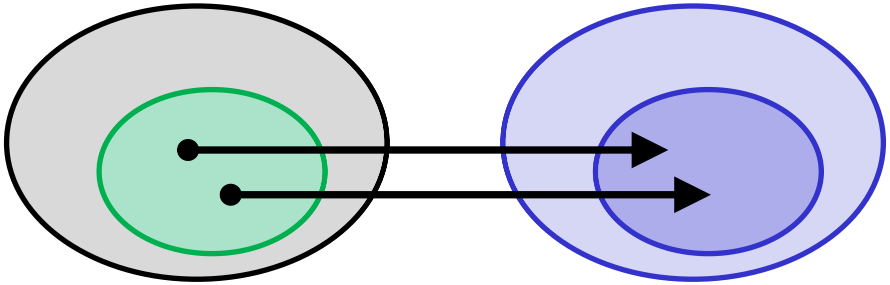
Charset Abstraction Function

```
class CharSet {
   // Rep invariant:
   //   this.elts has no nulls and no duplicates
   // Abstraction Function:
   // AF(this) = { c | c is contained in this.elts }
   private List<Character> elts = …
     …
 }
```

- Defined in terms of the representation `(this.elts)`
- Internal comment (not javadoc)
  - located just inside of the class definition at the very beginning

- Now we can re-implement `insert` to respect the AF

Data Abstraction: Summary



Representation Invariant describes what makes the concrete representation valid (green area)

Abstraction Function maps valid concrete values to abstract values

- Neither one is part of the ADT's specification
- Both are needed to reason an implementation satisfies the specification

Closing

Closing Announcements

- HW2 due tonight 10 pm
- HW3 due Thursday, July 5 at 10 pm
- Quiz 3 (coming soon!) due Thursday, July 5 at 10 pm

- Happy Independence Day!