There are 9 questions worth a total of 110 points. Please budget your time so you get to all of the questions. Keep your answers brief and to the point.

The exam is closed book, closed notes, closed electronics, closed mouth, open mind.

Many of the questions have short solutions, even if the question is somewhat long. Don't be alarmed.

For all of the questions involving proofs, assertions, invariants, and so forth, you should assume that all numeric quantities are unbounded integers (i.e., overflow can not happen) and that integer division is truncating division as in Java, i.e., 5/3 evaluates to 1.

If you don't remember the exact syntax of some command or the format of a command's output, make the best attempt you can. We will make allowances when grading.

Relax, you are here to learn.

Please wait to turn the page until everyone is told to begin.

Score _____ / 110

1	/ 10	6/ 15
2	/ 12	7/ 9
3	/ 17	8/ 15
4	/ 12	9/ 10
5.	/ 10	

Remember: For all of the questions involving proofs, assertions, invariants, and so forth, you should assume that all numeric quantities are unbounded integers (i.e., overflow can not happen) and that integer division is truncating division as in Java, i.e., 5/3 => 1.

Question 1. (10 points) (Forward reasoning) Using forward reasoning, write an assertion in each blank space indicating what is known about the program state at that point, given the precondition and the previously executed statements. Your final answers should be simplified. Be as specific as possible, but be sure to retain all relevant information.

(a)	$\{ x > 7 \}$			
	x = x + 4;			
	{	}		
	y = x - 2;			
	{	}		
	x = 2 * x;			
	{	}		
(b)				
(0)	{ X > 5 }			
	if $(x > 0)$			
	{		}	
	x = 3 - x;			
	{		}	
	else			
	{		}	
	x = x - 1;			
	{		}	
	{			}

Question 2. (12 points) (assertions) Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your final answers if possible.

(a) (5 points)



(b) (7 points)



Question 3. (17 points) Loops. The code on the next page is alleged to sort an array of integers using the algorithm "pancake sort". The only operation that modifies the array is flip (a, k), which reverses the elements in the array section a[k..a.length-1]. For example, if a = [5, 2, 6, 3, 2, 4] then flip (a, 2) would reverse the section starting at a[2] and change a to [5, 2, 4, 2, 3, 6] (the reversed elements are shown in *italics*). You do not need to implement this operation or prove that it works – assume that it is implemented correctly elsewhere.

Prove that pancakeSort successfully sorts its array argument by adding appropriate assertions and invariants to the code below. You may assume that the array parameter a is not null and has at least one element.

To save writing, you may write "a[i..j] is sorted" to mean $a[i] \le a[i+1] \le ... \le a[j]$. You may also use notations like "x = max(a[i..j])" to mean "x is the largest value in a[i..j]" and use similar notations to describe minimum values in sections of the array.

Use the space below for scratch work, then provide your proof on the next page, which should have enough room for your work.

You may remove this page from the exam if you wish.

(We announced during the exam that to save time it was not necessary to provide a complete correctness proof of the inner loop as long as an appropriate postcondition for that loop was given.)

Question 3. (cont.) Write your proof that the pancake sort method is correct by adding assertions and invariants below. You might not need to put an assertion between every two lines of code, but do not omit any essential details or steps.

```
\{ pre: a != null \&\& a.length >= 1 \}
public static void pancakeSort(int[] a) {
  int i = 0;
  while (i != a.length) {
    int minloc = i;
    int j = i + 1;
    while (j != a.length) {
      if (a[j] < a[minLoc]) {
        minLoc = j;
      }
      j = j + 1;
    }
    flip(a, minLoc);
    flip(a, i);
    i = i + 1;
  } // end of while loop
  { post: a[0..a.length-1] is sorted }
}
```

A *bag* or *multiset* is a set that allows duplicate elements. The next several questions concern a class called IntBag that implements a bag of integer values. The representation of data in this bag is an unordered linked list, where each node in the list contains an integer value and the number of times (> 0) that the integer value occurs in the bag. For instance, one possible representation of the multiset { 2, 3, 3, 2, 2, 1, 2 } is the following linked list:



Here is the beginning of the code for this class, showing the instance variable declarations (the rep) and a zero-argument constructor that initializes an empty IntBag. Also included are a pair of observer functions to return information about the contents of an IntBag.

```
public class IntBag {
  // Rep: an IntBag is stored as an unordered linked list of
  // nodes, each with a unique value and a positive count of
  // how many times that value occurs in the IntBag.
 private static class Link {
    int val; // data in this node
   int ncopies; // number of copies > 0 of val in this IntBag
   Link next; // next node in the list or null if none
  }
  // List of values stored in this IntBag
 private Link elts;
  /** Construct a new empty IntBag. */
 public IntBag() {
   elts = null;
  }
  /** Return the size of this IntBag.
     (Example: for { 1 2 2 2 3 }, the value returned is 5.) */
 public int size() {
    // implementation omitted
  }
  /** Return number of copies of n in this IntBag or 0 if n
   * is not contained in this IntBag. */
 public int contains(int n) {
     // implementation omitted
  }
```

Answer questions about this ADT on the next few pages. You may remove this page from the test for convenience if you wish.

Question 4. (12 points) (a) (3 points) Give a suitable abstract description of the class as would be written in the JavaDoc comment above the IntBag class heading.

(b) (5 points) Give a suitable Representation Invariant (RI) for this class.

(c) (4 points) Give a suitable Abstraction Function (AF) for this class relating the RI to the abstract value of a IntBag.

Question 5. (10 points) Specification. One function we need in our IntBag ADT is a method to add an integer value to an IntBag, which might, of course, simply increase the number of copies of that value in the IntBag. For consistency with other Java collection add methods, this method should return true if it alters the collection, which will always happen for a multiset like our IntBag. We'll get to the implementation on the next page, but we first need to properly specify this method below.

Complete the JavaDoc comments for the IntBag add method to provide the most suitable specification. Leave any unneeded parts blank. There is space at the beginning (before @param) where you should write a summary description of add as is done at the beginning of every JavaDoc method specification. Hint: the answer probably won't need nearly this much space.

```
/**
 *
 *
 *
 *
 *
 *
   @param n
 *
 *
 *
   @requires
 *
 *
 *
 *
 *
   @modifies
 *
 *
 *
 *
   @effects
 *
 *
 *
 *
   @throws
 *
 *
 *
 *
   @returns
 *
 *
 */
public boolean add(int n) {
   // Implementation omitted
}
```

Question 6. (15 points) Implementation. Give an implementation of method add for IntBag below using the linked-list representation described on previous pages. Your implementation should satisfy the specification of the method given in your answer to the previous question, and should be consistent with the rep invariant and abstraction functions you gave earlier.

public boolean add(int n) {

Question 7. (9 points) Describe three separate, distinct "black box" tests for your IntBag add method. For each test give the input values and expected result(s). You do not need to write JUnit tests or other Java code. Reminder: there are size() and contains() observer methods defined for this class that might be useful.

(a)

(b)

(c)

Question 8. (15 points) Equality. Give an implementation of a proper equals method for class IntBag. You need to include the method heading, but do not need to write Javadoc comments. Two IntBags a and b are equal if they contain exactly the same elements – i.e., every integer that is found in one IntBag occurs exactly the same number of times in the other IntBag. Note: you are only being asked to implement equals. You are not being asked to provide a hashCode function.

Hint: the solution does not need to be all that long, particularly if you take advantage of some of the other methods in the class – in particular, the sample solution didn't require a nested loop. However, you may not assume that there are additional methods available in IntBag besides the ones already given previously or implemented in earlier problems, or that would be inherited by IntBag from class Object.

Question 9. (10 points) Comparing specifications. This (final) question does not concern the IntBag ADT from the previous questions.

Here are parts of four possible specifications for a method that returns a prime number. (Recall that prime numbers have no integer divisors other than themselves and 1. The first few primes are 2, 3, 5, 7, 11, 13, \dots .)

- A. @param x
 @requires x is prime
 @return the next prime number greater than x
- B. @param x
 @requires x > 0
 @return the next prime number greater than x
- C. @param x
 @throws IllegalArgumentException if x < 3 or x is not prime
 @return the next prime number greater than x
- D. @param x
 @requires x is prime
 @return the next integer greater than x (i.e., x+1)

(a) List all of the specification that are stronger than A.
(b) List all of the specification that are stronger than B.
(c) List all of the specification that are stronger than C.
(d) List all of the specification that are stronger than D.
(e) Is it possible for a single method to satisfy A and B? (yes or no)
(f) Is it possible for a single method to satisfy A and C? (yes or no)