Warmup

What do PCs and Air-Conditioning units have in common?

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What do PCs and Air-Conditioning units have in common?

They both stop working when you open Windows™

Section 3:
HW4, ADTs, and more

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Agenda

Announcements
- HW3: due Friday (tomorrow!) at 11pm
- Don’t forget to commit/push your changes

Polynomial arithmetic

Abstract data types (ADT)

Representation invariants (RI)

Abstraction Functions

HW4: Polynomial Graphing Calculator

Problem 0: Write pseudocode algorithms for polynomial operations

Problem 1: Answer questions about RatNum

Problem 2: Implement RatTerm

Problem 3: Implement RatPoly

Problem 4: Implement RatPolyStack

Problem 5: Try out the calculator

RatThings

RatNum
- ADT for a Rational Number
- Has NaN

RatTerm
- Single polynomial term
- Coefficient (RatNum) & degree

RatPoly
- Sum of RatTerms

RatPolyStack
- Ordered collection of RatPolys

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

$$\begin{array}{rrrrrr} & & 5x^4 & + 4x^3 & - x^2 & & + 5 \\ + & 3x^5 & & - 2x^3 & & + x & - 5 \end{array}$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr} & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\ + & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5 \end{array}$$

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

$$
\begin{array}{rcccccc}
 & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\
+ & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5 \\
\hline
 & 3x^5 & + 5x^4 & + 2x^3 & - x^2 & + x & + 0
\end{array}
$$

Polynomial Subtraction

$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$

$$\begin{array}{rrrrrr} & & 5x^4 + 4x^3 & - x^2 & & + 5 \\ - & 3x^5 & - 2x^3 & & + x & - 5 \end{array}$$

Polynomial Subtraction

$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$

$$\begin{array}{rrrrrrr} & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\ - & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5 \end{array}$$

Polynomial Subtraction

$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$

$$
\begin{array}{r}
5x^4 + 4x^3 - x^2 \quad 0x \quad + 5 \\
- \quad 3x^5 \quad 0x^4 \quad - 2x^3 \quad 0x^2 + \quad x \quad - 5 \\
\hline
-3x^5 + 5x^4 + 6x^3 - x^2 - \quad x \quad + 10
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{r}
4x^3 - x^2 + 5 \\
* \qquad x - 5 \\
\hline
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{r}
4x^3 - x^2 + 5 \\
* \qquad\qquad\qquad x - 5 \\
\hline
-20x^3 + 5x^2 \qquad - 25
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{r}
4x^3 - x^2 + 5 \\
* \qquad\qquad x - 5 \\
\hline
-20x^3 + 5x^2 \qquad - 25 \\
4x^4 \quad -x^3 \qquad + 5x \qquad
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{lrrrrr}
 & & 4x^3 & - x^2 & & + 5 \\
* & & & & x & - 5 \\
\hline
 & & -20x^3 & + 5x^2 & & - 25 \\
+ & 4x^4 & -x^3 & & + 5x & \\
\hline
 & \mathbf{4x^4} & \mathbf{-21x^3} & \mathbf{+ 5x^2} & \mathbf{+ 5x} & \mathbf{- 25}
\end{array}
$$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5) \;/\; (x^3 - 2x - 5)$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5)$ / $(x^3 - 2x - 5)$

$$x^3 - 2x - 5 \;\overline{\big)\; 5x^6 + 4x^4 - x^3 + 5}$$

Poly Division

```
1  0 -2 -5 | 5  0  4  -1  0  0  5
```

Poly Division

5

1 0 -2 -5 | 5 0 4 -1 0 0 5

Poly Division

```
                                   5
1  0 -2 -5  |  5  0  4  -1  0  0  5
               5  0-10 -25
```

Poly Division

```
                      5
             ___________________________
1  0 -2 -5  |  5  0  4  -1  0  0  5
               5  0-10 -25
               ____________
               0  0 14  24
```

Poly Division

```
                    5
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              0  0 14  24
                   14  24 0
```

Poly Division

```
                          5   0
1  0 -2 -5 |  5   0   4  -1   0   0   5
              5   0-10 -25
              ------------
              0   0 14   24
                    14   24 0
```

Poly Division

```
                        5   0
1   0 -2 -5  |  5   0   4  -1   0   0   5
                5   0-10 -25
                0   0 14  24
                      14  24 0
                      14  24 0   0
```

Poly Division

```
                        5   0 14
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10 -25
              0   0 14   24
                    14   24 0
                    14   24 0   0
```

Poly Division

```
                        5   0 14
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10  -25
              0   0 14   24
                    14   24 0
                    14   24 0   0
                    14   0 -28 -70
```

Poly Division

```
                             5   0  14
1   0  -2  -5  |  5   0   4   -1   0   0   5
                  5   0-10  -25
                  0   0  14   24
                         14   24  0
                         14   24  0   0
                         14   0  -28  -70
                          0   24   28   70
```

Poly Division

```
                     5   0  14
          ___________________________
1  0 -2 -5 |  5   0   4  -1   0   0   5
              5   0-10 -25
              ____________
              0   0  14   24
                     14   24  0
                     14   24  0   0
                     14   0 -28 -70
                     _____________________
                     0   24   28   70
                         24   28   70  5
```

Poly Division

5 0 14 24

1 0 -2 -5 | 5 0 4 -1 0 0 5

5 0-10 -25

0 0 14 24

14 24 0

14 24 0 0

14 0 -28 -70

0 24 28 70

24 28 70 5

24 0 -48 -120

Poly Division

```
                      5   0  14  24
1  0 -2 -5  |  5   0   4  -1   0   0   5
               5   0-10 -25
               0   0  14   24
                      14   24  0
                      14   24  0   0
                      14   0 -28  -70
                      0   24   28   70
                          24   28   70  5
                            24   0   -48  -120
                            0    28  118  125
```

Poly Division

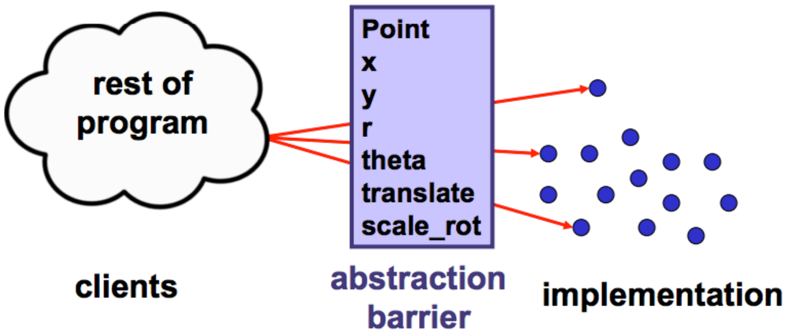
$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

$5x^3 + 14x + 24$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5) \,/\, (x^3 - 2x - 5)$

$$5x^3 + 14x + 24 + \frac{28x^2 + 118x + 125}{x^3 - 2x - 5}$$




```
class Point {
  public float x;
  public float y;
}
```

```
class Point {
  public float r;
  public float theta;
}
```

Abstract Data Types

A set of operations:

- Abstracts from the organization of data to the meaning of that data
- Abstracts from structure to use

Abstraction Barrier

- Representation/Implementation doesn’t matter to clients
- Hiding the details allows us to change them

The “concept” of 2-D point is the same for either implementation!

Abstract vs. Concrete

Abstract Representation: ADTs

1. **Abstract State:** What does the state of the data *represent*?
   What do the **fields** represent?

2. **Abstract Operations:** *What* operations can you do with the data?
   What **methods** are present, and what do they do?

- How the **client** views the data:
  - Independent of underlying code

Concrete Representation: Data Structures

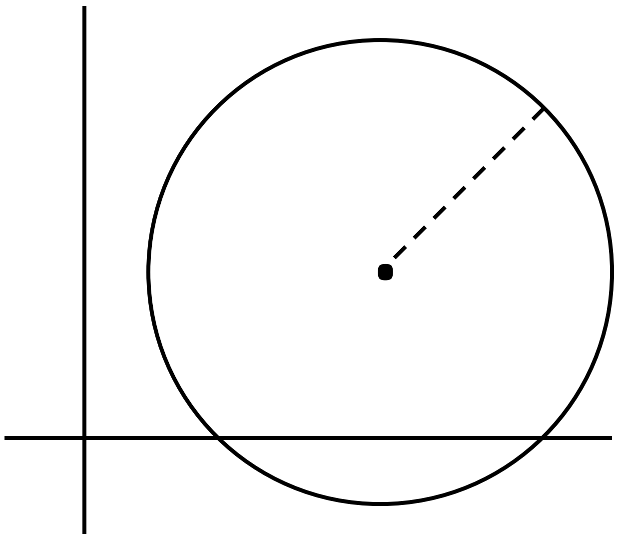
1. **Concrete State:** What *is* the state of the data?
   What are the **fields**?

2. **Concrete Operations:** *How* do you implement those operations to do that?
   How do you implement those **methods**?

- How the **implementer** views the data:
  - The actual underlying code

ADT Example: Circle

Circle on the Cartesian coordinate plane



Circle: Class Specification

What represents the abstract state of a Circle?

How can we describe a circle? What are some properties of a circle we can determine?

How can we implement this?

What are some ways to “break” a circle?

Representation Invariants

Indicates if an instance is *well-formed* or *valid*

Defines the set of valid concrete values

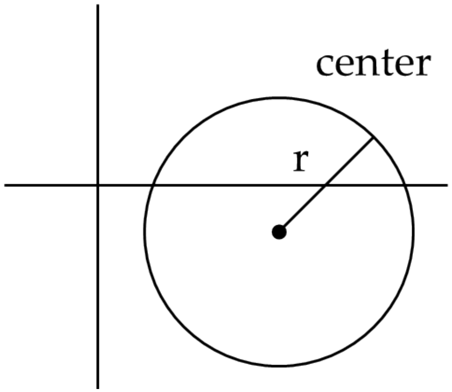
Maps **concrete representation** of object ➔ **boolean B**

If representation invariant is false/violated, the object is “broken” – doesn’t map to any abstract value

For implementors/debuggers/maintainers of the abstraction: No object should *<u>ever</u>* violate the rep invariant

Circle Implementation 1

```
public class Circle1 {
      private Point center;
      private double rad;

      // Rep invariant:
      //

      //  ...
}
```

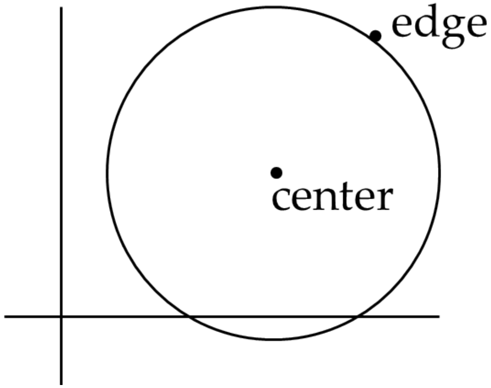


Circle Implementation 1

```
public class Circle1 {
      private Point center;
      private double rad;

      // Rep invariant:
      // center != null && rad > 0

      //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
      private Point center;
      private Point edge;

      // Rep invariant:
      //

      //    ...
}
```

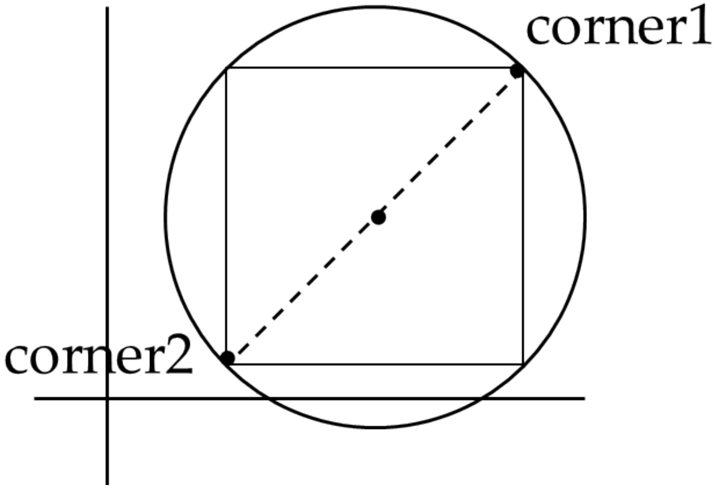


Circle Implementation 2

```
public class Circle2 {
      private Point center;
      private Point edge;

      // Rep invariant:
      // center != null &&
      // edge != null &&
      // !center.equals(edge)
      //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
      private Point corner1, corner2;

      // Rep invariant:
      //


      //     ...
}
```



Circle Implementation 3

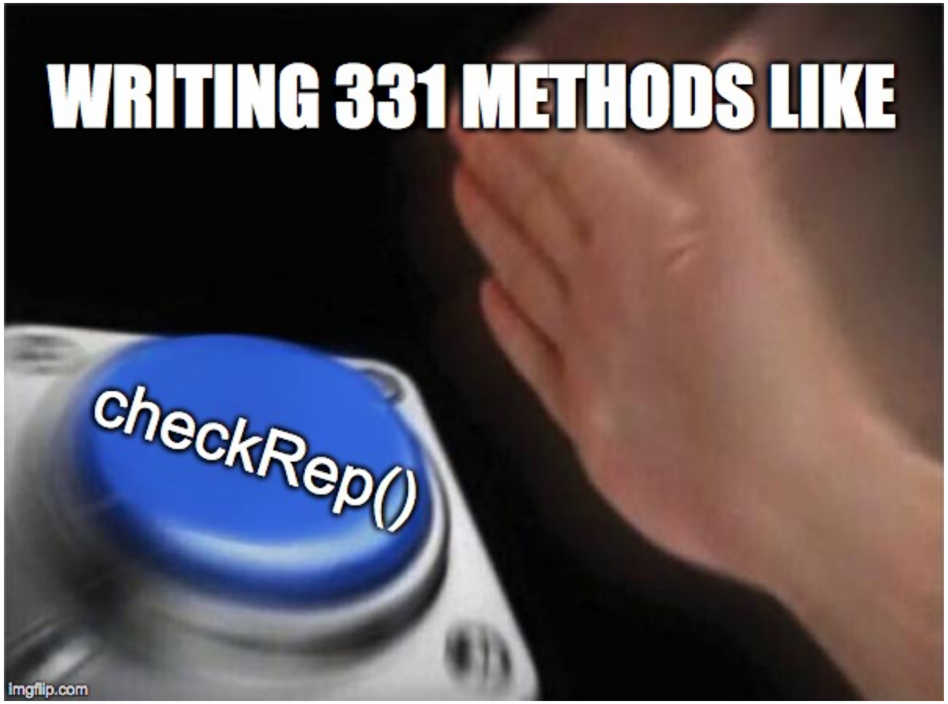
```
public class Circle3 {
      private Point corner1, corner2;

      // Rep invariant:
      // corner1 != null &&
      // corner2 != null &&
      // !corner1.equals(corner2)
      //    ...
}
```

Checking Rep Invariants

- Representation invariant should hold before and after every public method

Write and use `checkRep()`
- Call before and after public methods
- Make use of Java's assert syntax!
- OK that it adds extra code
  - Asserts won't be included on release builds
  - Important for finding bugs
- If some checks are expensive, you can use a global boolean variable to conditionally perform them

Takeaway for Rep Invariants



checkRep() Example with Asserts

```
public class Circle1 {
        private Point center;
        private double rad;

        private void checkRep() {
                assert center != null : “This does not have a
                                  center”;
                assert radius > 0 : “This circle has a negative
                             radius”;
        }
}
```

Using Asserts

To enable asserts: Go to Run->Run Configurations…->Arguments. Then put **–ea** in VM arguments section

- Do this for every main class

Abstraction Function

Abstraction function: a **mapping** from **internal state** to **abstract value**

Abstract fields may not map directly to representation fields

- Circle has **radius** but not necessarily

```
private int radius;
```

Internal representation can be anything as long as it somehow encodes the abstract value

Representation Invariant excludes values for which the abstraction function has no meaning

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center =
        //     c.radius =

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center = this.center
        //     c.radius = this.rad

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
       private Point center;
       private Point edge;

       // Abstraction function:
       // AF(this) = a circle c such that
       //     c.center =
       //     c.radius =

       // Rep invariant:
       // center != null && edge ! null &&
                                             !center.equals(edge)
      //    ...
}
```

Circle Implementation 2

```
public class Circle2 {
       private Point center;
       private Point edge;

       // Abstraction function:
       // AF(this) = a circle c such that
       //      c.center = this.center
       //      c.radius = sqrt((center.x-edge.x)^2 +
                                          (center.y-edge.y)^2)

       // Rep invariant:
       // center != null && edge ! null &&
                                           !center.equals(edge)
       //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
      private Point corner1, corner2;

      // Abstraction function:
      // AF(this) = a circle c such that
      //     c.center =
      //     c.radius =

      // Rep invariant:
      // corner1 != null && corner2 != null &&
                                   !corner1.equals(corner2)

      //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
       private Point corner1, corner2;

       // Abstraction function:
       // AF(this) = a circle c such that
       //     c.center = <(corner1.x + corner2.x) / 2,
                                  (corner.y + corner2.y) / 2>

       //     c.radius = (1/2)*sqrt((corner1.x-corner2.x)^2 +
                                    (corner1.y-corner2.y)^2)

       // Rep invariant:
       // corner1 != null && corner2 != null &&
                                   !corner1.equals(corner2)

       //    ...
}
```

checkRep() demo

Solutions to Worksheet:
NonNullStringList – Implementation 1

```
public class NonNullStringList {
// Abstraction function:
//      AF(this) = A list lst of strings with size s such that
//      lst.get(i) = this.arr[i] for all 0 < i < (s-1)
//      (Note you can use .get as it is part of the ADT for lst)
//      s = this.count

// Rep invariant:
//      arr[0,count-1] != null &&
//      count >=0 && arr != null

private String[] arr;
private int count;

public void add(String s) { ... }
public boolean remove(String s) { ... }
public String get(int i) { ... } }
```

Solutions to Worksheet: NonNullStringList – Implementation 2

```
public class NonNullStringList {
// Abstraction function:
// AF(this) = A list lst of strings with size s such that
//       lst.get(i) = this.head.(i times)next for all 0 < i < (s-1)
//       (Note you can use .get as it is part of the ADT for lst)

// Value in the nth node after head contains the
// nth item in the list

// Rep invariant:
// head.val != null, head.next.val != null, …
// No cycle in ListNodes


public ListNode head;
 public void add(String s) { ... }
 public boolean remove(String s) { ... }
 public String get(int i) { ... }
```