Lecture 7
Abstraction Functions
Connecting implementations to specs

**Representation Invariant**: maps Object $\rightarrow$ boolean
- Indicates if an instance is *well-formed*
- Defines the set of valid concrete values
- Only values in the valid set make sense as implementations of an abstract value
- **For implementors/debuggers/maintainers of the abstraction**: no object should ever violate the rep invariant
  - Such an object has no useful meaning

**Abstraction Function**: maps Object $\rightarrow$ abstract value
- What the data structure *means* as an abstract value
- How the data structure is to be interpreted
- Only defined on objects meeting the rep invariant
- **For implementors/debuggers/maintainers of the abstraction**: Each procedure should meet its spec (abstract values) by “doing the right thing” with the concrete representation
Rep inv. constrains structure, not meaning

An implementation of `insert` that preserves the rep invariant:

```java
public void insert(Character c) {
    Character cc = new Character(encrypt(c));
    if (!elts.contains(cc))
        elts.addElement(cc);
}
```

```
CharSet s = new CharSet();
s.insert('a');
if (s.member('a'))
    ...
```

Program is still wrong

- Clients observe incorrect behavior
- What client code exposes the error?
- Where is the error?
- We must consider the *meaning*
- The *abstraction function* helps us
Abstraction function: rep → abstract value

The abstraction function maps the concrete representation to the abstract value it represents.

AF: Object → abstract value

AF(CharSet this) = { c | c is contained in this.elts }

“set of Characters contained in this.elts”

Not executable because abstract values are “just” conceptual

The abstraction function lets us reason about what [concrete] methods do in terms of the clients’ [abstract] view.
Abstraction function and \texttt{insert}

Goal is to satisfy the specification of \texttt{insert}:

\begin{verbatim}
// modifies: this
// effects: this_{post} = this_{pre} U \{c\}
public void insert (Character c) {...}
\end{verbatim}

The AF tells us what the rep means, which lets us place the blame

\[
AF(\text{CharSet this}) = \{ c \mid c \text{ is contained in this.elts} \}
\]

Consider a call to \texttt{insert}:

On \textit{entry}, meaning is \(AF(\text{this}_{pre}) = \text{elts}_{pre}\)

On \textit{exit}, meaning is \(AF(\text{this}_{post}) = AF(\text{this}_{pre}) U \{\text{encrypt('a')}\}\)

What if we used this abstraction function instead?

\[
AF(\text{this}) = \{ c \mid \text{encrypt}(c) \text{ is contained in this.elts} \}
\]

\[
= \{ \text{decrypt}(c) \mid c \text{ is contained in this.elts} \}
\]
The abstraction function is a function

Why do we map concrete to abstract and not vice versa?

• It’s not a function in the other direction
  – Example: lists \([a, b]\) and \([b, a]\) might each represent the set \(\{a, b\}\)

• It’s not as useful in the other direction
  – Purpose is to reason about whether our methods are manipulating concrete representations correctly in terms of the abstract specifications
Stack AF example

Abstract stack with array and “top” index implementation

<table>
<thead>
<tr>
<th>new()</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>push(17)</td>
<td>17</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>pop()</td>
<td>17</td>
<td>-9</td>
<td>0</td>
</tr>
</tbody>
</table>

Abstract states are the same stack = <17> = <17>

Concrete states are different

<17,0,0>, top=1 ≠ <17,-9,0>, top=1

AF is a function
Inverse of AF is not a function
Benevolent side effects

Different implementation of `member`:

```java
boolean member(Character c1) {
    int i = elts.indexOf(c1);
    if (i == -1)
        return false;
    // move-to-front optimization
    Character c2 = elts.elementAt(0);
    elts.set(0, c1);
    elts.set(i, c2);
    return true;
}
```

- Move-to-front speeds up repeated membership tests
- Mutates rep, but does not change `abstract` value
  - `AF maps both reps to the same abstract value`
  - Precise reasoning/explanation for “clients can’t tell”
For any correct operation...
Writing an abstraction function

**Domain:** all representations that satisfy the rep invariant

**Range:** can be tricky to denote

- For mathematical entities like sets: easy
- For more complex abstractions: give names to specification
  - AF defines the value of each “specification field”

Overview section of the specification should provide a notation of writing abstract values

- Could implement a method for printing in this notation
  - Useful for debugging
  - Often a good choice for `toString`
Data Abstraction: Summary

Rep invariant
   – Which concrete values represent abstract values

Abstraction function
   – For each concrete value, which abstract value it represents

Together, they modularize the implementation
   – Neither one is part of the ADT’s specification
   – Both are needed to reason an implementation satisfies the specification

In practice, representation invariants are documented more often and more carefully than abstraction functions
   – A more widely understood and appreciated concept